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                          **The BBS Signature Scheme**

## Abstract

BBS is a digital signature scheme categorized as a form of short group signature that supports several unique properties. Notably, the scheme supports signing multiple messages whilst producing a single output digital signature. Through this capability, the possessor of a signature is able to generate proofs that selectively disclose subsets of the originally signed set of messages, whilst preserving the verifiable authenticity and integrity of the messages. Furthermore, these proofs are said to be zero-knowledge in nature as they do not reveal the underlying signature; instead, what they reveal is a proof of knowledge of the undisclosed signature.

## Discussion Venues

This note is to be removed before publishing as an RFC.

Source for this draft and an issue tracker can be found at <https://github.com/decentralized-identity/bbs-signature>.

## Status of This Memo

This Internet-Draft is submitted in full conformance with the provisions of BCP 78 and BCP 79.

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## 1. Introduction

A digital signature scheme is a fundamental cryptographic primitive that is used to provide data integrity and verifiable authenticity in various protocols. The core premise of digital signature technology is built upon asymmetric cryptography where-by the possessor of a private key is able to sign a message, where anyone in possession of the corresponding public key matching that of the private key is able to verify the signature.

The name BBS is derived from the authors of the original academic work of Dan Boneh, Xavier Boyen, and Hovav Shacham, where the scheme was first described.

Beyond the core properties of a digital signature scheme, BBS signatures provide multiple additional unique properties, three key ones are:

**Selective Disclosure** - The scheme allows a signer to sign multiple messages and produce a single -constant size- output signature. A holder/prover then possessing the messages and the signature can generate a proof whereby they can choose which messages to disclose, while revealing no-information about the undisclosed messages. The proof itself guarantees the integrity and authenticity of the disclosed messages (e.g. that they were originally signed by the signer).

**Unlinkable Proofs** - The proofs generated by the scheme are known as zero-knowledge, proofs-of-knowledge of the signature, meaning a verifying party in receipt of a proof is unable to determine which signature was used to generate the proof, removing a common source of correlation. In general, each proof generated is indistinguishable from random even for two proofs generated from the same signature.

**Proof of Possession** - The proofs generated by the scheme prove to a verifier that the party who generated the proof (holder/prover) was in possession of a signature without revealing it. The scheme also supports binding a presentation header to the generated proof. The presentation header can include arbitrary information such as a cryptographic nonce, an audience/domain identifier and or time based validity information.

Refer to the [Appendix B](#) for an elaboration on situations where these properties are useful

Below is a basic diagram describing the main entities involved in the scheme

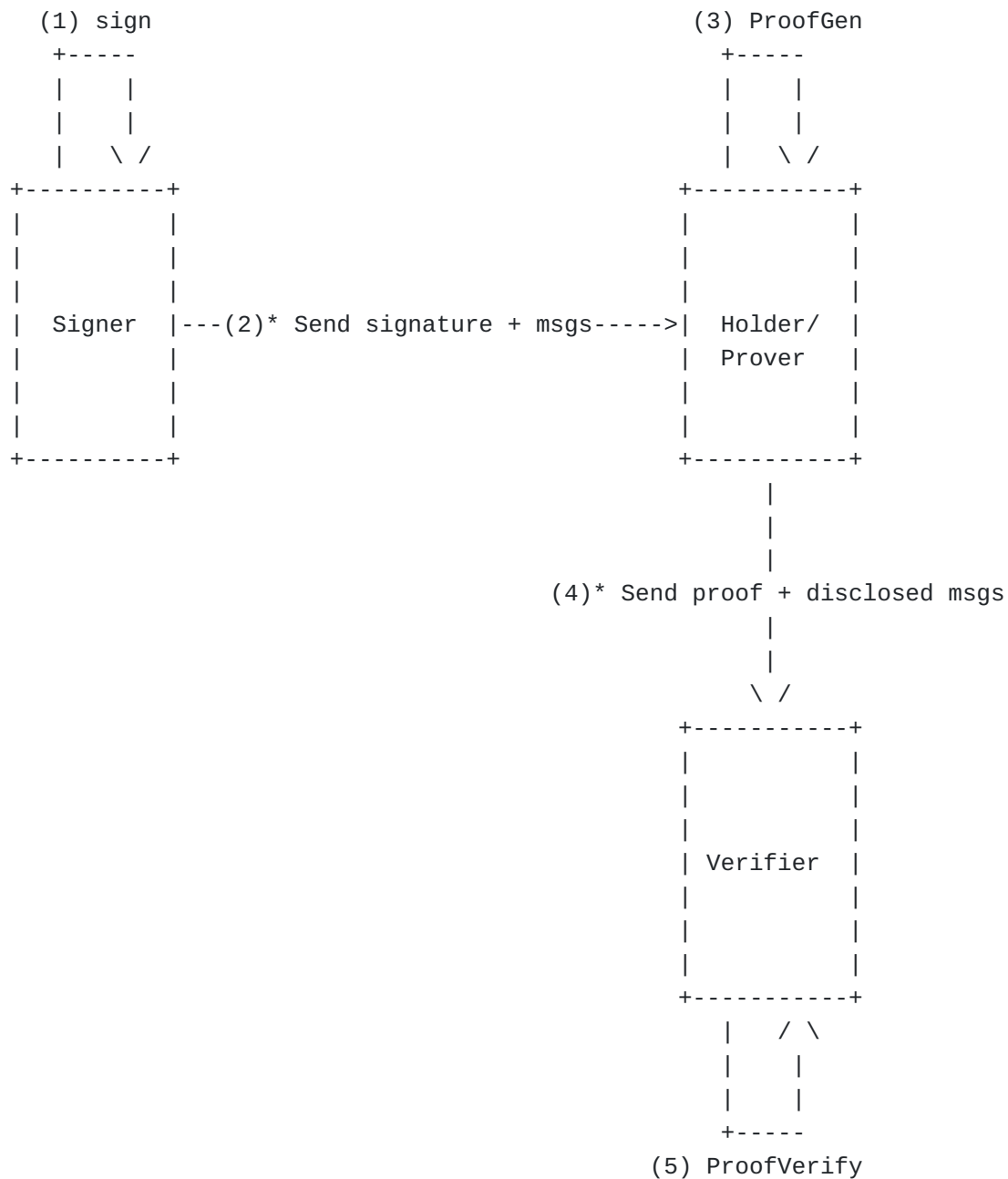


Figure 1: Basic diagram capturing the main entities involved in using the scheme

**Note** The protocols implied by the items annotated by an asterisk are out of scope for this specification

### 1.1. Terminology

The following terminology is used throughout this document:

**SK** The secret key for the signature scheme.

**PK**

The public key for the signature scheme.

**L** The total number of signed messages.

**R** The number of message indexes that are disclosed (revealed) in a proof-of-knowledge of a signature.

**U** The number of message indexes that are undisclosed in a proof-of-knowledge of a signature.

**msg** An input message to be signed by the signature scheme.

**generator** A valid point on the selected subgroup of the curve being used that is employed to commit a value.

**signature** The digital signature output.

**nonce** A cryptographic nonce

**presentation\_header (ph)** A payload generated and bound to the context of a specific spk.

**nizk** A non-interactive zero-knowledge proof from fiat-shamir heuristic.

**dst** The domain separation tag.

**I2OSP** As defined by Section 4 of [[RFC8017](#)]

**OS2IP** As defined by Section 4 of [[RFC8017](#)].

## 1.2. Notation

The following notation and primitives are used:

**a || b** Denotes the concatenation of octet strings a and b.

**I \ J** For sets I and J, denotes the difference of the two sets i.e., all the elements of I that do not appear in J, in the same order as they were in I.

**X[a..b]** Denotes a slice of the array X containing all elements from and including the value at index a until and including the value at index b. Note when this syntax is applied to an octet string, each element in the array X is assumed to be a single byte.

**range(a, b)** For integers a and b, with  $a \leq b$ , denotes the ascending ordered list of all integers between a and b inclusive (i.e., the integers "i" such that  $a \leq i \leq b$ ).

**utf8(ascii\_string)**

Encoding the inputted ASCII string to an octet string using UTF-8 character encoding.

**length(input)** Takes as input either an array or an octet string. If the input is an array, returns the number of elements of the array. If the input is an octet string, returns the number of bytes of the inputted octet string.

Terms specific to pairing-friendly elliptic curves that are relevant to this document are restated below, originally defined in [[I-D.irtf-cfrg-pairing-friendly-curves](#)]

**E1, E2** elliptic curve groups defined over finite fields. This document assumes that E1 has a more compact representation than E2, i.e., because E1 is defined over a smaller field than E2.

**G1, G2** subgroups of E1 and E2 (respectively) having prime order  $r$ .

**GT** a subgroup, of prime order  $r$ , of the multiplicative group of a field extension.

**e**  $G1 \times G2 \rightarrow GT$ : a non-degenerate bilinear map.

**r** The prime order of the G1 and G2 subgroups.

**P1, P2** points on G1 and G2 respectively. For a pairing-friendly curve, this document denotes operations in E1 and E2 in additive notation, i.e.,  $P + Q$  denotes point addition and  $x * P$  denotes scalar multiplication. Operations in GT are written in multiplicative notation, i.e.,  $a * b$  is field multiplication.

