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**SPAKE2+, an Augmented PAKE**  
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Abstract

This document describes SPAKE2+, a Password Authenticated Key Exchange (PAKE) protocol run between two parties for deriving a strong shared key with no risk of disclosing the password. SPAKE2+ is an augmented PAKE protocol, as only one party has knowledge of the password. This method is simple to implement, compatible with any prime order group and is computationally efficient.

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## [1.](#) Introduction

This document describes SPAKE2+, a Password Authenticated Key Exchange (PAKE) protocol run between two parties for deriving a strong shared key with no risk of disclosing the password. SPAKE2+ is an augmented PAKE protocol, as only one party makes direct use of the password during the execution of the protocol. The other party only needs a verification value at the time of the protocol execution instead of the password. The verification value can be computed once, during an offline initialization phase. The party using the password directly would typically be a client, and acts as a prover, while the other party would be a server, and acts as verifier.

The protocol is augmented in the sense that it provides some resilience to the compromise or extraction of the verification value. The design of the protocol forces the adversary to recover the password from the verification value to successfully execute the protocol. Hence this protocol can be advantageously combined with a salted Password Hashing Function to increase the cost of the recovery and slow down attacks. The verification value cannot be used directly to successfully run the protocol as a prover, making this protocol more robust than balanced PAKEs which don't benefit from Password Hashing Functions to the same extent.

This augmented property is especially valuable in scenarios where the execution of the protocol is constrained and the adversary can not query the salt of the password hash function ahead of the attack. Constraints may consist in being in physical proximity through a local network or when initiation of the protocol requires a first authentication factor.



This password-based key exchange protocol is compatible with any group. It only relies on group operations making it simple and computationally efficient. It also has a security proof. Predetermined parameters for a selection of commonly used groups are also provided.

This document has content split out from a related document specifying SPAKE2 [[SPAKE2](#)].

## **2. Requirements Notation**

The key words "MUST", "MUST NOT", "REQUIRED", "SHALL", "SHALL NOT", "SHOULD", "SHOULD NOT", "RECOMMENDED", "NOT RECOMMENDED", "MAY", and "OPTIONAL" in this document are to be interpreted as described in [BCP 14](#) [[RFC2119](#)] [[RFC8174](#)] when, and only when, they appear in all capitals, as shown here.

## **3. Definition of SPAKE2+**

### **3.1. Offline Initialization**

Let  $G$  be a group in which the computational Diffie-Hellman (CDH) problem is hard. Suppose  $G$  has order  $p \cdot h$  where  $p$  is a large prime;  $h$  will be called the cofactor. Let  $I$  be the unit element in  $G$ , e.g., the point at infinity if  $G$  is an elliptic curve group. We denote the operations in the group additively. We assume there is a representation of elements of  $G$  as byte strings: common choices would be SEC1 uncompressed or compressed [[SEC1](#)] for elliptic curve groups or big endian integers of a fixed (per-group) length for prime field DH. We fix two elements  $M$  and  $N$  in the prime-order subgroup of  $G$  as defined in the table in this document for common groups, as well as a generator  $P$  of the (large) prime-order subgroup of  $G$ .  $P$  is specified in the document defining the group, and so we do not repeat it here.

$||$  denotes concatenation of strings. We also let  $\text{len}(S)$  denote the length of a string in bytes, represented as an eight-byte little-endian number. Finally, let  $\text{nil}$  represent an empty string, i.e.,  $\text{len}(\text{nil}) = 0$ .

KDF is a key-derivation function that takes as input a salt, intermediate keying material (IKM), info string, and derived key length  $L$  to derive a cryptographic key of length  $L$ . MAC is a Message Authentication Code algorithm that takes a secret key and message as input to produce an output. Let Hash be a hash function from arbitrary strings to bit strings of a fixed length. Common choices for Hash are SHA256 or SHA512 [[RFC6234](#)]. Let PBKDF be a Password-Based Key Derivation Function designed to slow down brute-force attackers. Brute-force resistance may be obtained through various



computation hardness parameters such as memory or CPU cycles, and are typically configurable. Scrypt [[RFC7914](#)] and Argon2 are common examples of PBKDF functions. PBKDF and hardness parameters selection for the PBKDF are out of scope of this document. [Section 5](#) specifies variants of KDF, MAC, and Hash suitable for use with the protocols contained herein.

