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Authors: M. Abdalla
DFINITY - Zurich
B. Haase
Endress + Hauser Liquid Analysis - Gerlingen
J. Hesse
IBM Research Europe - Zurich

CPace, a balanced composable PAKE

Abstract

This document describes CPace which is a protocol that allows two parties that share a low-entropy secret (password) to derive a strong shared key without disclosing the secret to offline dictionary attacks. The CPace protocol was tailored for constrained devices, is compatible with any cyclic group of prime- and non-prime order.

Discussion Venues

This note is to be removed before publishing as an RFC.

Discussion of this document takes place on the Crypto Forum Research Group mailing list (cfrg@ietf.org), which is archived at https://mailarchive.ietf.org/arch/search/?email_list=cfrg.

Source for this draft and an issue tracker can be found at https://github.com/cfrg/draft-irtf-cfrg-cpace.

Status of This Memo

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1. Introduction

This document describes CPace which is a balanced Password-Authenticated-Key-Establishment (PAKE) protocol for two parties where both parties derive a cryptographic key of high entropy from a shared secret of low-entropy. CPace protects the passwords against offline dictionary attacks by requiring adversaries to actively interact with a protocol party and by allowing for at most one single password guess per active interaction.

The CPace design was tailored considering the following main objectives:

*Efficiency: Deployment of CPace is feasible on resource-constrained devices.

*Versatility: CPace supports different application scenarios via versatile input formats, and by supporting applications with and without clear initiator and responder roles.

*Implementation error resistance: CPace avoids common implementation pitfalls already by-design, and it does not offer incentives for insecure speed-ups. For smooth integration into different cryptographic library ecosystems, this document provides a variety of cipher suites.

*Post-quantum annoyance: CPace comes with mitigations with respect to adversaries that become capable of breaking the discrete logarithm problem on elliptic curves.

1.1. Outline of this document

*Section 3 describes the expected properties of an application using CPace, and discusses in particular which application-level aspects are relevant for CPace's security.

*Section 4 gives an overview over the recommended cipher suites for CPace which were optimized for different types of cryptographic library ecosystems.

*Section 5 introduces the notation used throughout this document.

*Section 6 specifies the CPace protocol.

*The final section provides explicit reference implementations and test vectors of all of the functions defined for CPace in the appendix.

As this document is primarily written for implementers and application designers, we would like to refer the theory-inclined reader to the scientific paper [AHH21] which covers the detailed security analysis of the different CPace instantiations as defined in this document via the cipher suites.

2. Requirements Notation

The key words "MUST", "MUST NOT", "REQUIRED", "SHALL", "SHALL NOT", "SHOULD", "SHOULD NOT", "RECOMMENDED", "NOT RECOMMENDED", "MAY", and "OPTIONAL" in this document are to be interpreted as described in BCP 14 [RFC2119] [RFC8174] when, and only when, they appear in all capitals, as shown here.

3. High-level application perspective

CPace enables balanced password-authenticated key establishment. CPace requires a shared secret octet string, the password-related string (PRS), is available for both parties A and B. PRS can be a low-entropy secret itself, for instance a clear-text password encoded according to [RFC8265], or any string derived from a common secret, for instance by use of a password-based key derivation function.

Applications with clients and servers where the server side is storing account and password information in its persistent memory are recommended to use augmented PAKE protocols such as OPAQUE [I-D.irtf-cfrg-opaque].

In the course of the CPace protocol, A sends one message MSGa to B and B sends one message MSGb to A. CPace does not mandate any ordering of these two messages. We use the term "initiator-responder" for CPace where A always speaks first, and the term "symmetric" setting where anyone can speak first.

CPace's output is an intermediate session key (ISK), but any party might abort in case of an invalid received message. A and B will produce the same ISK value only if both sides did initiate the

protocol using the same protocol inputs, specifically the same PRS string and the same value for the optional input parameters CI, ADa, ADb and sid that will be specified in the upcoming sections.

The naming of ISK key as "intermediate" session key highlights the fact that it is RECOMMENDED that applications process ISK by use of a suitable strong key derivation function KDF (such as defined in [RFC5869]) before using the key in a higher-level protocol.

3.1. Optional CPace inputs

For accomodating different application settings, CPace offers the following OPTIONAL inputs, i.e. inputs which MAY also be the empty string:

* Channel identifier (CI). CI can be used to bind a session key exchanged with CPace to a specific networking channel which interconnects the protocol parties. Both parties are required to have the same view of CI. CI will not be publicly sent on the wire and may also include confidential information.

* Associated data fields (ADa and ADb). These fields can be used to authenticate public associated data alongside the CPace protocol. The values ADa (and ADb, respectively) are guaranteed to be authenticated in case both parties agree on a key.

  ADa and ADb can for instance include party identities or protocol version information of an application protocol (e.g. to avoid downgrade attacks).

  If party identities are not encoded as part of CI, party identities SHOULD be included in ADa and ADB (see Section 9.1). In a setting with clear initiator and responder roles, identity information in ADa sent by the initiator can be used by the responder for choosing the right PRS string (respectively password) for this identity.

* Session identifier (sid). CPace comes with a security analysis [AHH21] in the framework of universal composability. This framework allows for modular analysis of a larger application protocol which uses CPace as a building block. For such analysis the CPace protocol is bound to a specific session of the larger protocol by use of a sid string that is globally unique. As a result, when used with a unique sid, CPace instances remain secure when running concurrently with other CPace instances, and even arbitrary other protocols.

  For this reason, it is RECOMMENDED that applications establish a unique session identifier sid prior to running the CPace protocol. This can be implemented by concatenating random bytes

produced by A with random bytes produced by B. If such preceding round is not an option but parties are assigned clear initiator-responder roles, it is RECOMMENDED to let the initiator A choose a fresh random sid and send it to B together with the first message. If a sid string is used it SHOULD HAVE a length of at least 8 bytes.

3.2. Responsibilities of the application layer

The following tasks are out of the scope of this document and left to the application layer

* Setup phase:

  - The application layer is responsible for the handshake that makes parties agree on a common CPace cipher suite.

  - The application layer needs to specify how to encode the CPace byte strings Ya/Yb and ADa/ADb defined in section Section 6 for transfer over the network. For CPace it is RECOMMENDED to encode network messages by using MSGa = lv_cat(Ya,ADa) and MSGb = lv_cat(Yb,ADb) using the length-value concatenation function lv_cat speficied in Section 5.3. This document provides test vectors for lv_cat-encoded messages. Alternative network encodings, e.g., the encoding method used for the client hello and server hello messages of the TLS protocol, MAY be used when considering the guidance given in Section 9.

* This document does not specify which encodings applications use for the mandatory PRS input and the optional inputs CI, sid, ADa and ADb. If PRS is a clear-text password or an octet string derived from a clear-text password, e.g. by use of a key-derivation function, the clear-text password SHOULD BE encoded according to [RFC8265].

* The application needs to settle whether CPace is used in the initiator-responder or the symmetric setting, as in the symmetric setting transcripts must be generated using ordered string concatenation. In this document we will provide test vectors for both, initiator-responder and symmetric settings.

4. CPace cipher suites

In the setup phase of CPace, both communication partners need to agree on a common cipher suite. Cipher suites consist of a combination of a hash function H and an elliptic curve environment G.

For naming cipher suites we use the convention "CPACE-G-H". We RECOMMEND the following cipher suites:

- CPACE-X25519-SHA512. This suite uses the group environment G_X25519 defined in Section 7.2 and SHA-512 as hash function. This cipher suite comes with the smallest messages on the wire and a low computational cost.

- CPACE-P256_XMD:SHA-256_SSWU_NU_-SHA256. This suite instantiates the group environment G as specified in Section 7.4 using the encode_to_curve function P256_XMD:SHA-256_SSWU_NU_ from [I-D.irtf-cfrg-hash-to-curve] on curve NIST-P256, and hash function SHA-256.

The following RECOMMENDED cipher suites provide higher security margins.

- CPACE-X448-SHAKE256. This suite uses the group environment G_X448 defined in Section 7.2 and SHAKE-256 as hash function.

- CPACE-P384_XMD:SHA-384_SSWU_NU_-SHA384. This suite instantiates G as specified in Section 7.4 using the encode_to_curve function P384_XMD:SHA-384_SSWU_NU_ from [I-D.irtf-cfrg-hash-to-curve] on curve NIST-P384 with H = SHA-384.

- CPACE-P521_XMD:SHA-512_SSWU_NU_-SHA512. This suite instantiates G as specified in Section 7.4 using the encode_to_curve function P521_XMD:SHA-384_SSWU_NU_ from [I-D.irtf-cfrg-hash-to-curve] on curve NIST-P384 with H = SHA-512.

CPace can also securely be implemented using the cipher suites CPACE-RISTR255-SHA512 and CPACE-DECAF448-SHAKE256 defined in Section 7.3. Section 9 gives guidance on how to implement CPace on further elliptic curves.

5. Definitions and notation

5.1. Hash function H

Common choices for H are SHA-512 [RFC6234] or SHAKE-256 [FIPS202]. (I.e. the hash function outputs octet strings, and not group elements.) For considering both variable-output-length hashes and fixed-output-length hashes, we use the following convention. In case that the hash function is specified for a fixed-size output, we define H.hash(m,l) such that it returns the first l octets of the output.

We use the following notation for referring to the specific properties of a hash function H:

- H.hash(m,l) is a function that operates on an input octet string m and returns a hashing result of l octets.

- H.b_in_bytes denotes the default output size in bytes corresponding to the symmetric security level of the hash function. E.g. H.b_in_bytes = 64 for SHA-512 and SHAKE-256 and H.b_in_bytes = 32 for SHA-256 and SHAKE-128. We use the notation H.hash(m) = H.hash(m, H.b_in_bytes) and let the hash operation output the default length if no explicit length parameter is given.