**Identity\_G1, Identity\_G2, Identity\_GT** The identity element for the G1, G2, and GT subgroups respectively.

**hash\_to\_curve\_g1(ostr, dst) -> P** A cryptographic hash function that takes an arbitrary octet string as input and returns a point in G1, using the hash\_to\_curve operation defined in [[I-D.irtf-cfrg-hash-to-curve](#)] and the inputted dst as the domain separation tag for that operation (more specifically, the inputted dst will

become the DST parameter for the hash\_to\_field operation, called by hash\_to\_curve).

**point\_to\_octets\_g1(P) -> ostr, point\_to\_octets\_g2(P) -> ostr**

returns the canonical representation of the point P for the respective subgroup as an octet string. This operation is also known as serialization.

**octets\_to\_point\_g1(ostr) -> P, octets\_to\_point\_g2(ostr) -> P**

returns the point P for the respective subgroup corresponding to the canonical representation ostr, or INVALID if ostr is not a valid output of the respective point\_to\_octets\_g\* function. This operation is also known as deserialization.

**subgroup\_check(P) -> VALID or INVALID** returns VALID when the point P is an element of the subgroup of order r, and INVALID otherwise. This function can always be implemented by checking that  $r * P$  is equal to the identity element. In some cases, faster checks may also exist, e.g., [Bowe19].

### 1.3. Organization of this document

This document is organized as follows:

- \*[Scheme Definition](#) defines the core operations and parameters for the BBS signature scheme.

- \*[Utility Operations](#) defines utilities used by the BBS signature scheme.

- \*[Security Considerations](#) describes a set of security considerations associated to the signature scheme.

- \*[Ciphersuites](#) defines the format of a ciphersuite, alongside a concrete ciphersuite based on the BLS12-381 curve.

## 2. Conventions

The keywords **MUST**, **MUST NOT**, **REQUIRED**, **SHALL**, **SHALL NOT**, **SHOULD**, **SHOULD NOT**, **RECOMMENDED**, **MAY**, and **OPTIONAL**, when they appear in this document, are to be interpreted as described in [RFC2119].

## 3. Scheme Definition

This section defines the BBS signature scheme, including the parameters required to define a concrete ciphersuite.



### 3.1. Parameters

The schemes operations defined in this section depend on the following parameters:

- \*A pairing-friendly elliptic curve, plus associated functionality given in [Section 1.2](#).

- \*A hash-to-curve suite as defined in [[I-D.irtf-cfrg-hash-to-curve](#)], using the aforementioned pairing-friendly curve. This defines the `hash_to_curve` and `expand_message` operations, used by this document.

- \*PRF(n): a pseudo-random function similar to [[RFC4868](#)]. Returns n pseudo randomly generated bytes.

### 3.2. Considerations

#### 3.2.1. Subgroup Selection

In definition of this signature scheme there are two possible variations based upon the sub-group selection, namely where public keys are defined in G2 and signatures in G1 OR the opposite where public keys are defined in G1 and signatures in G2. Some pairing cryptography based digital signature schemes such as [[I-D.irtf-cfrg-bls-signature](#)] elect to allow for both variations, because they optimize for different things. However, in the case of this scheme, due to the operations involved in both signature and proof generation being computational in-efficient when performed in G2 and in the pursuit of simplicity, the scheme is limited to a construction where public keys are in G2 and signatures in G1.

#### 3.2.2. Messages and generators

Throughout the operations of this signature scheme, each message that is signed is paired with a specific generator (point in G1). Specifically, if a generator `H_1` is multiplied with `msg_1` during signing, then `H_1` MUST be multiplied with `msg_1` in all other operations (signature verification, proof generation and proof verification).

Aside from the message generators, the scheme uses two additional generators: `Q_1` and `Q_2`. The first (`Q_1`), is used for the blinding value (s) of the signature. The second generator (`Q_2`), is used to sign the signature's domain, which binds both the signature and generated proofs to a specific context and cryptographically protects any potential application-specific information (for example, messages that must always be disclosed etc.).

### 3.3. Key Generation Operations

#### 3.3.1. KeyGen

This operation generates a secret key (SK) deterministically from a secret octet string (IKM).

KeyGen uses an HKDF [[RFC5869](#)] instantiated with the hash function hash.

For security, IKM MUST be infeasible to guess, e.g. generated by a trusted source of randomness.

IKM MUST be at least 32 bytes long, but it MAY be longer.

Because KeyGen is deterministic, implementations MAY choose either to store the resulting SK or to store IKM and call KeyGen to derive SK when necessary.

KeyGen takes an optional parameter, `key_info`. This parameter MAY be used to derive multiple independent keys from the same IKM. By default, `key_info` is the empty string.

SK = KeyGen(IKM, key\_info)

Inputs:

- IKM (REQUIRED), a secret octet string. See requirements above.
- key\_info (OPTIONAL), an octet string. if this is not supplied, it MUST default to an empty string.

Definitions:

- HKDF-Extract is as defined in [RFC5869], instantiated with hash func
- HKDF-Expand is as defined in [RFC5869], instantiated with hash funct
- I2OSP and OS2IP are as defined in [RFC8017], Section 4.
- L is the integer given by  $\text{ceil}((3 * \text{ceil}(\log_2(r))) / 16)$ .
- INITSALT is the ASCII string "BBS-SIG-KEYGEN-SALT-".

Outputs:

- SK, a uniformly random integer such that  $0 < SK < r$ .

Procedure:

1. salt = INITSALT
2. SK = 0
3. while SK == 0:
4.     salt = hash(salt)
5.     PRK = HKDF-Extract(salt, IKM || I2OSP(0, 1))
6.     OKM = HKDF-Expand(PRK, key\_info || I2OSP(L, 2), L)
7.     SK = OS2IP(OKM) mod r
8. return SK

**Note** This operation is the RECOMMENDED way of generating a secret key, but its use is not required for compatibility, and implementations MAY use a different KeyGen procedure. For security, such an alternative MUST output a secret key that is statistically close to uniformly random in the range  $0 < SK < r$ .

### 3.3.2. SkToPk

This operation takes a secret key (SK) and outputs a corresponding public key (PK).

$PK = SkToPk(SK)$

Inputs:

- SK (REQUIRED), a secret integer such that  $0 < SK < r$ .

Outputs:

- PK, a public key encoded as an octet string.

Procedure:

1.  $W = SK * P2$
2. return `point_to_octets_g2(W)`

### **3.4. Core Operations**

The operations in this section make use of a "Precomputations" set of steps. The "Precomputations" steps must be executed before the steps in the "Procedure" of each operation and include computations that can be cached and re-used multiple times (like creating the generators etc.) or procedural steps like de-structuring inputted arrays.

#### **3.4.1. Sign**

This operation computes a deterministic signature from a secret key (SK) and optionally over a header and or a vector of messages.

signature = Sign(SK, PK, header, messages)

Inputs:

- SK (REQUIRED), a non negative integer mod  $r$  outputted by the KeyGen operation.
- PK (REQUIRED), an octet string of the form outputted by the SkToPk operation provided the above SK as input.
- header (OPTIONAL), an octet string containing context and application specific information. If not supplied, it defaults to an empty string.
- messages (OPTIONAL), a vector of scalars. If not supplied, it defaults to the empty array `[]`.

Parameters:

- ciphersuite\_id, ASCII string. The unique ID of the ciphersuite.
- generator\_seed, ASCII string. The generators seed defined by the ciphersuite

Definitions:

- $L$ , is the non-negative integer representing the number of messages to be signed e.g `length(messages)`. If no messages are supplied as an input, the value of  $L$  MUST evaluate to zero (0).

Outputs:

- signature, a signature encoded as an octet string.

Precomputations:

1. `msg_1, ..., msg_L = messages[1], ..., messages[L]`
2. `(Q_1, Q_2, H_1, ..., H_L) = create_generators(generator_seed, L+2)`

Procedure:

1. `dom_array = (PK, L, Q_1, Q_2, H_1, ..., H_L, ciphersuite_id, header)`
2. `dom_for_hash = encode_for_hash(dom_array)`
3. if `dom_for_hash` is INVALID, return INVALID
4. `domain = hash_to_scalar(dom_for_hash, 1)`
5. `e_s_for_hash = encode_for_hash((SK, domain, msg_1, ..., msg_L))`
6. if `e_s_for_hash` is INVALID, return INVALID
7. `(e, s) = hash_to_scalar(e_s_for_hash, 2)`
8.  $B = P_1 + Q_1 * s + Q_2 * domain + H_1 * msg_1 + \dots + H_L * msg_L$
9.  $A = B * (1 / (SK + e))$
10. `signature_octets = signature_to_octets(A, e, s)`
11. return `signature_octets`

**Note** When computing step 9 of the above procedure there is an extremely small probability (around  $2^{-r}$ ) that the condition  $(SK + e) = 0 \bmod r$  will be met. How implementations evaluate the inverse of the scalar value 0 may vary, with some returning an error and others returning 0 as a result. If the returned value from the inverse operation  $1/(SK + e)$  does evaluate to 0 the value of A will equal Identity\_G1 thus an invalid signature. Implementations MAY elect to check  $(SK + e) = 0 \bmod r$  prior to step 9, and or  $A \neq \text{Identity\_G1}$  after step 9 to prevent the production of invalid signatures.