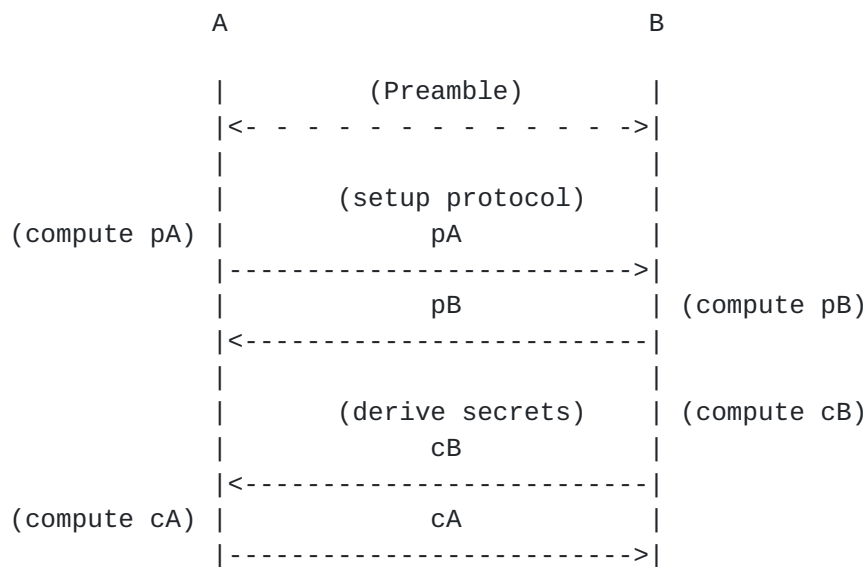
Let A and B be two parties. A and B may also have digital representations of the parties' identities such as Media Access Control addresses or other names (hostnames, usernames, etc). A and B may share Additional Authenticated Data (AAD) of length at most  $2^{16} - 1$  bits that is separate from their identities which they may want to include in the protocol execution. One example of AAD is a list of supported protocol versions if SPAKE2+ were used in a higher-level protocol which negotiates the use of a particular PAKE. Including this list would ensure that both parties agree upon the same set of supported protocols and therefore prevent downgrade attacks.

### [3.2.](#) Protocol Flow

SPAKE2+ is a two round protocol that establishes a shared secret with an additional round for key confirmation. Prior to invocation, A and B are provisioned with information such as the input password needed to run the protocol. A preamble exchange may occur in order to communicate identities, protocol version and PBKDF parameters related to the verification value. Details of the preamble phase is out of scope of this document. During the first round, A, the prover, sends a public share  $p_A$  to B, the verifier, and B responds with its own public share  $p_B$ . Both A and B then derive a shared secret used to produce encryption and authentication keys. The latter are used during the second round for key confirmation. ([Section 4](#) details the key derivation and confirmation steps.) In particular, B sends a key confirmation message  $c_B$  to A, and A responds with its own key confirmation message  $c_A$ . (Note that  $p_B$  and  $c_B$  MAY be sent in the same message.) Both parties MUST NOT consider the protocol complete prior to receipt and validation of these key confirmation messages.

This sample trace is shown below.





### 3.3. SPAKE2+

This protocol appears in [TDH]. Let  $w_0$  and  $w_1$  be two integers derived by hashing the password  $pw$  with the identities of the two participants, A and B. Specifically,  $w_0s \parallel w_1s = \text{PBKDF}(\text{len}(pw) \parallel pw \parallel \text{len}(A) \parallel A \parallel \text{len}(B) \parallel B)$ , and then computing  $w_0 = w_0s \bmod p$  and  $w_1 = w_1s \bmod p$ . If both identities A and B are absent, then  $w_0s \parallel w_1s = \text{PBKDF}(pw)$ , i.e., the length prefix is omitted as in [Section 3.1](#). The party B stores the verification value pair  $L=w_1*P$  and  $w_0$ .