- H.bmax_in_bytes denotes the *maximum* output size in octets supported by the hash function. In case of fixed-size hashes such as SHA-256, this is the same as H.b_in_bytes, while there is no such limit for hash functions such as SHAKE-256.

- H.s_in_bytes denotes the *input block size* used by H. For instance, for SHA-512 the input block size s_in_bytes is 128, while for SHAKE-256 the input block size amounts to 136 bytes.

5.2. Group environment G

The group environment G specifies an elliptic curve group (also denoted G for convenience) and associated constants and functions as detailed below. In this document we use multiplicative notation for the group operation.

- G.calculate_generator(H,PRS,CI,sid) denotes a function that outputs a representation of a generator (referred to as "generator" from now on) of the group which is derived from input octet strings PRS, CI, and sid and with the help of the hash function H.

- G.sample_scalar() is a function returning a representation of a scalar (referred to as "scalar" from now on) appropriate as a private Diffie-Hellman key for the group.

- G.scalar_mult(y,g) is a function operating on a scalar y and a group element g. It returns an octet string representation of the group element Y = g^y.

- G.I denotes a unique octet string representation of the neutral element of the group. G.I is used for detecting and signaling certain error conditions.

- G.scalar_mult_vfy(y,g) is a function operating on a scalar y and a group element g. It returns an octet string representation of

the group element g^y. Additionally, scalar_mult_vfy specifies validity conditions for y,g and g^y and outputs G.I in case they are not met.

*G.DSI denotes a domain-separation identifier string which SHALL be uniquely identifying the group environment G.

5.3. Notation for string operations

*bytes1 || bytes2 and denotes concatenation of octet strings.

*len(S) denotes the number of octets in a string S.

*nil denotes an empty octet string, i.e., len(nil) = 0.

*prepend_len(octet_string) denotes the octet sequence that is obtained from prepending the length of the octet string to the string itself. The length shall be prepended by using an LEB128 encoding of the length. This will result in a single-byte encoding for values below 128. (Test vectors and reference implementations for prepend_len and the LEB128 encodings are given in the appendix.)

*lv_cat(a0,a1, ...) is the "length-value" encoding function which returns the concatenation of the input strings with an encoding of their respective length prepended. E.g. lv_cat(a0,a1) returns prepend_len(a0) || prepend_len(a1). The detailed specification of lv_cat and a reference implementations are given in the appendix.

*network_encode(Y,AD) denotes the function specified by the application layer that outputs an octet string encoding of the input octet strings Y and AD for transfer on the network. The implementation of MSG = network_encode(Y,AD) SHALL allow the receiver party to parse MSG for the individual subcomponents Y and AD. For CPace we RECOMMEND to implement network_encode(Y,AD) as network_encode(Y,AD) = lv_cat(Y,AD).

Other prefix-free encodings, such as the network encoding used for the client-hello messages in TLS MAY also be used. We give guidance in the security consideration sections. but here the guidance given in the security consideration section MUST be considered.

*sample_random_bytes(n) denotes a function that returns n octets uniformly distributed between 0 and 255.

*zero_bytes(n) denotes a function that returns n octets with value 0.

*oCat(bytes1,bytes2) denotes ordered concatenation of octet strings, which places the lexiographically larger octet string first. (Explicit reference code for this function is given in the appendix.)

*transcript(MSGa,MSGb) denotes function outputing a string for the protocol transcript with messages MSGa and MSGb. In applications where CPace is used without clear initiator and responder roles, i.e. where the ordering of messages is not enforced by the protocol flow, transcript(MSGa,MSGb) = oCat(MSGa,MSGb) SHOULD be used. In the initiator-responder setting transcript(MSGa,MSGb) SHOULD BE implemented such that the later message is appended to the earlier message, i.e., transcript(MSGa,MSGb) = MSGa||MSGb if MSGa is sent first.

5.4. Notation for group operations

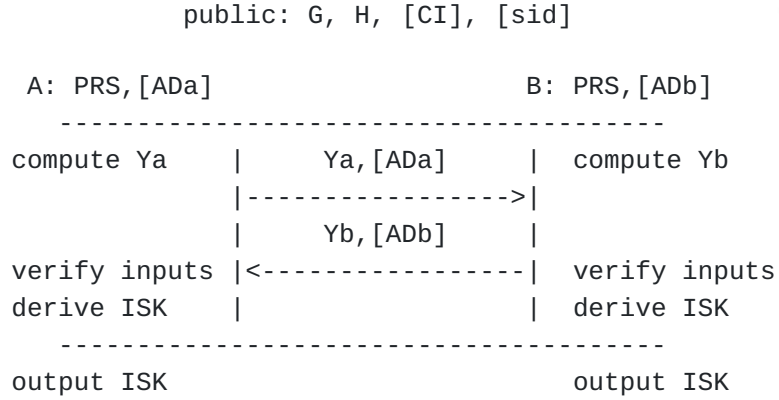
We use multiplicative notation for the group, i.e., X^2 denotes the element that is obtained by computing X*X, for group element X and group operation *.

6. The CPace protocol

CPace is a one round protocol between two parties, A and B. At invocation, A and B are provisioned with PRS,G,H and OPTIONAL CI,sid,ADa (for A) and CI,sid,ADb (for B). A sends a message MSGa to B. MSGa contains the public share Ya and OPTIONAL associated data ADa (i.e. an ADa field that MAY have a length of 0 bytes). Likewise, B sends a message MSGb to A. MSGb contains the public share Yb and OPTIONAL associated data ADb (i.e. an ADb field that MAY have a length of 0 bytes). Both A and B use the received messages for deriving a shared intermediate session key, ISK.

6.1. Protocol flow

Optional parameters and messages are denoted with [].



6.2. CPace protocol instructions

A computes a generator g = G.calculate_generator(H,PRS,CI,sid), scalar ya = G.sample_scalar() and group element Ya = G.scalar_mult (ya,g). A then transmits MSGa = network_encode(Ya, ADa) with optional associated data ADa to B.

B computes a generator g = G.calculate_generator(H,PRS,CI,sid), scalar yb = G.sample_scalar() and group element Yb = G.scalar_mult(yb,g). B sends MSGb = network_encode(Yb, ADb) with optional associated data ADb to A.

Upon reception of MSGa, B checks that MSGa was properly generated conform with the chosen encoding of network messages (notably correct length fields). If this parsing fails, then B MUST abort. (Testvectors of examples for invalid messages when using lv_cat() as network_encode function for CPace are given in the appendix.) B then computes K = G.scalar_mult_vfy(yb,Ya). B MUST abort if K=G.I. Otherwise B returns ISK = H.hash(prefix_free_cat(G.DSI || "_ISK", sid, K)||transcript(MSGa, MSGb)). B returns ISK and terminates.

Likewise upon reception of MSGb, A parses MSGb for Yb and ADb and checks for a valid encoding. If this parsing fails, then A MUST abort. A then computes K = G.scalar_mult_vfy(ya,Yb). A MUST abort if K=G.I. Otherwise A returns ISK = H.hash(lv_cat(G.DSI || "_ISK", sid, K) || transcript(MSGa, MSGb)). A returns ISK and terminates.

The session key ISK returned by A and B is identical if and only if the supplied input parameters PRS, CI and sid match on both sides and transcript view (containing of MSGa and MSGb) of both parties match.

Note that in case of a symmetric protocol execution without clear initiator/responder roles, ordered concatenation needs to be used for generating a matching view of the transcript by both parties.

7. Implementation of recommended CPace cipher suites

7.1. Common function for computing generators

The different cipher suites for CPace defined in the upcoming sections share the same method for deterministically combining the individual strings PRS, CI, sid and the domain-separation identifier DSI to a generator string that we describe here. Let CPACE-G-H denote the cipher suite.

- *generator_string(G.DSI, PRS, CI, sid, s_in_bytes) denotes a function that returns the string lv_cat(G.DSI, PRS, zero_bytes(len_zpad), CI, sid).

*len_zpad = MAX(0, s_in_bytes - len(prepend_len(PRS)) -
 len(prepend_len(G.DSI)) - 1)

The zero padding of length len_zpad is designed such that the encoding of G.DSI and PRS together with the zero padding field completely fills the first input block (of length s_in_bytes) of the hash. As a result the number of bytes to hash becomes independent of the actual length of the password (PRS). (A reference implementation and test vectors are provided in the appendix.)

The introduction of a zero-padding within the generator string also helps mitigating attacks of a side-channel adversary that analyzes correlations between publicly known variable information with the low-entropy PRS string. Note that the hash of the first block is intentionally made independent of session-specific inputs, such as sid or CI.

7.2. CPace group objects G_X25519 and G_X448 for single-coordinate Ladders on Montgomery curves

In this section we consider the case of CPace when using the X25519 and X448 Diffie-Hellman functions from [RFC7748] operating on the Montgomery curves Curve25519 and Curve448 [RFC7748]. CPace implementations using single-coordinate ladders on further Montgomery curves SHALL use the definitions in line with the specifications for X25519 and X448 and review the guidance given in Section 9.

For the group environment G_X25519 the following definitions apply:

*G_X25519.field_size_bytes = 32

*G_X25519.field_size_bits = 255

*G_X25519.sample_scalar() =
 sample_random_bytes(G.field_size_bytes)

*G_X25519.scalar_mult(y,g) = G.scalar_mult_vfy(y,g) = X25519(y,g)

*G_X25519.I = zero_bytes(G.field_size_bytes)

*G_X25519.DSI = "CPace255"

CPace cipher suites using G_X25519 MUST use a hash function producing at least H.b_max_in_bytes >= 32 bytes of output. It is RECOMMENDED to use G_X25519 in combination with SHA-512.