#### 3.4.2. Verify

This operation checks that a signature is valid for a given header and vector of messages against a supplied public key (PK). The messages MUST be supplied in this operation in the same order they were supplied to [Sign](#) when creating the signature.

```
result = Verify(PK, signature, header, messages)
```

Inputs:

- PK (REQUIRED), an octet string of the form outputted by the SkToPk operation.
- signature (REQUIRED), an octet string of the form outputted by the Sign operation.
- header (OPTIONAL), an octet string containing context and application specific information. If not supplied, it defaults to an empty string.
- messages (OPTIONAL), a vector of scalars. If not supplied, it defaults to the empty array "()".

Parameters:

- ciphersuite\_id, ASCII string. The unique ID of the ciphersuite.
- generator\_seed, ASCII string. The generators seed defined by the ciphersuite.

Definitions:

- L, is the non-negative integer representing the number of messages to be signed e.g length(messages). If no messages are supplied as an input, the value of L MUST evaluate to zero (0).

Outputs:

- result, either VALID or INVALID.

Precomputations:

1. (msg\_1, ..., msg\_L) = messages
2. (Q\_1, Q\_2, H\_1, ..., H\_L) = create\_generators(generator\_seed, L+2)

Procedure:

1. signature\_result = octets\_to\_signature(signature)
2. if signature\_result is INVALID, return INVALID
3. (A, e, s) = signature\_result
4. W = octets\_to\_pubkey(PK)
5. if W is INVALID, return INVALID
6. dom\_array = (PK, L, Q\_1, Q\_2, H\_1, ..., H\_L, ciphersuite\_id, header)
7. dom\_for\_hash = encode\_for\_hash(dom\_array)
8. if dom\_for\_hash is INVALID, return INVALID
9. domain = hash\_to\_scalar(dom\_for\_hash, 1)
10.  $B = P_1 + Q_1 * s + Q_2 * domain + H_1 * msg_1 + \dots + H_L * msg_L$
11. if  $e(A, W + P_2 * e) * e(B, -P_2) \neq \text{Identity\_GT}$ , return INVALID
12. return VALID

### 3.4.3. ProofGen

This operation computes a zero-knowledge proof-of-knowledge of a signature, while optionally selectively disclosing from the original set of signed messages. The "prover" may also supply a presentation header, see [Presentation header selection](#) for more details.

The messages supplied in this operation MUST be in the same order as when supplied to [Sign](#). To specify which of those messages will be disclosed, the prover can supply the list of indexes (disclosed\_indexes) that the disclosed messages have in the array of signed messages. Each element in disclosed\_indexes MUST be a non-negative integer, in the range from 1 to length(messages).



```
proof = ProofGen(PK, signature, header, ph, messages, disclosed_indexes)
```

#### Inputs:

- PK (REQUIRED), an octet string of the form outputted by the SkToPk operation.
- signature (REQUIRED), an octet string of the form outputted by the Sign operation.
- header (OPTIONAL), an octet string containing context and application specific information. If not supplied, it defaults to an empty string.
- ph (OPTIONAL), octet string containing the presentation header. If not supplied, it defaults to an empty string.
- messages (OPTIONAL), a vector of scalars. If not supplied, it defaults to the empty array "()".
- disclosed\_indexes (OPTIONAL), vector of unsigned integers in ascending order. Indexes of disclosed messages. If not supplied, it defaults to the empty array "()".

#### Parameters:

- ciphersuite\_id, ASCII string. The unique ID of the ciphersuite.
- generator\_seed, ASCII string. The generators seed defined by the ciphersuite.

#### Definitions:

- L, is the non-negative integer representing the number of messages, i.e.,  $L = \text{length}(\text{messages})$ . If no messages are supplied, the value of L MUST evaluate to zero (0).
- R, is the non-negative integer representing the number of disclosed (revealed) messages, i.e.,  $R = \text{length}(\text{disclosed\_indexes})$ . If no messages are disclosed, R MUST evaluate to zero (0).
- U, is the non-negative integer representing the number of undisclosed messages, i.e.,  $U = L - R$ .
- $\text{prf\_len} = \text{ceil}(\text{ceil}(\log_2(r))/8)$ , where r defined by the ciphersuite.

#### Outputs:

- proof, octet string; or INVALID.

#### Precomputations:

1.  $(i_1, \dots, i_R) = \text{disclosed\_indexes}$
2.  $(j_1, \dots, j_U) = \text{range}(1, L) \setminus \text{disclosed\_indexes}$
3.  $(\text{msg}_1, \dots, \text{msg}_L) = \text{messages}$
4.  $(\text{msg}_{i_1}, \dots, \text{msg}_{i_R}) = (\text{messages}[i_1], \dots, \text{messages}[i_R])$
5.  $(\text{msg}_{j_1}, \dots, \text{msg}_{j_U}) = (\text{messages}[j_1], \dots, \text{messages}[j_U])$
6.  $(Q_1, Q_2, \text{MsgGenerators}) = \text{create\_generators}(\text{generator\_seed}, L+2)$

7.  $(H_1, \dots, H_L) = \text{MsgGenerators}$
8.  $(H_{j1}, \dots, H_{jU}) = (\text{MsgGenerators}[j1], \dots, \text{MsgGenerators}[jU])$

Procedure:

1.  $\text{signature\_result} = \text{octets\_to\_signature}(\text{signature})$
2. if  $\text{signature\_result}$  is INVALID, return INVALID
3.  $(A, e, s) = \text{signature\_result}$
4.  $\text{dom\_array} = (\text{PK}, L, Q_1, Q_2, H_1, \dots, H_L, \text{ciphersuite\_id}, \text{header})$
5.  $\text{dom\_for\_hash} = \text{encode\_for\_hash}(\text{dom\_array})$
6. if  $\text{dom\_for\_hash}$  is INVALID, return INVALID
7.  $\text{domain} = \text{hash\_to\_scalar}(\text{dom\_for\_hash}, 1)$
8.  $(r1, r2, e\sim, r2\sim, r3\sim, s\sim) = \text{hash\_to\_scalar}(\text{PRF}(\text{prf\_len}), 6)$
9.  $(m\sim_{j1}, \dots, m\sim_{jU}) = \text{hash\_to\_scalar}(\text{PRF}(\text{prf\_len}), U)$
10.  $B = P1 + Q_1 * s + Q_2 * \text{domain} + H_1 * \text{msg}_1 + \dots + H_L * \text{msg}_L$
11.  $r3 = r1 \wedge -1 \bmod r$
12.  $A' = A * r1$
13.  $\text{Abar} = A' * (-e) + B * r1$
14.  $D = B * r1 + Q_1 * r2$
15.  $s' = r2 * r3 + s \bmod r$
16.  $C1 = A' * e\sim + Q_1 * r2\sim$
17.  $C2 = D * (-r3\sim) + Q_1 * s\sim + H_{j1} * m\sim_{j1} + \dots + H_{jU} * m\sim_{jU}$
18.  $\text{c\_array} = (A', \text{Abar}, D, C1, C2, R, i1, \dots, iR, \text{msg}_{i1}, \dots, \text{msg}_{iR}, \text{domain}, \text{ph})$
19.  $\text{c\_for\_hash} = \text{encode\_for\_hash}(\text{c\_array})$
20. if  $\text{c\_for\_hash}$  is INVALID, return INVALID
21.  $c = \text{hash\_to\_scalar}(\text{c\_for\_hash}, 1)$
22.  $e^\wedge = c * e + e\sim \bmod r$
23.  $r2^\wedge = c * r2 + r2\sim \bmod r$
24.  $r3^\wedge = c * r3 + r3\sim \bmod r$
25.  $s^\wedge = c * s' + s\sim \bmod r$
26. for  $j$  in  $(j1, \dots, jU)$ :  $m^\wedge_j = c * \text{msg}_j + m\sim_j \bmod r$
27.  $\text{proof} = (A', \text{Abar}, D, c, e^\wedge, r2^\wedge, r3^\wedge, s^\wedge, (m^\wedge_{j1}, \dots, m^\wedge_{jU}))$
28. return  $\text{proof\_to\_octets}(\text{proof})$

#### 3.4.4. ProofVerify

This operation checks that a proof is valid for a header, vector of disclosed messages (along side their index corresponding to their original position when signed) and presentation header against a public key (PK).

The operation accepts the list of messages the prover indicated to be disclosed. Those messages MUST be in the same order as when supplied to [Sign](#) (as a subset of the signed messages list). The operation also requires the total number of signed messages (L). Lastly, it also accepts the indexes that the disclosed messages had in the original array of messages supplied to [Sign](#) (i.e., the disclosed\_indexes list supplied to [ProofGen](#)). Every element in this list MUST be a non-negative integer in the range from 1 to L, in ascending order.

```
result = ProofVerify(PK, proof, L, header, ph,  
                    disclosed_messages,  
                    disclosed_indexes)
```

#### Inputs:

- PK (REQUIRED), an octet string of the form outputted by the SkToPk operation.
- proof (REQUIRED), an octet string of the form outputted by the ProofGen operation.
- L (REQUIRED), non-negative integer. The number of signed messages.
- header (OPTIONAL), an optional octet string containing context and application specific information. If not supplied, it defaults to an empty string.
- ph (OPTIONAL), octet string containing the presentation header. If not supplied, it defaults to an empty string.
- disclosed\_messages (OPTIONAL), a vector of scalars. If not supplied, it defaults to the empty array "()".
- disclosed\_indexes (OPTIONAL), vector of unsigned integers in ascending order. Indexes of disclosed messages. If not supplied, it defaults to the empty array "()".