Note that standards such as NIST.SP.800-56Ar3 suggest taking mod  $p$  of a hash value that is 64 bits longer than that needed to represent  $p$  to remove statistical bias introduced by the modulation. Protocols using this specification must define the method used to compute  $w_0$  and  $w_1$ : it may be necessary to carry out various forms of normalization of the password before hashing [RFC8265]. The hashing algorithm SHOULD be a PBKDF so as to slow down brute-force attackers.

When executing SPAKE2+, A selects  $x$  uniformly at random from the numbers in the range  $[0, p)$ , and lets  $X=x*P+w_0*M$ , then transmits  $pA=X$  to B. Upon receipt of  $X$ , A computes  $h*X$  and aborts if the result is equal to  $I$ . B then selects  $y$  uniformly at random from the numbers in  $[0, p)$ , then computes  $Y=y*P+w_0*N$ , and transmits  $pB=Y$  to A.

A computes  $Z$  as  $h*x*(Y-w_0*N)$ , and  $V$  as  $h*w_1*(Y-w_0*N)$ . B computes  $Z$  as  $h*y*(X-w_0*M)$  and  $V$  as  $h*y*L$ . Both share  $Z$  and  $V$  as common values. It is essential that both  $Z$  and  $V$  be used in combination with the transcript to derive the keying material. The protocol transcript encoding is shown below.





```

TT = len(A) || A || len(B) || B || len(X) || X
    || len(Y) || Y || len(Z) || Z || len(V) || V
    || len(w0) || w0

```

If an identity is absent, it is omitted from the transcript entirely. For example, if both A and B are absent, then  $TT = \text{len}(X) || X || \text{len}(Y) || Y || \text{len}(Z) || Z || \text{len}(w0) || w0$ . Likewise, if only A is absent,  $TT = \text{len}(B) || B || \text{len}(X) || X || \text{len}(Y) || Y || \text{len}(Z) || Z || \text{len}(w0) || w0$ . This must only be done for applications in which identities are implicit. Otherwise, the protocol risks Unknown Key Share attacks (discussion of Unknown Key Share attacks in a specific protocol is given in [\[I-D.ietf-mmusic-sdp-uks\]](#)).

Upon completion of this protocol, A and B compute shared secrets  $K_e$ ,  $K_{cA}$ , and  $K_{cB}$  as specified in [Section 4](#). B MUST send A a key confirmation message  $F_b$  so both parties agree upon these shared secrets. This confirmation message  $F_b$  is computed as a MAC over the received share ( $p_A$ ) using  $K_{cB}$ . Specifically, B computes  $F_b = \text{MAC}(K_{cB}, p_A)$ . After receipt and verification of B's confirmation message, A MUST send B a confirmation message using a MAC computed equivalently except with the use of  $p_B$  and  $K_{cA}$ . Key confirmation verification requires computing  $F$  and checking for equality against that which was received.

#### 4. Key Schedule and Key Confirmation

The protocol transcript  $TT$ , as defined in [Section 3.3](#), is unique and secret to A and B. Both parties use  $TT$  to derive shared symmetric secrets  $K_e$  and  $K_a$  as  $K_e || K_a = \text{Hash}(TT)$ . The length of each key is equal to half of the digest output, e.g.,  $|K_e| = |K_a| = 128$  bits for SHA-256.

Both endpoints use  $K_a$  to derive subsequent MAC keys for key confirmation messages. Specifically, let  $K_{cA}$  and  $K_{cB}$  be the MAC keys used by A and B, respectively. A and B compute them as  $K_{cA} || K_{cB} = \text{KDF}(\text{nil}, K_a, \text{"ConfirmationKeys"} || \text{AAD})$ , where AAD is the associated data each given to each endpoint, or nil (empty string) if none was provided. AAD may also include a string identifying the protocol, ciphersuite and all its parameters, including the definition of the group, and the element  $M$  and  $N$ . It may be omitted. The length of each of  $K_{cA}$  and  $K_{cB}$  is equal to half of the KDF output, e.g.,  $|K_{cA}| = |K_{cB}| = 128$  bits for HKDF with SHA256.

The resulting key schedule for this protocol, given transcript  $TT$  and additional associated data AAD, is as follows.

```

TT -> Hash(TT) = Ka || Ke
AAD -> KDF(nil, Ka, "ConfirmationKeys" || AAD) = KcA || KcB

```



A and B output  $K_e$  as the shared secret from the protocol.  $K_a$  and its derived keys ( $K_{cA}$  and  $K_{cB}$ ) are not used for anything except key confirmation.