For X448 the following definitions apply:

*G_X448.field_size_bytes = 56

*G_X448.field_size_bits = 448

*G_X448.sample_scalar() = sample_random_bytes(G.field_size_bytes)

*G_X448.scalar_mult(y,g) = G.scalar_mult_vfy(y,g) = X448(y,g)

*G_X448.I = zero_bytes(G.field_size_bytes)

*G_X448.DSI = "CPace448"

CPace cipher suites using G_X448 MUST use a hash function producing at least H.b_max_in_bytes >= 56 bytes of output. It is RECOMMENDED to use G_X448 in combination with SHAKE-256.

For both G_X448 and G_X25519 the G.calculate_generator(H, PRS,sid,CI) function shall be implemented as follows.

*First gen_str = generator_string(G.DSI,PRS,CI,sid, H.s_in_bytes) SHALL BE calculated using the input block size of the chosen hash function.

*This string SHALL then BE hashed to the required length gen_str_hash = H.hash(gen_str, G.field_size_bytes). Note that this implies that the permissible output length H.maxb_in_bytes MUST BE larger or equal to the field size of the group G for making a hashing function suitable.

*This result is then considered as a field coordinate using the u = decodeUCoordinate(gen_str_hash, G.field_size_bits) function from [RFC7748] which we repeat in the appendix for convenience.

*The result point g is then calculated as (g,v) = map_to_curve_elligator2(u) using the function from [I-D.irtf-cfrg-hash-to-curve]. Note that the v coordinate produced by the map_to_curve_elligator2 function is not required for CPace and discarded. The appendix repeats the definitions from [I-D.irtf-cfrg-hash-to-curve] for convenience.

In the appendix we show sage code that can be used as reference implementation.

7.2.1. Verification tests

For single-coordinate Montgomery ladders on Montgomery curves verification tests according to Section 8 SHALL consider the u coordinate values that encode a low-order point on either, the curve or the quadratic twist.

In addition to that in case of G_X25519 the tests SHALL also verify that the implementation of G.scalar_mult_vfy(y,g) produces the

expected results for non-canonical u coordinate values with bit #255 set, which also encode low-order points.

Corresponding test vectors are provided in the appendix.

7.3. CPace group objects G_Ristretto255 and G_Decaf448 for prime-order group abstractions

In this section we consider the case of CPace using the Ristretto255 and Decaf448 group abstractions [I-D.draft-irtf-cfrg-ristretto255-decaf448]. These abstractions define an encode and decode function, group operations using an internal encoding and a one-way-map. With the group abstractions there is a distinction between an internal representation of group elements and an external encoding of the same group element. In order to distinguish between these different representations, we prepend an underscore before values using the internal representation within this section.

For Ristretto255 the following definitions apply:

- *G_Ristretto255.DSI = "CPaceRistretto255"

- *G_Ristretto255.field_size_bytes = 32

- *G_Ristretto255.group_size_bits = 252

- *G_Ristretto255.group_order = 2^252 +
  27742317777372353535851937790883648493

CPace cipher suites using G_Ristretto255 MUST use a hash function producing at least H.b_max_in_bytes >= 64 bytes of output. It is RECOMMENDED to use G_Ristretto255 in combination with SHA-512.

For decaf448 the following definitions apply:

- *G_Decaf448.DSI = "CPaceDecaf448"

- *G_Decaf448.field_size_bytes = 56

- *G_Decaf448.group_size_bits = 445

- *G_Decaf448.group_order = l = 2^446 -
  13818066809895115352007386748515426
  88033669247488217860989454750388 5

CPace cipher suites using G_Decaf448 MUST use a hash function producing at least H.b_max_in_bytes >= 112 bytes of output. It is RECOMMENDED to use G_Decaf448 in combination with SHAKE-256.

For both abstractions the following definitions apply:

* It is RECOMMENDED to implement G.sample_scalar() as follows.

  - Set scalar = sample_random_bytes(G.group_size_bytes).

  - Then clear the most significant bits larger than G.group_size_bits.

  - Interpret the result as the little-endian encoding of an integer value and return the result.

* Alternatively, if G.sample_scalar() is not implemented according to the above recommendation, it SHALL be implemented using uniform sampling between 1 and (G.group_order - 1). Note that the more complex uniform sampling process can provide a larger side-channel attack surface for embedded systems in hostile environments.

* G.scalar_mult(y,_g) SHALL operate on a scalar y and a group element _g in the internal representation of the group abstraction environment. It returns the value Y = encode((_g)^y), i.e. it returns a value using the public encoding.

* G.I = is the public encoding representation of the identity element.

* G.scalar_mult_vfy(y,X) operates on a value using the public encoding and a scalar and is implemented as follows. If the decode(X) function fails, it returns G.I. Otherwise it returns encode(decode(X)^y).

* The G.calculate_generator(H, PRS,sid,CI) function SHALL return a decoded point and SHALL BE implemented as follows.

  - First gen_str = generator_string(G.DSI,PRS,CI,sid, H.s_in_bytes) is calculated using the input block size of the chosen hash function.

  - This string is then hashed to the required length gen_str_hash = H.hash(gen_str, 2 * G.field_size_bytes). Note that this implies that the permissible output length H.maxb_in_bytes MUST BE larger or equal to twice the field size of the group G for making a hash function suitable.

  - Finally the internal representation of the generator _g is calculated as _g = one_way_map(gen_str_hash) using the one-way map function from the abstraction.

Note that with these definitions the scalar_mult function operates on a decoded point _g and returns an encoded point, while the scalar_mult_vfy(y,X) function operates on an encoded point X (and also returns an encoded point).

7.3.1. Verification tests

For group abstractions verification tests according to Section 8 SHALL consider encodings of the neutral element and an octet string that does not decode to a valid group element.

7.4. CPace group objects for curves in Short-Weierstrass representation

The group environment objects G defined in this section for use with Short-Weierstrass curves, are parametrized by the choice of an elliptic curve and by choice of a suitable encode_to_curve(str) function. encode_to_curve(str) must map an octet string str to a point on the curve.

7.4.1. Curves and associated functions

Elliptic curves in Short-Weierstrass form are considered in [IEEE1363]. [IEEE1363] allows for both, curves of prime and non-prime order. However, for the procedures described in this section any suitable group MUST BE of prime order.

The specification for the group environment objects specified in this section closely follow the ECKAS-DH1 method from [IEEE1363]. I.e. we use the same methods and encodings and protocol substeps as employed in the TLS [RFC5246] [RFC8446] protocol family.

For CPace only the uncompressed full-coordinate encodings from [SEC1] (x and y coordinate) SHOULD be used. Commonly used curve groups are specified in [SEC2] and [RFC5639]. A typical representative of such a Short-Weierstrass curve is NIST-P256. Point verification as used in ECKAS-DH1 is described in Annex A.16.10. of [IEEE1363].

For deriving Diffie-Hellman shared secrets ECKAS-DH1 from [IEEE1363] specifies the use of an ECSVDP-DH method. We use ECSVDP-DH in combination with the identy map such that it either returns "error" or the x-coordinate of the Diffie-Hellman result point as shared secret in big endian format (fixed length output by FE2OSP without truncating leading zeros).

7.4.2. Suitable encode_to_curve methods

All the encode_to_curve methods specified in [I-D.irtf-cfrg-hash-to-curve] are suitable for CPace. For Short-Weierstrass curves it is

RECOMMENDED to use the non-uniform variant of the SSWU mapping primitive from [I-D.irtf-cfrg-hash-to-curve] if a SSWU mapping is available for the chosen curve. (We recommend non-uniform maps in order to give implementations the flexibility to opt for x-coordinate-only scalar multiplication algorithms.)

7.4.3. Definition of the group environment G for Short-Weierstrass curves

In this paragraph we use the following notation for defining the group object G for a selected curve and encode_to_curve method:

* With group_order we denote the order of the elliptic curve which MUST BE a prime.

* With is_valid(X) we denote a method which operates on an octet stream according to [SEC1] of a point on the group and returns true if the point is valid or false otherwise. This is_valid(X) method SHALL be implemented according to Annex A.16.10. of [IEEE1363]. I.e. it shall return false if X encodes either the neutral element on the group or does not form a valid encoding of a point on the group.

* With encode_to_curve(str) we denote a selected mapping function from [I-D.irtf-cfrg-hash-to-curve]. I.e. a function that maps octet string str to a point on the group. [I-D.irtf-cfrg-hash-to-curve] considers both, uniform and non-uniform mappings based on several different strategies. It is RECOMMENDED to use the nonuniform variant of the SSWU mapping primitive within [I-D.irtf-cfrg-hash-to-curve].

* G.DSI denotes a domain-separation identifier string. G.DSI which SHALL BE obtained by the concatenation of "CPace" and the associated name of the cipher suite used for the encode_to_curve function as specified in [I-D.irtf-cfrg-hash-to-curve]. E.g. when using the map with the name "P384_XMD:SHA-384_SSWU_NU_" on curve NIST-P384 the resulting value SHALL BE G.DSI = "CPaceP384_XMD:SHA-384_SSWU_NU_".

Using the above definitions, the CPace functions required for the group object G are defined as follows.

* G.sample_scalar() SHALL return a value between 1 and (G.group_order - 1). The value sampling MUST BE uniformly random. It is RECOMMENDED to use rejection sampling for converting a uniform bitstring to a uniform value between 1 and (G.group_order - 1).

*G.calculate_generator(H, PRS,sid,CI) function SHALL be implemented as follows.

  -First gen_str = generator_string(G.DSI,PRS,CI,sid, H.s_in_bytes) is calculated.

  -Then the output of a call to encode_to_curve(gen_str) is returned, using the selected function from [I-D.irtf-cfrg-hash-to-curve].