#### Parameters:

- ciphersuite\_id, ASCII string. The unique ID of the ciphersuite.
- generator\_seed, ASCII string. The generators seed defined by the ciphersuite.

#### Definitions:

- R, is the non-negative integer representing the number of disclosed (revealed) messages, i.e.,  $R = \text{length}(\text{disclosed\_indexes})$ . If no messages are disclosed, the value of R MUST evaluate to zero (0).
- U, is the non-negative integer representing the number of undisclosed messages, i.e.,  $U = L - R$ .

#### Outputs:

- result, either VALID or INVALID.

#### Precomputations:

1.  $(i_1, \dots, i_R) = \text{disclosed\_indexes}$
2.  $(j_1, \dots, j_U) = \text{range}(1, L) \setminus \text{disclosed\_indexes}$
3.  $(\text{msg}_{i_1}, \dots, \text{msg}_{i_R}) = \text{disclosed\_messages}$
4.  $(Q_1, Q_2, \text{MsgGenerators}) = \text{create\_generators}(\text{generator\_seed}, L+2)$
5.  $(H_1, \dots, H_L) = \text{MsgGenerators}$
6.  $(H_{i_1}, \dots, H_{i_R}) = (\text{MsgGenerators}[i_1], \dots, \text{MsgGenerators}[i_R])$
7.  $(H_{j_1}, \dots, H_{j_U}) = (\text{MsgGenerators}[j_1], \dots, \text{MsgGenerators}[j_U])$

Preconditions:

1. for  $i$  in  $(i1, \dots, iR)$ , if  $i < 1$  or  $i > L$ , return INVALID
2. if  $\text{length}(\text{disclosed\_messages}) \neq R$ , return INVALID

Procedure:

1.  $\text{proof\_result} = \text{octets\_to\_proof}(\text{proof})$
2. if  $\text{proof\_result}$  is INVALID, return INVALID
3.  $(A', \text{Abar}, D, c, e^\wedge, r2^\wedge, r3^\wedge, s^\wedge, (m^\wedge_{j1}, \dots, m^\wedge_{jU})) = \text{proof\_result}$
4.  $W = \text{octets\_to\_pubkey}(\text{PK})$
5. if  $W$  is INVALID, return INVALID
6.  $\text{dom\_array} = (\text{PK}, L, Q_1, Q_2, H_1, \dots, H_L, \text{ciphersuite\_id}, \text{header})$
7.  $\text{dom\_for\_hash} = \text{encode\_for\_hash}(\text{dom\_array})$
8. if  $\text{dom\_for\_hash}$  is INVALID, return INVALID
9.  $\text{domain} = \text{hash\_to\_scalar}(\text{dom\_for\_hash}, 1)$
10.  $C1 = (\text{Abar} - D) * c + A' * e^\wedge + Q_1 * r2^\wedge$
11.  $T = P1 + Q_2 * \text{domain} + H_{i1} * \text{msg}_{i1} + \dots + H_{iR} * \text{msg}_{iR}$
12.  $C2 = T * c - D * r3^\wedge + Q_1 * s^\wedge + H_{j1} * m^\wedge_{j1} + \dots + H_{jU} * m^\wedge_{jU}$
13.  $\text{cv\_array} = (A', \text{Abar}, D, C1, C2, R, i1, \dots, iR, \text{msg}_{i1}, \dots, \text{msg}_{iR}, \text{domain}, \text{ph})$
14.  $\text{cv\_for\_hash} = \text{encode\_for\_hash}(\text{cv\_array})$
15. if  $\text{cv\_for\_hash}$  is INVALID, return INVALID
16.  $\text{cv} = \text{hash\_to\_scalar}(\text{cv\_for\_hash}, 1)$
17. if  $c \neq \text{cv}$ , return INVALID
18. if  $A' == \text{Identity\_G1}$ , return INVALID
19. if  $e(A', W) * e(\text{Abar}, -P2) \neq \text{Identity\_GT}$ , return INVALID
20. return VALID

## 4. Utility Operations

### 4.1. Generator point computation

This operation defines how to create a set of generators that form a part of the public parameters used by the BBS Signature scheme to accomplish operations such as [Sign](#), [Verify](#), [ProofGen](#) and [ProofVerify](#). It takes one input, the number of generator points to create, which is determined in part by the number of signed messages.

As an optimization, implementations MAY cache the result of `create_generators` for a specific `generator_seed` (determined by the `ciphersuite`) and count. The values `n` and `v` MAY also be cached in order to efficiently extend a existing list of generator points.

```
generators = create_generators(count)
```

#### Inputs:

- count (REQUIRED), unsigned integer. Number of generators to create.

#### Parameters:

- hash\_to\_curve\_suite, the hash to curve suite id defined by the ciphersuite.
- hash\_to\_curve\_g1, the hash\_to\_curve operation for the G1 subgroup, defined by the suite specified by the hash\_to\_curve\_suite parameter.
- expand\_message, the expand\_message operation defined by the suite specified by the hash\_to\_curve\_suite parameter.
- generator\_seed, octet string. A seed value selected by the ciphersuite.

#### Definitions:

- seed\_dst, the octet string representing the ASCII encoded characters: "BBS\_" || hash\_to\_curve\_suite || "SIG\_GENERATOR\_SEED\_".
- generator\_dst, the octet string representing: "BBS\_" || hash\_to\_curve\_suite || "SIG\_GENERATOR\_DST\_", in the ASCII characters encoding.
- seed\_len =  $\text{ceil}((\text{ceil}(\log_2(r)) + k)/8)$ , where r and k are defined by the ciphersuite.

#### Outputs:

- generators, an array of generators.

#### Procedure:

1.  $v = \text{expand\_message}(\text{generator\_seed}, \text{seed\_dst}, \text{seed\_len})$
2.  $n = 1$
3. for i in range(1, count):
4.      $v = \text{expand\_message}(v \parallel \text{I2OSP}(n, 4), \text{seed\_dst}, \text{seed\_len})$
5.      $n = n + 1$
6.      $\text{generator\_i} = \text{Identity\_G1}$
7.      $\text{candidate} = \text{hash\_to\_curve\_g1}(v, \text{generator\_dst})$
8.     if candidate in (P1, generator\_1, ..., generator\_i):
9.         go back to step 4
10.      $\text{generator\_i} = \text{candidate}$
11. return (generator\_1, ..., generator\_count)

## 4.2. MapMessageToScalar

There are multiple ways in which messages can be mapped to their respective scalar values, which is their required form to be used with the [Sign](#), [Verify](#), [ProofGen](#) and [ProofVerify](#) operations.

### 4.2.1. MapMessageToScalarAsHash

This operation takes an input message and maps it to a scalar value via a cryptographic hash function for the given curve.

```
result = MapMessageToScalarAsHash(msg, dst)
```

Inputs:

- msg (REQUIRED), octet string.
- dst (REQUIRED), octet string. Domain separation tag; note this is not defined as a function argument as per [!I-D.irtf-cfrg-hash-to-curve] but as a parameter.

Outputs:

- result, a scalar value.

Procedure:

1. If  $\text{length}(\text{dst}) > 2^8 - 1$  or  $\text{length}(\text{msg}) > 2^{64} - 1$ , return INVALID
2.  $\text{dst\_prime} = \text{I2OSP}(\text{length}(\text{dst}), 1) \parallel \text{dst}$
3.  $\text{msg\_prime} = \text{I2OSP}(\text{length}(\text{msg}), 8) \parallel \text{msg}$
4.  $\text{result} = \text{hash\_to\_scalar}(\text{msg\_prime} \parallel \text{dst\_prime}, 1)$
5. return result

## 4.3. Hash to Scalar

This operation describes how to hash an arbitrary octet string to  $n$  scalar values in the multiplicative group of integers mod  $r$  (i.e., values in the range  $[1, r-1]$ ). This procedure acts as a helper function, used internally in various places within the operations described in the spec. To map a message to a scalar that would be passed as input to the [Sign](#), [Verify](#), [ProofGen](#) and [ProofVerify](#) functions, one must use [MapMessageToScalarAsHash](#) instead.

This operation makes use of `expand_message` defined in [I-D.irtf-cfrg-hash-to-curve], in a similar way used by the `hash_to_field` operation of Section 5 from the same document (with the additional checks for getting a scalar that is 0). Note that, if an implementer wants to use `hash_to_field` instead, they MUST use the multiplicative group of integers mod  $r$  ( $\text{Fr}$ ), as the target group ( $F$ ). However, the `hash_to_curve` ciphersuites used by this document, make use of `hash_to_field` with the target group being the multiplicative group



of integers mod  $p$  ( $F_p$ ). For completeness, we define here the operation making use of the `expand_message` function, that will be defined by the hash-to-curve suite used. If someone also has a `hash_to_field` implementation available, with the target group been  $F_r$ , they can use this instead (adding the check for a scalar been 0).

```
scalars = hash_to_scalar(msg_octets, count)
```

Inputs:

- `msg_octets` (REQUIRED), octet string. The message to be hashed.
- `count` (REQUIRED), an integer greater or equal to 1. The number of scalars to output.

Parameters:

- `hash_to_curve_suite`, the hash to curve suite id defined by the ciphersuite.
- `expand_message`, the `expand_message` operation defined by the suite specified by the `hash_to_curve_suite` parameter.