## 5. Ciphersuites

This section documents SPAKE2+ ciphersuite configurations. A ciphersuite indicates a group, cryptographic hash algorithm, and pair of KDF and MAC functions, e.g., SPAKE2+-P256-SHA256-HKDF-HMAC. This ciphersuite indicates a SPAKE2+ protocol instance over P-256 that uses SHA256 along with HKDF [RFC5869] and HMAC [RFC2104] for G, Hash, KDF, and MAC functions, respectively.

G	Hash	KDF	MAC
P-256	SHA256 [RFC6234]	HKDF [RFC5869]	HMAC [RFC2104]
P-256	SHA512 [RFC6234]	HKDF [RFC5869]	HMAC [RFC2104]
P-384	SHA256 [RFC6234]	HKDF [RFC5869]	HMAC [RFC2104]
P-384	SHA512 [RFC6234]	HKDF [RFC5869]	HMAC [RFC2104]
P-512	SHA512 [RFC6234]	HKDF [RFC5869]	HMAC [RFC2104]
edwards25519 [RFC7748]	SHA256 [RFC6234]	HKDF [RFC5869]	HMAC [RFC2104]
edwards448 [RFC7748]	SHA512 [RFC6234]	HKDF [RFC5869]	HMAC [RFC2104]
P-256	SHA256 [RFC6234]	HKDF [RFC5869]	CMAC-AES-128 [RFC4493]
P-256	SHA512 [RFC6234]	HKDF [RFC5869]	CMAC-AES-128 [RFC4493]

Table 1: SPAKE2+ Ciphersuites

The following points represent permissible point generation seeds for the groups listed in the Table Table 1, using the algorithm presented



in [Appendix A](#). These bytestrings are compressed points as in [\[SEC1\]](#) for curves from [\[SEC1\]](#).

For P256:

M =  
02886e2f97ace46e55ba9dd7242579f2993b64e16ef3dcab95afd497333d8fa12f  
seed: 1.2.840.10045.3.1.7 point generation seed (M)

N =  
03d8bbd6c639c62937b04d997f38c3770719c629d7014d49a24b4f98baa1292b49  
seed: 1.2.840.10045.3.1.7 point generation seed (N)

For P384:

M =  
030ff0895ae5ebf6187080a82d82b42e2765e3b2f8749c7e05eba366434b363d3dc  
36f15314739074d2eb8613fceed2853  
seed: 1.3.132.0.34 point generation seed (M)

N =  
02c72cf2e390853a1c1c4ad816a62fd15824f56078918f43f922ca21518f9c543bb  
252c5490214cf9aa3f0baab4b665c10  
seed: 1.3.132.0.34 point generation seed (N)

For P521:

M =  
02003f06f38131b2ba2600791e82488e8d20ab889af753a41806c5db18d37d85608  
cfae06b82e4a72cd744c719193562a653ea1f119eef9356907edc9b56979962d7aa  
seed: 1.3.132.0.35 point generation seed (M)

N =  
0200c7924b9ec017f3094562894336a53c50167ba8c5963876880542bc669e494b25  
32d76c5b53dfb349fdf69154b9e0048c58a42e8ed04cef052a3bc349d95575cd25  
seed: 1.3.132.0.35 point generation seed (N)

For edwards25519:

M =  
d048032c6ea0b6d697ddc2e86bda85a33adac920f1bf18e1b0c6d166a5cecdaf  
seed: edwards25519 point generation seed (M)

N =  
d3bfb518f44f3430f29d0c92af503865a1ed3281dc69b35dd868ba85f886c4ab  
seed: edwards25519 point generation seed (N)

For edwards448:



M =  
b6221038a775ecd007a4e4dde39fd76ae91d3cf0cc92be8f0c2fa6d6b66f9a12  
942f5a92646109152292464f3e63d354701c7848d9fc3b8880  
seed: edwards448 point generation seed (M)

N =  
6034c65b66e4cd7a49b0edec3e3c9ccc4588afd8cf324e29f0a84a072531c4db  
f97ff9af195ed714a689251f08f8e06e2d1f24a0ffc0146600  
seed: edwards448 point generation seed (N)

## 6. Security Considerations

SPAKE2+ appears in [TDH] along with a path to a proof that server compromise does not lead to password compromise under the DH assumption (though the corresponding model excludes pre-computation attacks).