*G.scalar_mult(s,X) is a function that operates on a scalar s and an input point X. The input X shall use the same encoding as produced by the G.calculate_generator method above. G.scalar_mult(s,X) SHALL return an encoding of either the point X^s or the point X^(-s) according to [SEC1]. Implementations SHOULD use the full-coordinate format without compression, as important protocols such as TLS 1.3 removed support for compression. Implementations of scalar_mult(s,X) MAY output either X^s or X^(-s) as both points X^s and X^(-s) have the same x-coordinate and result in the same Diffie-Hellman shared secrets K. (This allows implementations to opt for x-coordinate-only scalar multiplication algorithms.)

*G.scalar_mult_vfy(s,X) merges verification of point X according to [IEEE1363] A.16.10. and the the ECSVDP-DH procedure from [IEEE1363]. It SHALL BE implemented as follows:

  -If is_valid(X) = False then G.scalar_mult_vfy(s,X) SHALL return "error" as specified in [IEEE1363] A.16.10 and 7.2.1.

  -Otherwise G.scalar_mult_vfy(s,X) SHALL return the result of the ECSVDP-DH procedure from [IEEE1363] (section 7.2.1). I.e. it shall either return "error" (in case that X^s is the neutral element) or the secret shared value "z" (otherwise). "z" SHALL be encoded by using the big-endian encoding of the x-coordinate of the result point X^s according to [SEC1].

*We represent the neutral element G.I by using the representation of the "error" result case from [IEEE1363] as used in the G.scalar_mult_vfy method above.

7.4.4. Verification tests

For Short-Weierstrass curves verification tests according to Section 8 SHALL consider encodings of the point at infinity and an encoding of a point not on the group.

8. Implementation verification

Any CPace implementation MUST be tested against invalid or weak point attacks. Implementation MUST be verified to abort upon conditions where G.scalar_mult_vfy functions outputs G.I. For testing an implementation it is RECOMMENDED to include weak or invalid points in MSGa and MSGb and introduce this in a protocol run. It SHALL be verified that the abort condition is properly handled.

Moreover any implementation MUST be tested with respect invalid encodings of MSGa and MSGb where the length of the message does not match the specified encoding (i.e. where the sum of the prepended length information does not match the actual length of the message).

Corresponding test vectors are given in the appendix for all recommended cipher suites.

9. Security Considerations

A security proof of CPace is found in [AHH21]. This proof covers all recommended cipher suites included in this document. In the following sections we describe how to protect CPace against several attack families, such as relay-, length extension- or side channel attacks. We also describe aspects to consider when deviating from recommended cipher suites.

9.1. Party identifiers and relay attacks

If unique strings identifying the protocol partners are included either as part of the channel identifier CI, the session id sid or the associated data fields ADa, ADb, the ISK will provide implicit authentication also regarding the party identities. Incorporating party identifier strings is important for fending off relay attacks. Such attacks become relevant in a setting where several parties, say, A, B and C, share the same password PRS. An adversary might relay messages from a honest user A, who aims at interacting with user B, to a party C instead. If no party identifier strings are used, and B and C use the same PRS value, A might be establishing a common ISK key with C while assuming to interact with party B. Including and checking party identifiers can fend off such relay attacks.

9.2. Network message encoding and hashing protocol transcripts

It is RECOMMENDED to encode the (Ya,ADa) and (Yb,ADb) fields on the network by using network_encode(Y,AD) = lv_cat(Y,AD). I.e. we RECOMMEND to prepend an encoding of the length of the subfields. As the length of the variable-size input strings are prepended this results in a so-called prefix-free encoding of transcript strings,

using terminology introduced in [CDMP05]. This property allows for disregarding length-extension imperfections that come with the commonly used Merkle-Damgard hash function constructions such as SHA256 and SHA512.

Other alternative network encoding formats which prepend an encoding of the length of variable-size data fields in the protocol messages are equally suitable. This includes, e.g., the type-length-value format specified in the DER encoding standard (X.690) or the protocol message encoding used in the TLS protocol family for the TLS client-hello or server-hello messages.

In case that an application whishes to use another form of network message encoding which is not prefix-free, the guidance given in [CDMP05] SHOULD BE considered (e.g. by replacing hash functions with the HMAC constructions from[RFC2104]).

9.3. Key derivation

Although already K is a shared value, it MUST NOT itself be used as an application key. Instead, ISK MUST BE used. Leakage of K to an adversary can lead to offline dictionary attacks.

As noted already in Section 6 it is RECOMMENDED to process ISK by use of a suitable strong key derivation function KDF (such as defined in [RFC5869]) first, before using the key in a higher-level protocol.

9.4. Key confirmation

In many applications it is advisable to add an explicit key confirmation round after the CPace protocol flow. However, as some applications might only require implicit authentication and as explicit authentication messages are already a built-in feature in many higher-level protocols (e.g. TLS 1.3) the CPace protocol described here does not mandate use of a key confirmation on the level of the CPace sub-protocol.

Already without explicit key confirmation, CPace enjoys weak forward security under the sCDH and sSDH assumptions [AHH21]. With added explicit confirmation, CPace enjoys perfect forward security also under the strong sCDH and sSDH assumptions [AHH21].

Note that in [ABKLX21] it was shown that an idealized variant of CPace also enjoys perfect forward security without explicit key confirmation. However this proof does not explicitly cover the recommended cipher suites in this document and requires the stronger assumption of an algebraic adversary model. For this reason, we recommend adding explicit key confirmation if perfect forward security is required.

When implementing explicit key confirmation, it is recommended to use an appropriate message-authentication code (MAC) such as HMAC [RFC2104] or CMAC [RFC4493] using a key mac_key derived from ISK.

One suitable option that works also in the parallel setting without message ordering is to proceed as follows.

- First calculate mac_key as as mac_key = H.hash(b"CPaceMac" || ISK).

- Then let each party send an authenticator tag Ta, Tb that is calculated over the protocol message that it has sent previously. I.e. let party A calculate its transmitted authentication code Ta as Ta = MAC(mac_key, MSGa) and let party B calculate its transmitted authentication code Tb as Tb = MAC(mac_key, MSGb).

- Let the receiving party check the remote authentication tag for the correct value and abort in case that it's incorrect.

9.5. Sampling of scalars

For curves over fields F_p where p is a prime close to a power of two, we recommend sampling scalars as a uniform bit string of length field_size_bits. We do so in order to reduce both, complexity of the implementation and reducing the attack surface with respect to side-channels for embedded systems in hostile environments. The effect of non-uniform sampling on security was demonstrated to be begning in [AHH21] for the case of Curve25519 and Curve448. This analysis however does not transfer to most curves in Short-Weierstrass form. As a result, we recommend rejection sampling if G is as in Section 7.4.

9.6. Single-coordinate CPace on Montgomery curves

The recommended cipher suites for the Montgomery curves Curve25519 and Curve448 in Section 7.2 rely on the following properties [AHH21]:

- The curve has order (p * c) with p prime and c a small cofactor. Also the curve's quadratic twist must be of order (p' * c') with p' prime and c' a cofactor.

- The cofactor c' of the twist MUST BE EQUAL to or an integer multiple of the cofactor c of the curve.

- Both field order q and group order p MUST BE close to a power of two along the lines of [AHH21], Appendix E.

- The representation of the neutral element G.I MUST BE the same for both, the curve and its twist.

*The implementation of G.scalar_mult_vfy(y,X) MUST map all c low-order points on the curve and all c' low-order points on the twist to G.I.

Montgomery curves other than the ones recommended here can use the specifications given in Section 7.2, given that the above properties hold.

9.7. Nonce values

Secret scalars ya and yb MUST NOT be reused. Values for sid SHOULD NOT be reused since the composability guarantees established by the simulation-based proof rely on the uniqueness of session ids [AHH21].

If CPace is used in a concurrent system, it is RECOMMENDED that a unique sid is generated by the higher-level protocol and passed to CPace. One suitable option is that sid is generated by concatenating ephemeral random strings contributed by both parties.

9.8. Side channel attacks

All state-of-the art methods for realizing constant-time execution SHOULD be used. In case that side channel attacks are to be considered practical for a given application, it is RECOMMENDED to pay special attention on computing the secret generator G.calculate_generator(PRS,CI,sid). The most critical substep to consider might be the processing of the first block of the hash that includes the PRS string. The zero-padding introduced when hashing the sensitive PRS string can be expected to make the task for a side-channel attack somewhat more complex. Still this feature alone is not sufficient for ruling out power analysis attacks.

9.9. Quantum computers

CPace is proven secure under the hardness of the strong computational Simultaneous Diffie-Hellmann (sSDH) and strong computational Diffie-Hellmann (sCDH) assumptions in the group G (as defined in [AHH21]). These assumptions are not expected to hold any longer when large-scale quantum computers (LSQC) are available. Still, even in case that LSQC emerge, it is reasonable to assume that discrete-logarithm computations will remain costly. CPace with ephemeral session id values sid forces the adversary to solve one computational Diffie-Hellman problem per password guess [ES21]. In this sense, using the wording suggested by Steve Thomas on the CFRG mailing list, CPace is "quantum-annoying".

10. IANA Considerations

No IANA action is required.

11. Acknowledgements

Thanks to the members of the CFRG for comments and advice. Any comment and advice is appreciated.