Definitions:

- `h2s_dst`, the octet string representing the ASCII encoded characters: "BBS\_" || `hash_to_curve_suite` || "HASH\_TO\_SCALAR\_".
- `expand_len` =  $\text{ceil}((\text{ceil}(\log_2(r)) + k) / 8)$ , where  $r$  and  $k$  are defined by the ciphersuite.

Outputs:

- `scalars`, an array of non-zero scalars mod  $r$ .

Procedure:

1. `len_in_bytes` = `count` \* `expand_len`
2. `t` = 0
3. `msg_prime` = `msg_octets` || `I2OSP(t, 1)` || `I2OSP(count, 4)`
4. `uniform_bytes` = `expand_message(msg_prime, h2s_dst, len_in_bytes)`
5. for `i` in (1, ..., `count`):
6.     `tv` = `uniform_bytes[(i-1)*expand_len..i*expand_len-1]`
7.     `scalar_i` = `OS2IP(tv)` mod  $r$
8. if 0 in (`scalar_1`, ..., `scalar_count`):
9.     `t` = `t` + 1
10. go back to step 3
11. return (`scalar_1`, ..., `scalar_count`)

## 4.4. Serialization

### 4.4.1. OctetsToSignature

This operation describes how to decode an octet string, validate it and return the underlying components that make up the signature.

```
signature = octets_to_signature(signature_octets)
```

Inputs:

- `signature_octets` (REQUIRED), octet string of the form output from `signature_to_octets` operation.

Outputs:

`signature`, a signature in the form  $(A, e, s)$ , where  $A$  is a point in  $G_1$  and  $e$  and  $s$  are non-zero scalars mod  $r$ .

Procedure:

1. `expected_len = octet_point_length + 2 * octet_scalar_length`
2. if `length(signature_octets) != expected_len`, return `INVALID`
3. `A_octets = signature_octets[0..(octet_point_length - 1)]`
4. `A = octets_to_point_g1(A_octets)`
5. if `A` is `INVALID`, return `INVALID`
6. if `A == Identity_G1`, return `INVALID`
7. `index = octet_point_length`
8. `end_index = index + octet_scalar_length - 1`
9. `e = OS2IP(signature_octets[index..end_index])`
10. if `e = 0` OR `e >= r`, return `INVALID`
11. `index += octet_scalar_length`
12. `end_index = index + octet_scalar_length - 1`
13. `s = OS2IP(signature_octets[index..end_index])`
14. if `s = 0` OR `s >= r`, return `INVALID`
15. return  $(A, e, s)$

### 4.4.2. SignatureToOctets

This operation describes how to encode a signature to an octet string.

*Note* this operation deliberately does not perform the relevant checks on the inputs  $A$ ,  $e$  and  $s$  because its assumed these are done prior to its invocation, e.g as is the case with the [Sign](#) operation.

```
signature_octets = signature_to_octets(signature)
```

Inputs:

- signature (REQUIRED), a valid signature, in the form (A, e, s), where  
A a point in G1 and e, s non-zero scalars mod r.

Outputs:

- signature\_octets, octet string.

Procedure:

1. (A, e, s) = signature
2. A\_octets = point\_to\_octets\_g1(A)
3. e\_octets = I2OSP(e, octet\_scalar\_length)
4. s\_octets = I2OSP(s, octet\_scalar\_length)
5. return (A\_octets || e\_octets || s\_octets)

#### **4.4.3. OctetsToProof**

This operation describes how to decode an octet string representing a proof, validate it and return the underlying components that make up the proof value.

The proof value outputted by this operation consists of the following components, in that order:

1. Three (3) valid points of the G1 subgroup, each of which must not equal the identity point.
2. Five (5) integers representing scalars in the range of 1 to r-1 inclusive.
3. A set of integers representing scalars in the range of 1 to r-1 inclusive, corresponding to the undisclosed from the proof message commitments. This set can be empty (i.e., "()).

```
proof = octets_to_proof(proof_octets)
```

Inputs:

- proof\_octets (REQUIRED), octet string of the form outputted from the proof\_to\_octets operation.

Parameters:

- r (REQUIRED), non-negative integer. The prime order of the G1 and G2 groups, defined by the ciphersuite.
- octet\_scalar\_length (REQUIRED), non-negative integer. The length of a scalar octet representation, defined by the ciphersuite.
- octet\_point\_length (REQUIRED), non-negative integer. The length of a point in G1 octet representation, defined by the ciphersuite.

Outputs:

- proof, a proof value in the form described above or INVALID

Procedure:

1. proof\_len\_floor = 3 \* octet\_point\_length + 5 \* octet\_scalar\_length
2. if length(proof\_octets) < proof\_len\_floor, return INVALID

// Points (i.e., (A', Abar, D) in ProofGen) de-serialization.

3. index = 0
4. for i in range(0, 2):
5.     end\_index = index + octet\_point\_length - 1
6.     A\_i = octets\_to\_point\_g1(proof\_octets[index..end\_index])
7.     if A\_i is INVALID or Identity\_G1, return INVALID
8.     index += octet\_point\_length

// Scalars (i.e., (c, e<sup>^</sup>, r2<sup>^</sup>, r3<sup>^</sup>, s<sup>^</sup>, (m<sup>^</sup>\_j1, ..., m<sup>^</sup>\_jU)) in ProofGen) de-serialization.

9. j = 0
10. while index < length(proof\_octets):
11.     end\_index = index + octet\_scalar\_length - 1
12.     s\_j = OS2IP(proof\_octets[index..end\_index])
13.     if s\_j = 0 or if s\_j >= r, return INVALID
14.     index += octet\_scalar\_length
15.     j += 1
16. if index != length(proof\_octets), return INVALID
17. msg\_commitments = ()
18. If j > 5, set msg\_commitments = (s\_5, ..., s\_(j-1))
19. return (A\_0, A\_1, A\_2, s\_0, s\_1, s\_2, s\_3, s\_4, msg\_commitments)

#### 4.4.4. ProofToOctets

This operation describes how to encode a proof, as computed at step 25 in [ProofGen](#), to an octet string. The input to the operation MUST be a valid proof.

The inputted proof value must consist of the following components, in that order:

1. Three (3) valid compressed points of the  $G_1$  subgroup, different from the identity point of  $G_1$  (i.e.,  $A'$ ,  $Abar$ ,  $D$ , in ProofGen)
2. Five (5) integers representing scalars in the range of 1 to  $r-1$  inclusive (i.e.,  $c$ ,  $e^\wedge$ ,  $r2^\wedge$ ,  $r3^\wedge$ ,  $s^\wedge$ , in ProofGen).
3. A number of integers representing scalars in the range of 1 to  $r-1$  inclusive, corresponding to the undisclosed from the proof messages (i.e.,  $m^\wedge_{j1}$ , ...,  $m^\wedge_{jU}$ , in ProofGen, where  $U$  the number of undisclosed messages).

```
proof_octets = proof_to_octets(proof)
```

Inputs:

- proof (REQUIRED), a BBS proof in the form calculated by ProofGen in step 25 (see above).

Parameters:

- octet\_scalar\_length (REQUIRED), non-negative integer. The length of a scalar octet representation, defined by the ciphersuite.

Outputs:

- proof\_octets, octet string.

Procedure:

1.  $(A', Abar, D, c, e^\wedge, r2^\wedge, r3^\wedge, s^\wedge, (m^\wedge_1, \dots, m^\wedge_U)) = \text{proof}$
2. Let proof\_octets be an empty octet string.

```
// Points Serialization.
```

3. for point in  $(A', Abar, D)$ :

4.     point\_octets = point\_to\_octets\_g1(point)

5.     proof\_octets = proof\_octets || point\_octets

```
// Scalar Serialization.
```

6. for scalar in  $(c, e^\wedge, r2^\wedge, r3^\wedge, s^\wedge, m^\wedge_1, \dots, m^\wedge_U)$ :

7.     scalar\_octets = I2OSP(scalar, octet\_scalar\_length)

8.     proof\_octets = proof\_octets || scalar\_octets

9. return proof\_octets

#### **4.4.5. OctetsToPublicKey**

This operation describes how to decode an octet string representing a public key, validates it and returns the corresponding point in G2. Steps 2 to 5 check if the public key is valid. As an optimization, implementations MAY cache the result of those steps, to avoid unnecessarily repeating validation for known public keys.

`W = octets_to_pubkey(PK)`

Inputs:

- PK, octet string. A public key in the form outputted by the SkToPK operation

Outputs:

- W, a valid point in G2 or INVALID

Procedure:

1. `W = octets_to_point_g2(PK)`
2. If W is INVALID, return INVALID
3. if `subgroup_check(W)` is INVALID, return INVALID
4. If `W == Identity_G2`, return INVALID
5. return W

#### **4.4.6. EncodeForHash**

This document uses the `hash_to_scalar` function to hash elements to scalars in the multiplicative group mod  $r$  (see [Section 5.3](#)). To avoid ambiguity, elements passed to that operation, must first be encoded appropriately using `encode_for_hash`. The following procedure describes how to encode each element accordingly by serializing it to an appropriate format depending on its type and concatenating the results. Specifically,

\*Points in G1 or G2 will be encoded using the `point_to_octets_g*` implementation for a particular ciphersuite.

\*Non-negative integers will be encoded using I2OSP with an output length of 8 bytes.

\*Scalars will be zero-extended to a fixed length, defined by a particular ciphersuite.