Elements received from a peer MUST be checked for group membership: failure to properly validate group elements can lead to attacks. Beyond the cofactor multiplication checks to ensure that these elements are in the prime order subgroup of G, it is essential that endpoints verify received points are members of G.

The choices of random numbers MUST BE uniform. Randomly generated values (e.g., x and y) MUST NOT be reused; such reuse may permit dictionary attacks on the password.

## 7. IANA Considerations

No IANA action is required.

## 8. Acknowledgments

Thanks to Ben Kaduk and Watson Ladd, from which this specification originally emanates.

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## Appendix A. Algorithm used for Point Generation

This section describes the algorithm that was used to generate the points (M) and (N) in the table in [Section 5](#).

For each curve in the table below, we construct a string using the curve OID from [\[RFC5480\]](#) (as an ASCII string) or its name, combined with the needed constant, for instance "1.3.132.0.35 point generation seed (M)" for P-512. This string is turned into a series of blocks by hashing with SHA256, and hashing that output again to generate the next 32 bytes, and so on. This pattern is repeated for each group and value, with the string modified appropriately.

A byte string of length equal to that of an encoded group element is constructed by concatenating as many blocks as are required, starting from the first block, and truncating to the desired length. The byte



string is then formatted as required for the group. In the case of Weierstrass curves, we take the desired length as the length for representing a compressed point (section 2.3.4 of [\[SEC1\]](#)), and use the low-order bit of the first byte as the sign bit. In order to obtain the correct format, the value of the first byte is set to 0x02 or 0x03 (clearing the first six bits and setting the seventh bit), leaving the sign bit as it was in the byte string constructed by concatenating hash blocks. For the [\[RFC8032\]](#) curves a different procedure is used. For edwards448 the 57-byte input has the least-significant 7 bits of the last byte set to zero, and for edwards25519 the 32-byte input is not modified. For both the [\[RFC8032\]](#) curves the (modified) input is then interpreted as the representation of the group element. If this interpretation yields a valid group element with the correct order ( $p$ ), the (modified) byte string is the output. Otherwise, the initial hash block is discarded and a new byte string constructed from the remaining hash blocks. The procedure of constructing a byte string of the appropriate length, formatting it as required for the curve, and checking if it is a valid point of the correct order, is repeated until a valid element is found.

The following python snippet generates the above points, assuming an elliptic curve implementation following the interface of `Edwards25519Point.stdbase()` and `Edwards448Point.stdbase()` in [Appendix A of \[RFC8032\]](#):



```

def iterated_hash(seed, n):
    h = seed
    for i in range(n):
        h = hashlib.sha256(h).digest()
    return h

def bighash(seed, start, sz):
    n = -(-sz // 32)
    hashes = [iterated_hash(seed, i) for i in range(start, start + n)]
    return b''.join(hashes)[:sz]

def canon_pointstr(ecname, s):
    if ecname == 'edwards25519':
        return s
    elif ecname == 'edwards448':
        return s[:-1] + bytes([s[-1] & 0x80])
    else:
        return bytes([(s[0] & 1) | 2]) + s[1:]

def gen_point(seed, ecname, ec):
    for i in range(1, 1000):
        hval = bighash(seed, i, len(ec.encode()))
        pointstr = canon_pointstr(ecname, hval)
        try:
            p = ec.decode(pointstr)
            if p != ec.zero_elem() and p * p.l() == ec.zero_elem():
                return pointstr, i
        except Exception:
            pass

```

## Appendix B. Test Vectors

This section contains test vectors for SPAKE2+ using the P256-SHA256-HKDF-HMAC ciphersuite. (Choice of PBKDF is omitted and values for  $w$  and  $w_0, w_1$  are provided directly.) All points are encoded using the uncompressed format, i.e., with a  $0x04$  octet prefix, specified in [SEC1] A and B identity strings are provided in the protocol invocation.