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Appendix A. CPace function definitions

A.1. Definition and test vectors for string utility functions

A.1.1. prepend_len function

```
def prepend_len(data):
    "prepend LEB128 encoding of length"
    length = len(data)
    length_encoded = b""
    while True:
        if length < 128:
            length_encoded += bytes([length])
        else:
            length_encoded += bytes([(length & 0x7f) + 0x80])
        length = int(length >> 7)
        if length == 0:
            break;
    return length_encoded + data
```

A.1.2. prepend_len test vectors

```
prepend_len(b""): (length: 1 bytes)
  00
prepend_len(b"1234"): (length: 5 bytes)
  0431323334
prepend_len(bytes(range(127))): (length: 128 bytes)
  7f000102030405060708090a0b0c0d0e0f101112131415161718191a1b
  1c1d1e1f202122232425262728292a2b2c2d2e2f303132333435363738
  393a3b3c3d3e3f404142434445464748494a4b4c4d4e4f505152535455
  565758595a5b5c5d5e5f606162636465666768696a6b6c6d6e6f707172
  737475767778797a7b7c7d7e
prepend_len(bytes(range(128))): (length: 130 bytes)
  8001000102030405060708090a0b0c0d0e0f101112131415161718191a
  1b1c1d1e1f202122232425262728292a2b2c2d2e2f3031323334353637
  38393a3b3c3d3e3f404142434445464748494a4b4c4d4e4f5051525354
  55565758595a5b5c5d5e5f606162636465666768696a6b6c6d6e6f7071
  72737475767778797a7b7c7d7e7f
```

A.1.3. lv_cat function

```
def lv_cat(*args):
    result = b""
    for arg in args:
        result += prepend_len(arg)
    return result
```

A.1.4. Testvector for lv_cat()

```
lv_cat(b"1234",b"5",b"",b"6789"): (length: 13 bytes)
  043132333401350004363738 39
```

A.1.5. Examples for messages not obtained from a lv_cat-based encoding

The following messages are examples which have invalid encoded length fields. I.e. they are examples where parsing for the sum of the length of subfields as expected for a message generated using lv_cat(Y,AD) does not give the correct length of the message. Parties MUST abort upon reception of such invalid messages as MSGa or MSGb.

```
Inv_MSG1 not encoded by lv_cat: (length: 3 bytes)
  ffffff
Inv_MSG2 not encoded by lv_cat: (length: 3 bytes)
  ffff03
Inv_MSG3 not encoded by lv_cat: (length: 4 bytes)
  00ffff03
Inv_MSG4 not encoded by lv_cat: (length: 4 bytes)
  00ffffff
```

A.2. Definition of generator_string function.

```
def generator_string(DSI,PRS,CI,sid,s_in_bytes):
    # Concat all input fields with prepended length information.
    # Add zero padding in the first hash block after DSI and PRS.
    len_zpad = max(0,s_in_bytes - 1 - len(prepend_len(PRS))
                    - len(prepend_len(DSI)))
    return lv_cat(DSI, PRS, zero_bytes(len_zpad),
                          CI, sid)
```

A.3. Definitions and test vector ordered concatenation

A.3.1. Definitions for lexiographical ordering

For ordered concatenation lexiographical ordering of byte sequences is used:

```
def lexiographically_larger(bytes1,bytes2):
   "Returns True if bytes1 > bytes2 using lexiographical ordering."
   min_len = min (len(bytes1), len(bytes2))
   for m in range(min_len):
       if bytes1[m] > bytes2[m]:
           return True;
       elif bytes1[m] < bytes2[m]:
           return False;
   return len(bytes1) > len(bytes2)
```

A.3.2. Definitions for ordered concatenation

With the above definition of lexiographical ordering ordered concatenation is specified as follows.

```
def oCAT(bytes1,bytes2):
    if lexiographically_larger(bytes1,bytes2):
        return bytes1 + bytes2
    else:
        return bytes2 + bytes1
```

A.3.3. Test vectors ordered concatenation

```
  string comparison for oCAT:
    lexiographically_larger(b"\0", b"\0\0") == False
    lexiographically_larger(b"\1", b"\0\0") == True
    lexiographically_larger(b"\0\0", b"\0") == True
    lexiographically_larger(b"\0\0", b"\1") == False
    lexiographically_larger(b"\0\1", b"\1") == False
    lexiographically_larger(b"ABCD", b"BCD") == False

  oCAT(b"ABCD",b"BCD"): (length: 7 bytes)
    42434441424344
  oCAT(b"BCD",b"ABCDE"): (length: 8 bytes)
    4243444142434445
```

A.4. Decoding and Encoding functions according to RFC7748

```
   def decodeLittleEndian(b, bits):
       return sum([b[i] << 8*i for i in range((bits+7)/8)])

   def decodeUCoordinate(u, bits):
       u_list = [ord(b) for b in u]
       # Ignore any unused bits.
       if bits % 8:
           u_list[-1] &= (1<<(bits%8))-1
       return decodeLittleEndian(u_list, bits)

   def encodeUCoordinate(u, bits):
       return ''.join([chr((u >> 8*i) & 0xff)
                       for i in range((bits+7)/8)])
```

A.5. Elligator 2 reference implementation

The Elligator 2 map requires a non-square field element Z which shall be calculated as follows.

```
    def find_z_ell2(F):
        # Find nonsquare for Elligator2
        # Argument: F, a field object, e.g., F = GF(2^255 - 19)
        ctr = F.gen()
        while True:
            for Z_cand in (F(ctr), F(-ctr)):
                # Z must be a non-square in F.
                if is_square(Z_cand):
                    continue
                return Z_cand
            ctr += 1
```

The values of the non-square Z only depend on the curve. The algorithm above results in a value of Z = 2 for Curve25519 and Z=-1 for Ed448.

The following code maps a field element r to an encoded field element which is a valid u-coordinate of a Montgomery curve with curve parameter A.

```
 def elligator2(r, q, A, field_size_bits):
     # Inputs: field element r, field order q,
     #         curve parameter A and field size in bits
     Fq = GF(q); A = Fq(A); B = Fq(1);

     # get non-square z as specified in the hash2curve draft.
     z = Fq(find_z_ell2(Fq))
     powerForLegendreSymbol = floor((q-1)/2)

     v = - A / (1 + z * r^2)
     epsilon = (v^3 + A * v^2 + B * v)^powerForLegendreSymbol
     x = epsilon * v - (1 - epsilon) * A/2
     return encodeUCoordinate(Integer(x), field_size_bits)
```

Appendix B. Test vectors

B.1. Test vector for CPace using group X25519 and hash SHA-512

B.1.1. Test vectors for calculate_generator with group X25519

```
  Inputs
    H   = SHA-512 with input block size 128 bytes.
    PRS = b'Password' ; ZPAD length: 109 ; DSI = b'CPace255'
    CI = b'\nAinitiator\nBresponder'
    CI = 0a41696e69746961746f720a42726573706f6e646572
    sid = 7e4b4791d6a8ef019b936c79fb7f2c57
  Outputs
    generator_string(G.DSI,PRS,CI,sid,H.s_in_bytes):
    (length: 168 bytes)
      0843506163653235350850617373776f72646d00000000000000000
      0000000000000000000000000000000000000000000000000000000
      0000000000000000000000000000000000000000000000000000000
      0000000000000000000000000000000000000000000000000000000
      0000000000000000000000000000000160a41696e69746961746f72
      0a42726573706f6e646572107e4b4791d6a8ef019b936c79fb7f2c57
    hash generator string: (length: 32 bytes)
      10047198e8c4cacf0ab8a6d0ac337b8ae497209d042f7f3a50945863
      94e821fc
    decoded field element of 255 bits: (length: 32 bytes)
      10047198e8c4cacf0ab8a6d0ac337b8ae497209d042f7f3a50945863
      94e8217c
    generator g: (length: 32 bytes)
      4e6098733061c0e8486611a904fe5edb049804d26130a44131a6229e
      55c5c321
```

B.1.2. Test vector for MSGa

```
  Inputs
    ADa = b'ADa'
    ya (little endian): (length: 32 bytes)
      45acf93116ae5d3dae995a7c627df2924321a8e857d9a200807131e3
      8839b0c2
  Outputs
    Ya: (length: 32 bytes)
      6f7fd31863b18b0cc9830fc842c60dea80120ccf2fd375498225e45a
      52065361
    MSGa = lv_cat(Ya,ADa): (length: 37 bytes)
      206f7fd31863b18b0cc9830fc842c60dea80120ccf2fd375498225e4
      5a5206536103414461
```

B.1.3. Test vector for MSGb

```
  Inputs
    ADb = b'ADb'
    yb (little endian): (length: 32 bytes)
      a145e914b347002d298ce2051394f0ed68cf3623dfe5db082c78ffa5
      a667acdc
  Outputs
    Yb: (length: 32 bytes)
      e1b730a4956c0f853d96c5d125cebeeea46952c07c6f66da65bd9ffd
      2f71a462
    MSGb = lv_cat(Yb,ADb): (length: 37 bytes)
      20e1b730a4956c0f853d96c5d125cebeeea46952c07c6f66da65bd9f
      fd2f71a46203414462
```

B.1.4. Test vector for secret points K

```
    scalar_mult_vfy(ya,Yb): (length: 32 bytes)
      2a905bc5f0b93ee72ac4b6ea8723520941adfc892935bf6f86d9e199
      befa6024
    scalar_mult_vfy(yb,Ya): (length: 32 bytes)
      2a905bc5f0b93ee72ac4b6ea8723520941adfc892935bf6f86d9e199
      befa6024
```

B.1.5. Test vector for ISK calculation initiator/responder

```
    unordered cat of transcript : (length: 74 bytes)
      206f7fd31863b18b0cc9830fc842c60dea80120ccf2fd375498225e4
      5a52065361034144612 0e1b730a4956c0f853d96c5d125cebeeea469
      52c07c6f66da65bd9ffd2f71a46203414462
    DSI = G.DSI_ISK, b'CPace255_ISK': (length: 12 bytes)
      435061636532353 55f49534b
    lv_cat(DSI,sid,K)||MSGa||MSGb: (length: 137 bytes)
      0c435061636532353 55f49534b107e4b4791d6a8ef019b936c79fb7f
      2c57202a905bc5f0b93ee72ac4b6ea8723520941adfc892935bf6f86
      d9e199befa6024206f7fd31863b18b0cc9830fc842c60dea80120ccf
      2fd375498225e45a52065361034144612 0e1b730a4956c0f853d96c5
      d125cebeeea46952c07c6f66da65bd9ffd2f71a46203414462
    ISK result: (length: 64 bytes)
      99a9e0ff35acb94ad8af1cd6b32ac409dc7d00557ccd9a7d19d3b462
      9e5f1f084f9332096162438c7ecc78331b4eda17e1a229a47182eccc
      9ea58cd9cdcd8e9a
```