\*Octet strings will be zero-extended to a length that is a multiple of 8 bits. Then, the extended value is encoded directly.

\*ASCII strings will be transformed into octet strings using UTF-8 encoding.

After encoding, octet strings will be prepended with a value representing the length of their binary representation in the form of the number of bytes. This length must be encoded to octets using I2OSP with output length of 8 bytes. The combined value (encoded value + length prefix) binary representation is then encoded as a single octet string. For example, the string `0x14d` will be encoded

as 0x00000000000000002014d. If the length of the octet string is larger than  $2^{64} - 1$ , the octet string must be rejected. Similarly, ASCII strings, after encoded to octets (using utf8), will also be appended with the length of their octet-string representation.

Optional input/parameters to operations that feature in a call to `hash_to_scalar`, that are not supplied to the operation should default to an empty octet string. For example, if X is an optional input/parameter that is not supplied, whilst A and B are required, then the procedural step of `hash(A || X || B)` MUST be evaluated to `hash(A || "" || B)`.

The above is further described in the following procedure.

```
result = encode_for_hash(input_array)
```

Inputs:

- `input_array`, an array of elements to be hashed. All elements of this array that are octet strings MUST be multiples of 8 bits.

Parameters:

- `octet_scalar_length`, non-negative integer. The length of a scalar octet representation, defined by the ciphersuite.

Outputs:

- `result`, an octet string or INVALID.

Procedure:

1. let `octets_to_hash` be an empty octet string.
2. for `el` in `input_array`:
3.     if `el` is an ASCII string: `el = utf8(el)`
4.     if `el` is an octet string representing a public key: `el_octets = el`
5.     else if `el` is an octet string:
6.         if `length(el) > 2^{64} - 1`, return INVALID
7.         `el_octets = I2OSP(length(el), 8) || el`
8.     else if `el` is a Point in G1: `el_octets = point_to_octets_g1(el)`
9.     else if `el` is a Point in G2: `el_octets = point_to_octets_g2(el)`
10.    else if `el` is a Scalar: `el_octets = I2OSP(el, octet_scalar_length)`
11.    else if `el` is a non-negative integer: `el_octets = I2OSP(el, 8)`
12.    else: return INVALID
13.    `octets_to_hash = octets_to_hash || el_octets`
14. return `octets_to_hash`



## 5. Security Considerations

### 5.1. Validating public keys

It is RECOMMENDED for any operation in [Core Operations](#) involving public keys, that they deserialize the public key first using the [OctetsToPublicKey](#) operation, even if they only require the octet-string representation of the public key. If the `octets_to_pubkey` procedure (see the [OctetsToPublicKey](#) section) returns `INVALID`, the calling operation should also return `INVALID` and abort. An example of where this recommendation applies is the [Sign](#) operation. An example of where an explicit invocation to the `octets_to_pubkey` operation is already defined and therefore required is the [Verify](#) operation.

### 5.2. Point de-serialization

This document makes use of `octet_to_point_g*` to parse octet strings to elliptic curve points (either in  $G_1$  or  $G_2$ ). It is assumed (even if not explicitly described) that the result of this operation will not be `INVALID`. If `octet_to_point_g*` returns `INVALID`, then the calling operation should immediately return `INVALID` as well and abort the operation. Note that the only place where the output is assumed to be `VALID` implicitly is in the [EncodingForHash](#) section.

### 5.3. Skipping membership checks

Some existing implementations skip the `subgroup_check` invocation in [Verify](#), whose purpose is ensuring that the signature is an element of a prime-order subgroup. This check is REQUIRED of conforming implementations, for two reasons.

1. For most pairing-friendly elliptic curves used in practice, the pairing operation  $e$  [Section 1.2](#) is undefined when its input points are not in the prime-order subgroups of  $E_1$  and  $E_2$ . The resulting behavior is unpredictable, and may enable forgeries.
2. Even if the pairing operation behaves properly on inputs that are outside the correct subgroups, skipping the subgroup check breaks the strong unforgeability property [\[ADR02\]](#).

### 5.4. Side channel attacks

Implementations of the signing algorithm SHOULD protect the secret key from side-channel attacks. One method for protecting against certain side-channel attacks is ensuring that the implementation executes exactly the same sequence of instructions and performs exactly the same memory accesses, for any value of the secret key. In other words, implementations on the underlying pairing-friendly elliptic curve SHOULD run in constant time.

## 5.5. Randomness considerations

The IKM input to KeyGen MUST be infeasible to guess and MUST be kept secret. One possibility is to generate IKM from a trusted source of randomness. Guidelines on constructing such a source are outside the scope of this document.

Secret keys MAY be generated using other methods; in this case they MUST be infeasible to guess and MUST be indistinguishable from uniformly random modulo  $r$ .

BBS proofs are nondeterministic, meaning care must be taken against attacks arising from using bad randomness, for example, the nonce reuse attack on ECDSA [[HDWH12](#)]. It is RECOMMENDED that the presentation header used in this specification contain a nonce chosen at random from a trusted source of randomness, see the [Section 5.6](#) for additional considerations.

When a trusted source of randomness is used, signatures and proofs are much harder to forge or break due to the use of multiple nonces.

## 5.6. Presentation header selection

The signature proofs of knowledge generated in this specification are created using a specified presentation header. A verifier-specified cryptographically random value (e.g., a nonce) featuring in the presentation header provides strong protections against replay attacks, and is RECOMMENDED in most use cases. In some settings, proofs can be generated in a non-interactive fashion, in which case verifiers MUST be able to verify the uniqueness of the presentation header values.

## 5.7. Implementing hash\_to\_curve\_g1

The security analysis models hash\_to\_curve\_g1 as random oracles. It is crucial that these functions are implemented using a cryptographically secure hash function. For this purpose, implementations MUST meet the requirements of [[I-D.irtf-cfrg-hash-to-curve](#)].

In addition, ciphersuites MUST specify unique domain separation tags for hash\_to\_curve. Some guidance around defining this can be found in [Section 6](#).

## 5.8. Choice of underlying curve

BBS signatures can be implemented on any pairing-friendly curve. However care MUST be taken when selecting one that is appropriate, this specification defines a ciphersuite for using the BLS12-381 curve in [Section 6](#) which as a curve achieves around 117 bits of

security according to a recent NCC ZCash cryptography review [[ZCASH-REVIEW](#)].

### 5.9. Security of proofs generated by ProofGen

The proof, as returned by ProofGen, is a zero-knowledge proof-of-knowledge [[CDL16](#)]. This guarantees that no information will be revealed about the signature itself or the undisclosed messages, from the output of ProofGen. Note that the security proofs in [[CDL16](#)] work on type 3 pairing setting. This means that G1 should be different from G2 and with no efficient isomorphism between them.

## 6. Ciphersuites

This section defines the format for a BBS ciphersuite. It also gives concrete ciphersuites based on the BLS12-381 pairing-friendly elliptic curve [[I-D.irtf-cfrg-pairing-friendly-curves](#)].

### 6.1. Ciphersuite Format

#### 6.1.1. Ciphersuite ID

The following section defines the format of the unique identifier for the ciphersuite denoted `ciphersuite_id`. The REQUIRED format for this string is

```
"BBS_" || H2C_SUITE_ID || ADD_INFO
```

\*Strings in double quotes are ASCII-encoded literals.

\*H2C\_SUITE\_ID is the suite ID of the hash-to-curve suite used to define the hashtocurve function.

\*ADD\_INFO is an optional string indicating any additional information used to uniquely qualify the ciphersuite. When present this value MUST only contain ASCII characters between 0x21 and 0x7e (inclusive), and MUST end with an underscore (0x5f), other than the last character the string MUST not contain any other underscores (0x5f).

#### 6.1.2. Additional Parameters

The parameters that each ciphersuite needs to define are generally divided into three main categories; the basic parameters (a hash function etc.), the serialization operations (`point_to_octets_g1` etc.) and the generator parameters. See below for more details.

##### Basic parameters:

\*hash: a cryptographic hash function.

\*octet\_scalar\_length: Number of bytes to represent a scalar value, in the multiplicative group of integers mod  $r$ , encoded as an octet string. It is RECOMMENDED this value be set to  $\text{ceil}(\log_2(r)/8)$ .

\*octet\_point\_length: Number of bytes to represent a point encoded as an octet string outputted by the point\_to\_octets\_g\* function. It is RECOMMENDED that this value is set to  $\text{ceil}(\log_2(p)/8)$ .

\*hash\_to\_curve\_suite: The hash-to-curve ciphersuite id, in the form defined in [[I-D.irtf-cfrg-hash-to-curve](#)]. This defines the hash\_to\_curve\_g1 (the hash\_to\_curve operation for the G1 subgroup, see the [Notation](#) section) and the expand\_message (either expand\_message\_xmd or expand\_message\_xof) operations used in this document.

### **Serialization functions:**

\*point\_to\_octets\_g1: a function that returns the canonical representation of the point  $P$  for the G1 subgroup as an octet string.

\*point\_to\_octets\_g2: a function that returns the canonical representation of the point  $P$  for the G2 subgroup as an octet string.

\*octets\_to\_point\_g1: a function that returns the point  $P$  in the subgroup G1 corresponding to the canonical representation ostr, or INVALID if ostr is not a valid output of point\_to\_octets\_g1.