### B.1. SPAKE2+ Test Vectors

```

SPAKE2+(A='client', B='server')
w0 = 0x4f9e28322a64f9dc7a01b282cc51e2abc4f9ed568805ca84f4ed3ef806516
cf8
w1 = 0x8d73e4ca273859c873d809431d15f30e2b722007964e32699160b54fda3ee
855
L = 0x0491bb1e6672e71ad80b17d13f7a72ca2fe7f882d4bd734e2d140f67ab49d2
c3e76dbcf706954bd9ada4e3a7fc50cf9294729f93b130ada3d3a4ae98cc7e7b6971

```





```
X = 0x04879567d09560c02be565429036ed1d2fc3ca53f2eb6fadda4dba09eff3a0
096f032f0e227207ebebe05e1e95de325dfffe579c8aae76054030e5435fd5298c75
Y = 0x04b595a25588a2fba757195a756d289c191240296699f61fee8f15a7a741a4
23d48bd44cf544b409bbe4262a8045051e734567548ba43b3117efd6fb03acf41aff
Z = 0x047bb4661db7085d019cffa8495aba73d22f87ab8ba22e789477ef933b916f
412863aeb2dbc8003e4f1c2193290338ea0c7d786d30ca47a48eea273375a0c72ca1
V = 0x0417658e1e9707a29d429a4733d3bee703574aec222e781a6e7e5f5e504908
11aabf28e112fee32a37c228df9b53e6220468a2f6f07427604d8917870ac965eec7
TT = 0x06000000000000000636c69656e74060000000000000073657276657241000
0000000000004879567d09560c02be565429036ed1d2fc3ca53f2eb6fadda4dba09e
ff3a0096f032f0e227207ebebe05e1e95de325dfffe579c8aae76054030e5435fd52
98c75410000000000000004b595a25588a2fba757195a756d289c191240296699f61
fee8f15a7a741a423d48bd44cf544b409bbe4262a8045051e734567548ba43b3117e
fd6fb03acf41aff4100000000000000047bb4661db7085d019cffa8495aba73d22f8
7ab8ba22e789477ef933b916f412863aeb2dbc8003e4f1c2193290338ea0c7d786d3
0ca47a48eea273375a0c72ca141000000000000000417658e1e9707a29d429a4733d
3bee703574aec222e781a6e7e5f5e50490811aabf28e112fee32a37c228df9b53e62
20468a2f6f07427604d8917870ac965eec720000000000000004f9e28322a64f9dc7
a01b282cc51e2abc4f9ed568805ca84f4ed3ef806516cf8
Ka = 0xbf800062847c5182bf5c549b05ea6cce
Ke = 0xce9acf88ff9440777bda3e34fa4993cd
KcA = 0x73c6a5597096e99b8025172bb45b4a2f
KcB = 0x96a801673bd07b51d61fbaea03ef17cf
MAC(A) = 0xcab37c89192f9ad90ca5e6b8eadb130d313b51d24b7889e2536f7c800
26e076a
MAC(B) = 0xf7076a78a3d16f0c62cb9e40bd1a91b68dee144b87016e2dae81c36e9
73f3b2e
```