B.1.6. Test vector for ISK calculation parallel execution

```
ordered cat of transcript : (length: 74 bytes)
  20e1b730a4956c0f853d96c5d125cebeeea46952c07c6f66da65bd9f
  fd2f71a46203414462206f7fd31863b18b0cc9830fc842c60dea8012
  0ccf2fd375498225e45a5206536103414461
DSI = G.DSI_ISK, b'CPace255_ISK': (length: 12 bytes)
  435061636532353 55f49534b
lv_cat(DSI,sid,K)||oCAT(MSGa,MSGb): (length: 137 bytes)
  0c435061636532353 55f49534b107e4b4791d6a8ef019b936c79fb7f
  2c57202a905bc5f0b93ee72ac4b6ea8723520941adfc892935bf6f86
  d9e199befa602420e1b730a4956c0f853d96c5d125cebeeea46952c0
  7c6f66da65bd9ffd2f71a46203414462206f7fd31863b18b0cc9830f
  c842c60dea80120ccf2fd375498225e45a5206536103414461
ISK result: (length: 64 bytes)
  3cd6a9670fa3ff211d829b845baa0f5ba9ad580c3ba0ee790bd0e9cd
  556290a8ffce44419fbf94e4cb8e7fe9f454fd25dc13e689e4d6ab0a
  c2211c70a8ac0062
```

B.1.7. Corresponding ANSI-C initializers

```
const uint8_t tc_PRS[] = {
 0x50,0x61,0x73,0x73,0x77,0x6f,0x72,0x64,
};
const uint8_t tc_CI[] = {
 0x0a,0x41,0x69,0x6e,0x69,0x74,0x69,0x61,0x74,0x6f,0x72,0x0a,
 0x42,0x72,0x65,0x73,0x70,0x6f,0x6e,0x64,0x65,0x72,
};
const uint8_t tc_sid[] = {
 0x7e,0x4b,0x47,0x91,0xd6,0xa8,0xef,0x01,0x9b,0x93,0x6c,0x79,
 0xfb,0x7f,0x2c,0x57,
};
const uint8_t tc_g[] = {
 0x4e,0x60,0x98,0x73,0x30,0x61,0xc0,0xe8,0x48,0x66,0x11,0xa9,
 0x04,0xfe,0x5e,0xdb,0x04,0x98,0x04,0xd2,0x61,0x30,0xa4,0x41,
 0x31,0xa6,0x22,0x9e,0x55,0xc5,0xc3,0x21,
};
const uint8_t tc_ya[] = {
 0x45,0xac,0xf9,0x31,0x16,0xae,0x5d,0x3d,0xae,0x99,0x5a,0x7c,
 0x62,0x7d,0xf2,0x92,0x43,0x21,0xa8,0xe8,0x57,0xd9,0xa2,0x00,
 0x80,0x71,0x31,0xe3,0x88,0x39,0xb0,0xc2,
};
const uint8_t tc_ADa[] = {
 0x41,0x44,0x61,
};
const uint8_t tc_Ya[] = {
 0x6f,0x7f,0xd3,0x18,0x63,0xb1,0x8b,0x0c,0xc9,0x83,0x0f,0xc8,
 0x42,0xc6,0x0d,0xea,0x80,0x12,0x0c,0xcf,0x2f,0xd3,0x75,0x49,
 0x82,0x25,0xe4,0x5a,0x52,0x06,0x53,0x61,
};
const uint8_t tc_yb[] = {
 0xa1,0x45,0xe9,0x14,0xb3,0x47,0x00,0x2d,0x29,0x8c,0xe2,0x05,
 0x13,0x94,0xf0,0xed,0x68,0xcf,0x36,0x23,0xdf,0xe5,0xdb,0x08,
 0x2c,0x78,0xff,0xa5,0xa6,0x67,0xac,0xdc,
};
const uint8_t tc_ADb[] = {
 0x41,0x44,0x62,
};
const uint8_t tc_Yb[] = {
 0xe1,0xb7,0x30,0xa4,0x95,0x6c,0x0f,0x85,0x3d,0x96,0xc5,0xd1,
 0x25,0xce,0xbe,0xee,0xa4,0x69,0x52,0xc0,0x7c,0x6f,0x66,0xda,
 0x65,0xbd,0x9f,0xfd,0x2f,0x71,0xa4,0x62,
};
const uint8_t tc_K[] = {
 0x2a,0x90,0x5b,0xc5,0xf0,0xb9,0x3e,0xe7,0x2a,0xc4,0xb6,0xea,
 0x87,0x23,0x52,0x09,0x41,0xad,0xfc,0x89,0x29,0x35,0xbf,0x6f,
 0x86,0xd9,0xe1,0x99,0xbe,0xfa,0x60,0x24,
};
const uint8_t tc_ISK_IR[] = {
 0x99,0xa9,0xe0,0xff,0x35,0xac,0xb9,0x4a,0xd8,0xaf,0x1c,0xd6,
```

```
 0xb3,0x2a,0xc4,0x09,0xdc,0x7d,0x00,0x55,0x7c,0xcd,0x9a,0x7d,
 0x19,0xd3,0xb4,0x62,0x9e,0x5f,0x1f,0x08,0x4f,0x93,0x32,0x09,
 0x61,0x62,0x43,0x8c,0x7e,0xcc,0x78,0x33,0x1b,0x4e,0xda,0x17,
 0xe1,0xa2,0x29,0xa4,0x71,0x82,0xec,0xcc,0x9e,0xa5,0x8c,0xd9,
 0xcd,0xcd,0x8e,0x9a,
};
const uint8_t tc_ISK_SY[] = {
 0x3c,0xd6,0xa9,0x67,0x0f,0xa3,0xff,0x21,0x1d,0x82,0x9b,0x84,
 0x5b,0xaa,0x0f,0x5b,0xa9,0xad,0x58,0x0c,0x3b,0xa0,0xee,0x79,
 0x0b,0xd0,0xe9,0xcd,0x55,0x62,0x90,0xa8,0xff,0xce,0x44,0x41,
 0x9f,0xbf,0x94,0xe4,0xcb,0x8e,0x7f,0xe9,0xf4,0x54,0xfd,0x25,
 0xdc,0x13,0xe6,0x89,0xe4,0xd6,0xab,0x0a,0xc2,0x21,0x1c,0x70,
 0xa8,0xac,0x00,0x62,
};
```

B.1.8. Test vectors for G_X25519.scalar_mult_vfy: low order points

Test vectors for which G_X25519.scalar_mult_vfy(s_in,ux) must return the neutral element or would return the neutral element if bit #255 of field element representation was not correctly cleared. (The decodeUCoordinate function from RFC7748 mandates clearing bit #255 for field element representations for use in the X25519 function.).

```
u0: 0000000000000000000000000000000000000000000000000000000000000000
u1: 0100000000000000000000000000000000000000000000000000000000000000
u2: ecffffffffffffffffffffffffffffffffffffffffffffffffffffffffffff7f
u3: e0eb7a7c3b41b8ae1656e3faf19fc46ada098deb9c32b1fd866205165f49b800
u4: 5f9c95bca3508c24b1d0b1559c83ef5b04445cc4581c8e86d8224eddd09f1157
u5: edffffffffffffffffffffffffffffffffffffffffffffffffffffffffffff7f
u6: daffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffff
u7: eeffffffffffffffffffffffffffffffffffffffffffffffffffffffffffff7f
u8: dbffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffff
u9: d9ffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffff
ua: cdeb7a7c3b41b8ae1656e3faf19fc46ada098deb9c32b1fd866205165f49b880
ub: 4c9c95bca3508c24b1d0b1559c83ef5b04445cc4581c8e86d8224eddd09f11d7

u0 ... ub MUST be verified to produce the correct results q0 ... qb:

Additionally, u0,u1,u2,u3,u4,u5 and u7 MUST trigger the abort case
when included in MSGa or MSGb.

s = af46e36bf0527c9d3b16154b82465edd62144c0ac1fc5a18506a2244ba449aff
qN = G_X25519.scalar_mult_vfy(s, uX)
q0: 0000000000000000000000000000000000000000000000000000000000000000
q1: 0000000000000000000000000000000000000000000000000000000000000000
q2: 0000000000000000000000000000000000000000000000000000000000000000
q3: 0000000000000000000000000000000000000000000000000000000000000000
q4: 0000000000000000000000000000000000000000000000000000000000000000
q5: 0000000000000000000000000000000000000000000000000000000000000000
q6: d8e2c776bbacd510d09fd9278b7edcd25fc5ae9adfba3b6e040e8d3b71b21806
q7: 0000000000000000000000000000000000000000000000000000000000000000
q8: c85c655ebe8be44ba9c0ffde69f2fe10194458d137f09bbff725ce58803cdb38
q9: db64dafa9b8fdd136914e61461935fe92aa372cb056314e1231bc4ec12417456
qa: e062dcd5376d58297be2618c7498f55baa07d7e03184e8aada20bca28888bf7a
qb: 993c6ad11c4c29da9a56f7691fd0ff8d732e49de6250b6c2e80003ff4629a175
```