\*octets\_to\_point\_g2: a function that returns the point  $P$  in the subgroup G2 corresponding to the canonical representation ostr, or INVALID if ostr is not a valid output of point\_to\_octets\_g2.

### **Generator parameters:**

\*generator\_seed: The seed used to determine the generator points which form part of the public parameters used by the BBS signature scheme. Note there are multiple possible scopes for this seed, including: a globally shared seed (where the resulting message generators are common across all BBS signatures); a signer specific seed (where the message generators are specific to a signer); and a signature specific seed (where the message generators are specific per signature). The ciphersuite MUST define this seed OR how to compute it as a pre-cursor operation to any others.

## 6.2. BLS12-381 Ciphersuite

The following ciphersuite is based on the BLS12-381 elliptic curve defined in Section 4.2.1 of [[I-D.irtf-cfrg-pairing-friendly-curves](#)]. The targeted security level of the suite in bits is  $k = 128$ . The ciphersuite makes use of an extendable output function, and most specifically of SHAKE-256, as defined in Section 6.2 of [[SHA3](#)]. It also uses the hash-to-curve suite defined by this document in [Appendix A.1](#), which also makes use of the SHAKE-256 function.

### Basic parameters:

- \*Ciphersuite\_ID: "BBS\_BLS12381G1\_XOF:SHAKE-256\_SSWU\_R0\_"
- \*hash: SHAKE-256 as defined in [[SHA3](#)].
- \*octet\_scalar\_length: 32, based on the RECOMMENDED approach of  $\text{ceil}(\log_2(r)/8)$ .
- \*octet\_point\_length: 48, based on the RECOMMENDED approach of  $\text{ceil}(\log_2(p)/8)$ .
- \*hash\_to\_curve\_suite: "BLS12381G1\_XOF:SHAKE-256\_SSWU\_R0\_" as defined in [Appendix A.1](#) for the G1 subgroup.

### Serialization functions:

- \*point\_to\_octets\_g1: follows the format documented in Appendix C section 1 of [[I-D.irtf-cfrg-pairing-friendly-curves](#)] for the G1 subgroup, using compression (i.e., setting  $C_{\text{bit}} = 1$ ).
- \*point\_to\_octets\_g2: follows the format documented in Appendix C section 1 of [[I-D.irtf-cfrg-pairing-friendly-curves](#)] for the G2 subgroup, using compression (i.e., setting  $C_{\text{bit}} = 1$ ).
- \*octets\_to\_point\_g1: follows the format documented in Appendix C section 2 of [[I-D.irtf-cfrg-pairing-friendly-curves](#)] for the G1 subgroup.
- \*octets\_to\_point\_g2: follows the format documented in Appendix C section 2 of [[I-D.irtf-cfrg-pairing-friendly-curves](#)] for the G2 subgroup.

### Generator parameters:

- \*generator\_seed: A global seed value of "BBS\_BLS12381G1\_XOF:SHAKE-256\_SSWU\_R0\_MESSAGE\_GENERATOR\_SEED" which is used by the [create\\_generators](#) operation to compute the required set of message generators.

### 6.2.1. Test Vectors

The following section details a basic set of test vectors that can be used to confirm an implementations correctness

**NOTE** All binary data below is represented as octet strings encoded in hexadecimal format

**NOTE** These fixtures are a work in progress and subject to change

Further fixtures are available in [Appendix C](#)

#### 6.2.1.1. Message Generators

Following the procedure defined in [Section 4.1](#) with an input seed value of

BBS\_BLS12381G1\_XOF:SHAKE-256\_SSWU\_RO\_MESSAGE\_GENERATOR\_SEED

a dst of

BBS\_BLS12381G1\_XOF:SHAKE-256\_SSWU\_RO\_

and a length value of 10

Outputs the following values

{{ \$generators[0] }}

{{ \$generators[1] }}

{{ \$generators[2] }}

{{ \$generators[3] }}

{{ \$generators[4] }}

{{ \$generators[5] }}

{{ \$generators[6] }}

{{ \$generators[7] }}

{{ \$generators[8] }}

{{ \$generators[9] }}

#### 6.2.1.2. Key Pair

Following the procedure defined in [Section 3.3.1](#) with an input IKM value as follows

```
{{ $keyPair.seed }}
```

Outputs the following SK value

```
{{ $keyPair.keyPair.secretKey }}
```

Following the procedure defined in [Section 3.3.2](#) with an input SK value as above produces the following PK value

```
{{ $keyPair.keyPair.publicKey }}
```

#### **6.2.1.3. Valid Single Message Signature**

Using the following message

```
{{ $signatureFixtures.signature001.messages[0] }}
```

Along with the SK value as defined in [Section 6.2.1.2](#) as inputs into the Sign operations, yields the following output signature

```
{{ $signatureFixtures.signature001.signature }}
```

#### **6.2.1.4. Valid Multi-Message Signature**

Using the following messages (**Note** the ordering of the messages MUST be preserved)

```
{{ $signatureFixtures.signature004.messages[0] }}
```

```
{{ $signatureFixtures.signature004.messages[1] }}
```

```
{{ $signatureFixtures.signature004.messages[2] }}
```

```
{{ $signatureFixtures.signature004.messages[3] }}
```

```
{{ $signatureFixtures.signature004.messages[4] }}
```

```
{{ $signatureFixtures.signature004.messages[5] }}
```

```
{{ $signatureFixtures.signature004.messages[6] }}
```

```
{{ $signatureFixtures.signature004.messages[7] }}
```

```
{{ $signatureFixtures.signature004.messages[8] }}
```

```
{{ $signatureFixtures.signature004.messages[9] }}
```

Along with the SK value as defined in [Section 6.2.1.2](#) as inputs into the Sign operations, yields the following output signature

```
{{ $signatureFixtures.signature004.signature }}
```

## 7. IANA Considerations

This document does not make any requests of IANA.

## 8. Acknowledgements

The authors would like to acknowledge the significant amount of academic work that preceeded the development of this document. In particular the original work of [BBS04] which was subsequently developed in [ASM06] and in [CDL16]. This last academic work is the one mostly used by this document.

The current state of this document is the product of the work of the Decentralized Identity Foundation Applied Cryptography Working group, which includes numerous active participants. In particular, the following individuals contributed ideas, feedback and wording that influenced this specification:

Orie Steele, Christian Paquin, Alessandro Guggino and Tomislav Markovski

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## Appendix A. BLS12-381 hash\_to\_curve definition using SHAKE-256

The following defines a hash\_to\_curve suite [[I-D.irtf-cfrg-hash-to-curve](#)] for the BLS12-381 curve for both the G1 and G2 subgroups using the extendable output function (xof) of SHAKE-256 as per the guidance defined in section 8.9 of [[I-D.irtf-cfrg-hash-to-curve](#)].

Note the notation used in the below definitions is sourced from [[I-D.irtf-cfrg-hash-to-curve](#)].

### A.1. BLS12-381 G1

The suite of BLS12381G1\_XOF:SHAKE-256\_SSWU\_R0\_ is defined as follows:

- \* encoding type: hash\_to\_curve (Section 3 of  
[[@!I-D.irtf-cfrg-hash-to-curve](#)])
- \* E:  $y^2 = x^3 + 4$
- \* p: 0x1a0111ea397fe69a4b1ba7b6434bacd764774b84f38512bf6730d2a0f6b0f624  
1eabfffeb153ffffb9fefffffffffaaab
- \* r: 0x73eda753299d7d483339d80809a1d80553bda402fffe5bfefffffffff00000001
- \* m: 1
- \* k: 128
- \* expand\_message: expand\_message\_xof (Section 5.3.2 of  
[[@!I-D.irtf-cfrg-hash-to-curve](#)])
- \* hash: SHAKE-256
- \* L: 64
- \* f: Simplified SWU for  $AB == 0$  (Section 6.6.3 of  
[[@!I-D.irtf-cfrg-hash-to-curve](#)])
- \* Z: 11
- \* E':  $y'^2 = x'^3 + A' * x' + B'$ , where
  - A' = 0x144698a3b8e9433d693a02c96d4982b0ea985383ee66a8d8e8981aef  
d881ac98936f8da0e0f97f5cf428082d584c1d
  - B' = 0x12e2908d11688030018b12e8753eee3b2016c1f0f24f4070a0b9c14f  
cef35ef55a23215a316ceaa5d1cc48e98e172be0
- \* iso\_map: the 11-isogeny map from E' to E given in Appendix E.2 of  
[[@!I-D.irtf-cfrg-hash-to-curve](#)]
- \* h\_eff: 0xd201000000010001

Note that the h\_eff values for this suite are copied from that defined for the BLS12381G1\_XMD:SHA-256\_SSWU\_RO\_ suite defined in section 8.8.1 of [[I-D.irtf-cfrg-hash-to-curve](#)].

An optimized example implementation of the Simplified SWU mapping to the curve E' isogenous to BLS12-381 G1 is given in Appendix F.2 [[I-D.irtf-cfrg-hash-to-curve](#)].

## Appendix B. Use Cases

### B.1. Non-correlating Security Token

In the most general sense BBS signatures can be used in any application where a cryptographically secured token is required but correlation caused by usage of the token is un-desirable.

For example in protocols like OAuth2.0 the most commonly used form of the access token leverages the JWT format alongside conventional cryptographic primitives such as traditional digital signatures or HMACs. These access tokens are then used by a relying party to prove authority to a resource server during a request. However, because the access token is most commonly sent by value as it was issued by the authorization server (e.g in a bearer style scheme), the access token can act as a source of strong correlation for the relying party. Relevant prior art can be found [here](#).