SPAKE2+(A='client', B='')

```
w0 = 0x4f9e28322a64f9dc7a01b282cc51e2abc4f9ed568805ca84f4ed3ef806516
cf8
w1 = 0x8d73e4ca273859c873d809431d15f30e2b722007964e32699160b54fda3ee
855
L = 0x0491bb1e6672e71ad80b17d13f7a72ca2fe7f882d4bd734e2d140f67ab49d2
c3e76dbcf706954bd9ada4e3a7fc50cf9294729f93b130ada3d3a4ae98cc7e7b6971
X = 0x0426fbbedb3b9ccea93d609838dcc1d4baebdbb9c287763ed4cdb2d3cc76f78
8d3388db3da1f63e945f3f1ba17f7b986ab9ed3170359ee406cbb40f3e3719453b15
Y = 0x04d4960922990acb87809e734fed2c2ccb72fd26ed173e8207cdc6220073ac
5017660788e96db275f6edf2ba400d4e090273c24dc907d80ff9cad7f42fd9f79c3f
Z = 0x0421996ff4d9c05b2389ae05118c519679df5d6de258b31f2a17da7604c8e3
c17bb3c4aae2ae4217951aa82144cb8b677be8061f28893f70216c1e11ba2bacd50d
V = 0x04729f7c6c5bd68310345b1a10b84ea7db64c70441da2255992208b7a8e0b3
9d4f0e634acf7d440b4552a41df291ac6a409f8cf5a47cec9fed5f85fea1241379a4
TT = 0x06000000000000000636c69656e7441000000000000000426fbbedb3b9ccea9
3d609838dcc1d4baebdbb9c287763ed4cdb2d3cc76f788d3388db3da1f63e945f3f1
ba17f7b986ab9ed3170359ee406cbb40f3e3719453b15410000000000000004d4960
922990acb87809e734fed2c2ccb72fd26ed173e8207cdc6220073ac5017660788e96
db275f6edf2ba400d4e090273c24dc907d80ff9cad7f42fd9f79c3f4100000000000
```



```
0000421996ff4d9c05b2389ae05118c519679df5d6de258b31f2a17da7604c8e3c17
bb3c4aae2ae4217951aa82144cb8b677be8061f28893f70216c1e11ba2bacd50d410
0000000000000004729f7c6c5bd68310345b1a10b84ea7db64c70441da2255992208b
7a8e0b39d4f0e634acf7d440b4552a41df291ac6a409f8cf5a47cec9fed5f85fea12
41379a420000000000000004f9e28322a64f9dc7a01b282cc51e2abc4f9ed568805c
a84f4ed3ef806516cf8
```

```
Ka = 0xfd19104b836b0ba9dfaaeab88610be57
```

```
Ke = 0x90337374f974f673707de5ba1b98e5b8
```

```
KcA = 0x2e10249c566677c8826b48ad10b19bb5
```

```
KcB = 0x4fcdf8fd0bfcfaeeabb9d6f48e264e4a3
```

```
MAC(A) = 0xaaef200ea5f5c41e1fdb9b3455dde715cd8aa96f8afd3274f7159c3c5
4887f2c
```

```
MAC(B) = 0x926eadbf4b720b46ea622d7100e0013eb24d1591496846a604cf90c14
46fe0e4
```

```
SPAKE2+(A='', B='server')
```

```
w0 = 0x4f9e28322a64f9dc7a01b282cc51e2abc4f9ed568805ca84f4ed3ef806516
cf8
```

```
w1 = 0x8d73e4ca273859c873d809431d15f30e2b722007964e32699160b54fda3ee
855
```

```
L = 0x0491bb1e6672e71ad80b17d13f7a72ca2fe7f882d4bd734e2d140f67ab49d2
c3e76dbcf706954bd9ada4e3a7fc50cf9294729f93b130ada3d3a4ae98cc7e7b6971
```

```
X = 0x0463a7531acd204e7d83ac6562278d7ced01a715eff937a25520bd2220c626
33db0ea510591c5cd23159a7a97181ec24433aac6e628f16d42c455fcae668411e34
```

```
Y = 0x0433625217e2ccc0c545126f756d999c16df68b14b73b3fe473593c1d3a0d7
287b43b353177806c641588ec969852b56b17190d6ebe80313de74e5eee0c1403025
```

```
Z = 0x049ef5ea46e8ca42f3e822c598858ca347bf19cc74a8a1fbfd836ec4d77bee
7f0cd4d42f4f817caa3360c918d2538d7c96de5db47a72949ca2888d02c18ea6f92b
```

```
V = 0x0408a70fc9dca87b70a7d4a074bdcca0222806f0caa0542d8d62aecf535ea8
ffbc5e48419c5127a0f7f03685013c09d22f797523d26e7db159fecaccebc54ed2a7
```

```
TT = 0x0600000000000000073657276657241000000000000000463a7531acd204e7
d83ac6562278d7ced01a715eff937a25520bd2220c62633db0ea510591c5cd23159a
```

```
7a97181ec24433aac6e628f16d42c455fcae668411e344100000000000000433625
217e2ccc0c545126f756d999c16df68b14b73b3fe473593c1d3a0d7287b43b353177
```

```
806c641588ec969852b56b17190d6ebe80313de74e5eee0c14030254100000000000
000049ef5ea46e8ca42f3e822c598858ca347bf19cc74a8a1fbfd836ec4d77bee7f0
```

```
cd4d42f4f817caa3360c918d2538d7c96de5db47a72949ca2888d02c18ea6f92b410
000000000000000408a70fc9dca87b70a7d4a074bdcca0222806f0caa0542d8d62aec
```

```
f535ea8ffbc5e48419c5127a0f7f03685013c09d22f797523d26e7db159fecaccebc
54ed2a720000000000000004f9e28322a64f9dc7a01b282cc51e2abc4f9ed568805c
```

```
a84f4ed3ef806516cf8
```

```
Ka = 0x5c85900898b2079c9de09ebef63cebd1
```

```
Ke = 0x13c812476859e909682c3be7436bfef0
```

```
KcA = 0x77bd636ab9bf153339c5724ee04f87a7
```

```
KcB = 0x194325b27d7c291c94a689ddafeaaa3c
```

```
MAC(A) = 0x3bb61248a1fd2946743314848fc501eb3455eb113bd8966e200de14d5
e412688
```

```
MAC(B) = 0x3e7912bd2a85a1f56d36fbb16de29834b000d49e50d4c17f992942ee5
```