B.2. Test vector for CPace using group X448 and hash SHAKE-256

B.2.1. Test vectors for calculate_generator with group X448

```
  Inputs
    H   = SHAKE-256 with input block size 136 bytes.
    PRS = b'Password' ; ZPAD length: 117 ; DSI = b'CPace448'
    CI = b'\nAinitiator\nBresponder'
    CI = 0a41696e69746961746f720a42726573706f6e646572
    sid = 5223e0cdc45d6575668d64c552004124
  Outputs
    generator_string(G.DSI,PRS,CI,sid,H.s_in_bytes):
    (length: 176 bytes)
      0843506163653434380850617373776f726475000000000000000000
      0000000000000000000000000000000000000000000000000000000000
      0000000000000000000000000000000000000000000000000000000000
      0000000000000000000000000000000000000000000000000000000000
      0000000000000000000000000000000000000000000000000000000000
      000000000000000000000000000000000000000000000160a4169
      6e69746961746f720a42726573706f6e646572105223e0cdc45d6575
      668d64c552004124
    hash generator string: (length: 56 bytes)
      769e06d6c41c8cf1c87aa3df8e687167f6d0a2e41821e856276a0221
      d88272359d0b43204b546174c9179c83c107b707f296eafaa1c5a293
    decoded field element of 448 bits: (length: 56 bytes)
      769e06d6c41c8cf1c87aa3df8e687167f6d0a2e41821e856276a0221
      d88272359d0b43204b546174c9179c83c107b707f296eafaa1c5a293
    generator g: (length: 56 bytes)
      6fdae14718eb7506dd96e3f7797896efdb8db9ec0797485c9c48a192
      2e44961da097f2908b084a5de33ab671630660d27d79ffd6ee8ec846
```

B.2.2. Test vector for MSGa

```
  Inputs
    ADa = b'ADa'
    ya (little endian): (length: 56 bytes)
      21b4f4bd9e64ed355c3eb676a28ebedaf6d8f17bdc365995b3190971
      53044080516bd083bfcce66121a3072646994c8430cc382b8dc543e8
  Outputs
    Ya: (length: 56 bytes)
      396bd11daf223711e575cac6021e3fa31558012048a1cec7876292b9
      6c61eda353fe04f33028d2352779668a934084da776c1c51a58ce4b5
    MSGa = lv_cat(Ya,ADa): (length: 61 bytes)
      38396bd11daf223711e575cac6021e3fa31558012048a1cec7876292
      b96c61eda353fe04f33028d2352779668a934084da776c1c51a58ce4
      b503414461
```

B.2.3. Test vector for MSGb

```
  Inputs
    ADb = b'ADb'
    yb (little endian): (length: 56 bytes)
      848b0779ff415f0af4ea14df9dd1d3c29ac41d836c7808896c4eba19
      c51ac40a439caf5e61ec88c307c7d619195229412eaa73fb2a5ea20d
  Outputs
    Yb: (length: 56 bytes)
      53c519fb490fde5a04bda8c18b327d0fc1a9391d19e0ac00c59df9c6
      0422284e593d6b092eac94f5aa644ed883f39bd4f04e4beb6af86d58
    MSGb = lv_cat(Yb,ADb): (length: 61 bytes)
      3853c519fb490fde5a04bda8c18b327d0fc1a9391d19e0ac00c59df9
      c60422284e593d6b092eac94f5aa644ed883f39bd4f04e4beb6af86d
      5803414462
```

B.2.4. Test vector for secret points K

```
    scalar_mult_vfy(ya,Yb): (length: 56 bytes)
      e00af217556a40ccbc9822cc27a43542e45166a653aa4df746d5f8e1
      e8df483e9baff71c9eb03ee20a688ad4e4d359f70ac9ec3f6a659997
    scalar_mult_vfy(yb,Ya): (length: 56 bytes)
      e00af217556a40ccbc9822cc27a43542e45166a653aa4df746d5f8e1
      e8df483e9baff71c9eb03ee20a688ad4e4d359f70ac9ec3f6a659997
```

B.2.5. Test vector for ISK calculation initiator/responder

```
    unordered cat of transcript : (length: 122 bytes)
      38396bd11daf223711e575cac6021e3fa31558012048a1cec7876292
      b96c61eda353fe04f33028d2352779668a934084da776c1c51a58ce4
      b5034144613853c519fb490fde5a04bda8c18b327d0fc1a9391d19e0
      ac00c59df9c60422284e593d6b092eac94f5aa644ed883f39bd4f04e
      4beb6af86d5803414462
    DSI = G.DSI_ISK, b'CPace448_ISK': (length: 12 bytes)
      435061636534343 85f49534b
    lv_cat(DSI,sid,K)||MSGa||MSGb: (length: 209 bytes)
      0c435061636534343 85f49534b105223e0cdc45d6575668d64c55200
      412438e00af217556a40ccbc9822cc27a43542e45166a653aa4df746
      d5f8e1e8df483e9baff71c9eb03ee20a688ad4e4d359f70ac9ec3f6a
      65999738396bd11daf223711e575cac6021e3fa31558012048a1cec7
      876292b96c61eda353fe04f33028d2352779668a934084da776c1c51
      a58ce4b5034144613853c519fb490fde5a04bda8c18b327d0fc1a939
      1d19e0ac00c59df9c60422284e593d6b092eac94f5aa644ed883f39b
      d4f04e4beb6af86d5803414462
    ISK result: (length: 64 bytes)
      4030297722c1914711da6b2a224a44b53b30c05ab02c2a3d3ccc7272
      a3333ce3a4564c17031b634e89f65681f52d5c3d1df7baeb88523d2e
      481b3858aed86315
```

B.2.6. Test vector for ISK calculation parallel execution

```
ordered cat of transcript : (length: 122 bytes)
  3853c519fb490fde5a04bda8c18b327d0fc1a9391d19e0ac00c59df9
  c60422284e593d6b092eac94f5aa644ed883f39bd4f04e4beb6af86d
  580341446238396bd11daf223711e575cac6021e3fa31558012048a1
  cec7876292b96c61eda353fe04f33028d2352779668a934084da776c
  1c51a58ce4b503414461
DSI = G.DSI_ISK, b'CPace448_ISK': (length: 12 bytes)
  435061636534343 85f49534b
lv_cat(DSI,sid,K)||oCAT(MSGa,MSGb): (length: 209 bytes)
  0c435061636534343 85f49534b105223e0cdc45d6575668d64c55200
  412438e00af217556a40ccbc9822cc27a43542e45166a653aa4df746
  d5f8e1e8df483e9baff71c9eb03ee20a688ad4e4d359f70ac9ec3f6a
  6599973853c519fb490fde5a04bda8c18b327d0fc1a9391d19e0ac00
  c59df9c60422284e593d6b092eac94f5aa644ed883f39bd4f04e4beb
  6af86d580341446238396bd11daf223711e575cac6021e3fa3155801
  2048a1cec7876292b96c61eda353fe04f33028d2352779668a934084
  da776c1c51a58ce4b503414461
ISK result: (length: 64 bytes)
  925e95d1095dad1af6378d5ef8b9a998bd3855bfc7d36cb5ca05b0a7
  a93346abcb8cef04bceb28c38fdaf0cc608fd1dcd462ab523f3b7f75
  2c77c411be3ac8fb
```