BBS Signatures due to their unique properties removes this source of correlation but maintains the same set of guarantees required by a resource server to validate an access token back to its relevant authority (note that an approach to signing JSON tokens with BBS that may be of relevance is the [JWP](#) format and serialization). In the context of a protocol like OAuth2.0 the access token issued by the authorization server would feature a BBS Signature, however instead of the relying party providing this access token as issued, in their request to a resource server, they generate a unique proof from the original access token and include that in the request instead, thus removing this vector of correlation.

### B.2. Improved Bearer Security Token

Bearer based security tokens such as JWT based access tokens used in the OAuth2.0 protocol are a highly popular format for expressing authorization grants. However their usage has several security limitations. Notably a bearer based authorization scheme often has to rely on a secure transport between the authorized party (client) and the resource server to mitigate the potential for a MITM attack or a malicious interception of the access token. The scheme also has to assume a degree of trust in the resource server it is presenting an access token to, particularly when the access token grants more than just access to the target resource server, because in a bearer based authorization scheme, anyone who possesses the access token has authority to what it grants. Bearer based access tokens also suffer from the threat of replay attacks.

Improved schemes around authorization protocols often involve adding a layer of proof of cryptographic key possession to the presentation of an access token, which mitigates the deficiencies highlighted

above as well as providing a way to detect a replay attack. However, approaches that involve proof of cryptographic key possession such as DPOP (<https://datatracker.ietf.org/doc/html/draft-ietf-oauth-dpop-04>) suffer from an increase in protocol complexity. A party requesting authorization must pre-generate appropriate key material, share the public portion of this with the authorization server alongside proving possession of the private portion of the key material. The authorization server must also be able to accommodate receiving this information and validating it.

BBS Signatures offer an alternative model that solves the same problems that proof of cryptographic key possession schemes do for bearer based schemes, but in a way that doesn't introduce new up-front protocol complexity. In the context of a protocol like OAuth2.0 the access token issued by the authorization server would feature a BBS Signature, however instead of the client providing this access token as issued, in their request to a resource server, they generate a unique proof from the original access token and include that in the request instead. Because the access token is not shared in a request to a resource server, attacks such as MITM are mitigated. A resource server also obtains the ability to detect a replay attack by ensuring the proof presented is unique.

### **B.3. Selectively Disclosure Enabled Identity Credentials**

BBS signatures when applied to the problem space of identity credentials can help to enhance user privacy. For example a digital drivers license that is cryptographically signed with a BBS signature, allows the holder or subject of the license to disclose different claims from their drivers license to different parties. Furthermore, the unlinkable presentations property of proofs generated by the scheme remove an important possible source of correlation for the holder across multiple presentations.

## **Appendix C. Additional BLS12-381 Ciphersuite Test Vectors**

**NOTE** These fixtures are a work in progress and subject to change

### **C.1. Modified Message Signature**

Using the following message

```
{{ $signatureFixtures.signature002.messages[0] }}
```

And the following signature

```
{{ $signatureFixtures.signature002.signature }}
```

Along with the PK value as defined in [Section 6.2.1.2](#) as inputs into the Verify operation should fail signature validation due to the message value being different from what was signed

### **C.2. Extra Unsigned Message Signature**

Using the following messages

```
{{ $signatureFixtures.signature003.messages[0] }}
```

```
{{ $signatureFixtures.signature003.messages[1] }}
```

And the following signature

```
{{ $signatureFixtures.signature002.signature }}
```

Along with the PK value as defined in [Section 6.2.1.2](#) as inputs into the Verify operation should fail signature validation due to an additional message being supplied that was not signed

### **C.3. Missing Message Signature**

Using the following messages

```
{{ $signatureFixtures.signature005.messages[0] }}
```

```
{{ $signatureFixtures.signature005.messages[1] }}
```

And the following signature

```
{{ $signatureFixtures.signature005.signature }}
```

Along with the PK value as defined in [Section 6.2.1.2](#) as inputs into the Verify operation should fail signature validation due to missing messages that were originally present during the signing

### **C.4. Reordered Message Signature**

Using the following messages

```
{{ $signatureFixtures.signature006.messages[0] }}  
{{ $signatureFixtures.signature006.messages[1] }}  
{{ $signatureFixtures.signature006.messages[2] }}  
{{ $signatureFixtures.signature006.messages[3] }}  
{{ $signatureFixtures.signature006.messages[4] }}  
{{ $signatureFixtures.signature006.messages[5] }}  
{{ $signatureFixtures.signature006.messages[6] }}  
{{ $signatureFixtures.signature006.messages[7] }}  
{{ $signatureFixtures.signature006.messages[8] }}  
{{ $signatureFixtures.signature006.messages[9] }}
```

And the following signature

```
{{ $signatureFixtures.signature006.signature }}
```

Along with the PK value as defined in [Section 6.2.1.2](#) as inputs into the Verify operation should fail signature validation due to messages being re-ordered from the order in which they were signed

### **C.5. Wrong Public Key Signature**

Using the following messages

```
{{ $signatureFixtures.signature007.messages[0] }}  
{{ $signatureFixtures.signature007.messages[1] }}  
{{ $signatureFixtures.signature007.messages[2] }}  
{{ $signatureFixtures.signature007.messages[3] }}  
{{ $signatureFixtures.signature007.messages[4] }}  
{{ $signatureFixtures.signature007.messages[5] }}  
{{ $signatureFixtures.signature007.messages[6] }}  
{{ $signatureFixtures.signature007.messages[7] }}  
{{ $signatureFixtures.signature007.messages[8] }}  
{{ $signatureFixtures.signature007.messages[9] }}
```

And the following signature

```
{{ $signatureFixtures.signature007.signature }}
```

Along with the PK value as defined in [Section 6.2.1.2](#) as inputs into the Verify operation should fail signature validation due to public key used to verify is in-correct

## Appendix D. Proof Generation and Verification Algorithmic Explanation

The following section provides an explanation of how the ProofGen and ProofVerify operations work.

Let the prover be in possession of a BBS signature  $(A, e, s)$  on messages  $msg_1, \dots, msg_L$  and a domain value (see [Sign](#)). Let  $A = B * (1/(e + SK))$  where SK the signer's secret key and,

$$B = P1 + Q_1 * s + Q_2 * domain + H_1 * msg_1 + \dots + H_L * msg_L$$

Let  $(i1, \dots, iR)$  be the indexes of generators corresponding to messages the prover wants to disclose and  $(j1, \dots, jU)$  be the indexes corresponding to undisclosed messages (i.e.,  $(j1, \dots, jU) = \text{range}(1, L) \setminus (i1, \dots, iR)$ ). To prove knowledge of a signature on the disclosed messages, work as follows,

\*Hide the signature by randomizing it. To randomize the signature  $(A, e, s)$ , take uniformly random  $r1, r2$  in  $[1, r-1]$ , and calculate,

1.  $A' = A * r1$ ,
2.  $Abar = A' * (-e) + B * r1$
3.  $D = B * r1 + H0 * r2$ .

Also set,

4.  $r3 = r1^{-1} \text{ mod } r$
5.  $s' = r2 * r3 + s \text{ mod } r$ .

The values  $(A', Abar, D)$  will be part of the proof and are used to prove possession of a BBS signature, without revealing the signature itself. Note that;  $e(A', PK) = e(Abar, P2)$  where PK the signer's public key and P2 the base element in G2 (used to create the signer's PK, see [SkToPk](#)). This also serves to bind the proof to the signer's PK.

\*Set the following,

1.  $C1 = Abar - D$
2.  $C2 = P1 + Q_2 * domain + H_{i1} * msg_{i1} + \dots + H_{iR} * msg_{iR}$



Create a non-interactive zero-knowledge proof-of-knowledge (nizk) of the values  $e$ ,  $r_2$ ,  $r_3$ ,  $s'$  and  $\text{msg\_j1}$ , ...,  $\text{msg\_jU}$  (the undisclosed messages) so that both of the following equalities hold,

$$\text{EQ1. } C_1 = A' * (-e) - H_0 * r_2$$

$$\text{EQ2. } C_2 = H_0 * s' - D * r_3 + H_{j1} * \text{msg\_j1} + \dots + H_{jU} * \text{msg\_jU}.$$

Note that the verifier will know the elements in the left side of the above equations (i.e.,  $C_1$  and  $C_2$ ) but not in the right side (i.e.,  $s'$ ,  $r_3$  and the undisclosed messages:  $\text{msg\_j1}$ , ...,  $\text{msg\_jU}$ ). However, using the nizk, the prover can convince the verifier that they (the prover) know the elements that satisfy those equations, without disclosing them. Then, if both EQ1 and EQ2 hold, and  $e(A', PK) = e(\text{Abar}, P_2)$ , an extractor can return a valid BBS signature from the signer's SK, on the disclosed messages. The proof returned is  $(A', \text{Abar}, D, \text{nizk})$ . To validate the proof, a verifier checks that  $e(A', PK) = e(\text{Abar}, P_2)$  and verifies the nizk. Validating the proof, will guarantee the authenticity and integrity of the disclosed messages, as well as ownership of the undisclosed messages and of the signature.

## Appendix E. Document History

-00

\*Initial version

-01

\*Populated fixtures

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