9255f1e

SPAKE2+(A='', B='')

w0 = 0x4f9e28322a64f9dc7a01b282cc51e2abc4f9ed568805ca84f4ed3ef806516cf8

w1 = 0x8d73e4ca273859c873d809431d15f30e2b722007964e32699160b54fda3ee855

L = 0x0491bb1e6672e71ad80b17d13f7a72ca2fe7f882d4bd734e2d140f67ab49d2c3e76dbcf706954bd9ada4e3a7fc50cf9294729f93b130ada3d3a4ae98cc7e7b6971

X = 0x04f60f506cfa07506d4bfd2b3f56038b1c001fe6826374122c30e914747eab647988702cc70210eb2aa625e603d56961af16ec543ee3d4d2cb90d6fe2f3c1d1180

Y = 0x046898fafef34fff9936217608151af08313305cf8e6f9add10d721c04a018607f5b5aca327e150cd5d588de83e46491ec766e2cf87da9fb07dc3745c0630b03bb

Z = 0x042adeeea1417cc6c592fef772da8ba0f3aea69a5fb15923d0e9ae7c3301c7ff87e9ff9fba292ad410e4af71770858e9a314f1deb75f77bde276d3cc8b45ffd70c

V = 0x04845c130c8c20865828e21ed3400abea726b07fdeb7533fa6017accc37e0be4922241dad44846112e42bee999501fdb4d09fc798e4677d403d10bfa862928584e

TT = 0x410000000000000004f60f506cfa07506d4bfd2b3f56038b1c001fe6826374122c30e914747eab647988702cc70210eb2aa625e603d56961af16ec543ee3d4d2c

b90d6fe2f3c1d1180410000000000000046898fafef34fff9936217608151af08313305cf8e6f9add10d721c04a018607f5b5aca327e150cd5d588de83e46491ec766e2

cf87da9fb07dc3745c0630b03bb4100000000000000042adeeea1417cc6c592fef772da8ba0f3aea69a5fb15923d0e9ae7c3301c7ff87e9ff9fba292ad410e4af7177085

8e9a314f1deb75f77bde276d3cc8b45ffd70c410000000000000004845c130c8c20865828e21ed3400abea726b07fdeb7533fa6017accc37e0be4922241dad44846112e4

2bee999501fdb4d09fc798e4677d403d10bfa862928584e20000000000000004f9e28322a64f9dc7a01b282cc51e2abc4f9ed568805ca84f4ed3ef806516cf8

Ka = 0x850a18a77b14ef5e71b4a239413630a8

Ke = 0x4454819282b3e886a7e65b7b0de7cc62

KcA = 0x05df6196c12d6203768c73d875e2bfc5

KcB = 0xb58e61c322f685add02c125767e4fbb7

MAC(A) = 0x33e50d29f8eacc67bfdab4a6c46c88d75ac3308416c64dfbb0d7fb1c0feda5b0

MAC(B) = 0x55434e5e501ad2d476aa1ae334ef27ba437a5dea87683defac575a63b548ca64

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