B.2.7. Corresponding ANSI-C initializers

```
const uint8_t tc_PRS[] = {
 0x50,0x61,0x73,0x73,0x77,0x6f,0x72,0x64,
};
const uint8_t tc_CI[] = {
 0x0a,0x41,0x69,0x6e,0x69,0x74,0x69,0x61,0x74,0x6f,0x72,0x0a,
 0x42,0x72,0x65,0x73,0x70,0x6f,0x6e,0x64,0x65,0x72,
};
const uint8_t tc_sid[] = {
 0x52,0x23,0xe0,0xcd,0xc4,0x5d,0x65,0x75,0x66,0x8d,0x64,0xc5,
 0x52,0x00,0x41,0x24,
};
const uint8_t tc_g[] = {
 0x6f,0xda,0xe1,0x47,0x18,0xeb,0x75,0x06,0xdd,0x96,0xe3,0xf7,
 0x79,0x78,0x96,0xef,0xdb,0x8d,0xb9,0xec,0x07,0x97,0x48,0x5c,
 0x9c,0x48,0xa1,0x92,0x2e,0x44,0x96,0x1d,0xa0,0x97,0xf2,0x90,
 0x8b,0x08,0x4a,0x5d,0xe3,0x3a,0xb6,0x71,0x63,0x06,0x60,0xd2,
 0x7d,0x79,0xff,0xd6,0xee,0x8e,0xc8,0x46,
};
const uint8_t tc_ya[] = {
 0x21,0xb4,0xf4,0xbd,0x9e,0x64,0xed,0x35,0x5c,0x3e,0xb6,0x76,
 0xa2,0x8e,0xbe,0xda,0xf6,0xd8,0xf1,0x7b,0xdc,0x36,0x59,0x95,
 0xb3,0x19,0x09,0x71,0x53,0x04,0x40,0x80,0x51,0x6b,0xd0,0x83,
 0xbf,0xcc,0xe6,0x61,0x21,0xa3,0x07,0x26,0x46,0x99,0x4c,0x84,
 0x30,0xcc,0x38,0x2b,0x8d,0xc5,0x43,0xe8,
};
const uint8_t tc_ADa[] = {
 0x41,0x44,0x61,
};
const uint8_t tc_Ya[] = {
 0x39,0x6b,0xd1,0x1d,0xaf,0x22,0x37,0x11,0xe5,0x75,0xca,0xc6,
 0x02,0x1e,0x3f,0xa3,0x15,0x58,0x01,0x20,0x48,0xa1,0xce,0xc7,
 0x87,0x62,0x92,0xb9,0x6c,0x61,0xed,0xa3,0x53,0xfe,0x04,0xf3,
 0x30,0x28,0xd2,0x35,0x27,0x79,0x66,0x8a,0x93,0x40,0x84,0xda,
 0x77,0x6c,0x1c,0x51,0xa5,0x8c,0xe4,0xb5,
};
const uint8_t tc_yb[] = {
 0x84,0x8b,0x07,0x79,0xff,0x41,0x5f,0x0a,0xf4,0xea,0x14,0xdf,
 0x9d,0xd1,0xd3,0xc2,0x9a,0xc4,0x1d,0x83,0x6c,0x78,0x08,0x89,
 0x6c,0x4e,0xba,0x19,0xc5,0x1a,0xc4,0x0a,0x43,0x9c,0xaf,0x5e,
 0x61,0xec,0x88,0xc3,0x07,0xc7,0xd6,0x19,0x19,0x52,0x29,0x41,
 0x2e,0xaa,0x73,0xfb,0x2a,0x5e,0xa2,0x0d,
};
const uint8_t tc_ADb[] = {
 0x41,0x44,0x62,
};
const uint8_t tc_Yb[] = {
 0x53,0xc5,0x19,0xfb,0x49,0x0f,0xde,0x5a,0x04,0xbd,0xa8,0xc1,
 0x8b,0x32,0x7d,0x0f,0xc1,0xa9,0x39,0x1d,0x19,0xe0,0xac,0x00,
 0xc5,0x9d,0xf9,0xc6,0x04,0x22,0x28,0x4e,0x59,0x3d,0x6b,0x09,
```

```
 0x2e,0xac,0x94,0xf5,0xaa,0x64,0x4e,0xd8,0x83,0xf3,0x9b,0xd4,
 0xf0,0x4e,0x4b,0xeb,0x6a,0xf8,0x6d,0x58,
};
const uint8_t tc_K[] = {
 0xe0,0x0a,0xf2,0x17,0x55,0x6a,0x40,0xcc,0xbc,0x98,0x22,0xcc,
 0x27,0xa4,0x35,0x42,0xe4,0x51,0x66,0xa6,0x53,0xaa,0x4d,0xf7,
 0x46,0xd5,0xf8,0xe1,0xe8,0xdf,0x48,0x3e,0x9b,0xaf,0xf7,0x1c,
 0x9e,0xb0,0x3e,0xe2,0x0a,0x68,0x8a,0xd4,0xe4,0xd3,0x59,0xf7,
 0x0a,0xc9,0xec,0x3f,0x6a,0x65,0x99,0x97,
};
const uint8_t tc_ISK_IR[] = {
 0x40,0x30,0x29,0x77,0x22,0xc1,0x91,0x47,0x11,0xda,0x6b,0x2a,
 0x22,0x4a,0x44,0xb5,0x3b,0x30,0xc0,0x5a,0xb0,0x2c,0x2a,0x3d,
 0x3c,0xcc,0x72,0x72,0xa3,0x33,0x3c,0xe3,0xa4,0x56,0x4c,0x17,
 0x03,0x1b,0x63,0x4e,0x89,0xf6,0x56,0x81,0xf5,0x2d,0x5c,0x3d,
 0x1d,0xf7,0xba,0xeb,0x88,0x52,0x3d,0x2e,0x48,0x1b,0x38,0x58,
 0xae,0xd8,0x63,0x15,
};
const uint8_t tc_ISK_SY[] = {
 0x92,0x5e,0x95,0xd1,0x09,0x5d,0xad,0x1a,0xf6,0x37,0x8d,0x5e,
 0xf8,0xb9,0xa9,0x98,0xbd,0x38,0x55,0xbf,0xc7,0xd3,0x6c,0xb5,
 0xca,0x05,0xb0,0xa7,0xa9,0x33,0x46,0xab,0xcb,0x8c,0xef,0x04,
 0xbc,0xeb,0x28,0xc3,0x8f,0xda,0xf0,0xcc,0x60,0x8f,0xd1,0xdc,
 0xd4,0x62,0xab,0x52,0x3f,0x3b,0x7f,0x75,0x2c,0x77,0xc4,0x11,
 0xbe,0x3a,0xc8,0xfb,
};
```

B.2.8. Test vectors for G_X448.scalar_mult_vfy: low order points

Test vectors for which G_X448.scalar_mult_vfy(s_in,ux) must return the neutral element. This includes points that are non-canonicaly encoded, i.e. have coordinate values larger than the field prime.

Weak points for X448 smaller than the field prime (canonical)

u0: (length: 56 bytes)
0000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000
u1: (length: 56 bytes)
0100000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000
u2: (length: 56 bytes)
fefffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffe
ffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffff

Weak points for X448 larger or equal to the field prime (non-canonical)

u3: (length: 56 bytes)
ffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffff
ffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffff
u4: (length: 56 bytes)
00000000000000000000000000000000000000000000000000000000000000ff
ffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffffff

All of the above points u0 ... u4 MUST trigger the abort case when included in the protocol messages MSGa or MSGb.

Expected results for X448 resp. G_X448.scalar_mult_vfy

```
scalar s: (length: 56 bytes)
  af8a14218bf2a2062926d2ea9b8fe4e8b6817349b6ed2feb1e5d64d7a4
  523f15fceec70fb111e870dc58d191e66a14d3e9d482d04432cadd
G_X448.scalar_mult_vfy(s,u0): (length: 56 bytes)
  00000000000000000000000000000000000000000000000000000000
  00000000000000000000000000000000000000000000000000000000
G_X448.scalar_mult_vfy(s,u1): (length: 56 bytes)
  00000000000000000000000000000000000000000000000000000000
  00000000000000000000000000000000000000000000000000000000
G_X448.scalar_mult_vfy(s,u2): (length: 56 bytes)
  00000000000000000000000000000000000000000000000000000000
  00000000000000000000000000000000000000000000000000000000
G_X448.scalar_mult_vfy(s,u3): (length: 56 bytes)
  00000000000000000000000000000000000000000000000000000000
  00000000000000000000000000000000000000000000000000000000
G_X448.scalar_mult_vfy(s,u4): (length: 56 bytes)
  00000000000000000000000000000000000000000000000000000000
  00000000000000000000000000000000000000000000000000000000

 Test vectors for scalar_mult with nonzero outputs

scalar s: (length: 56 bytes)
  af8a14218bf2a2062926d2ea9b8fe4e8b6817349b6ed2feb1e5d64d7a4
  523f15fceec70fb111e870dc58d191e66a14d3e9d482d04432cadd
point coordinate u_curve on the curve: (length: 56 bytes)
  ab0c68d772ec2eb9de25c49700e46d6325e66d6aa39d7b65eb84a68c55
  69d47bd71b41f3e0d210f44e146dec8926b174acb3f940a0b82cab
G_X448.scalar_mult_vfy(s,u_curve): (length: 56 bytes)
  3b0fa9bc40a6fdc78c9e06ff7a54c143c5d52f365607053bf0656f5142
  0496295f910a101b38edc1acd3bd240fd55dcb7a360553b8a7627e

point coordinate u_twist on the twist: (length: 56 bytes)
  c981cd1e1f72d9c35c7d7cf6be426757c0dc8206a2fcfa564a8e7618c0
  3c0e61f9a2eb1c3e0dd97d6e9b1010f5edd03397a83f5a914cb3ff
G_X448.scalar_mult_vfy(s,u_twist): (length: 56 bytes)
  d0a2bb7e9c5c2c627793d8342f23b759fe7d9e3320a85ca4fd61376331
  50ffd9a9148a9b75c349fac43d64bec49a6e126cc92cbfbf353961
```

B.3. Test vector for CPace using group ristretto255 and hash SHA-512

B.3.1. Test vectors for calculate_generator with group ristretto255

```
  Inputs
    H   = SHA-512 with input block size 128 bytes.
    PRS = b'Password' ; ZPAD length: 100 ;
    DSI = b'CPaceRistretto255'
    CI = b'\nAinitiator\nBresponder'
    CI = 0a41696e69746961746f720a42726573706f6e646572
    sid = 7e4b4791d6a8ef019b936c79fb7f2c57
  Outputs
    generator_string(G.DSI,PRS,CI,sid,H.s_in_bytes):
    (length: 168 bytes)
      1143506163655269737472657474 6f32353085061737377 6f726464
      0000000000000000000000000000000000000000000000000000000000000000
      0000000000000000000000000000000000000000000000000000000000000000
      0000000000000000000000000000000000000000000000000000000000000000
      0000000000000000000000000000160a41696e69746961746f72
      0a42726573706f6e646572107e4b4791d6a8ef019b936c79fb7f2c57
    hash result: (length: 64 bytes)
      a5ce446f63a1ae6d1fee80fa67d0b4004a4b1283ec5549a462bf33a6
      c1ae06a0871f9bf48545f49b2a792eed255ac04f52758c9c60448306
      810b44e986e3dcbb
    encoded generator g: (length: 32 bytes)
      9c5712178570957204d89ac11acbef789dd076992ba361429acb2bc3
      8c71d14c
```

B.3.2. Test vector for MSGa

```
  Inputs
    ADa = b'ADa'
    ya (little endian): (length: 32 bytes)
      1433dd19359992d4e06d740d3993d429af6338ffb4531ce175d22449
      853a790b
  Outputs
    Ya: (length: 32 bytes)
      a8fc42c4d57b3c7346661011122a00563d0995fd72b62123ae244400
      e86d7b1a
    MSGa = lv_cat(Ya,ADa): (length: 37 bytes)
      20a8fc42c4d57b3c7346661011122a00563d0995fd72b62123ae2444
      00e86d7b1a03414461
```

B.3.3. Test vector for MSGb

```
  Inputs
    ADb = b'ADb'
    yb (little endian): (length: 32 bytes)
      0e6566d32d80a5a1135f99c27f2d637aa24da23027c3fa76b9d1cfd9
      742fdc00
  Outputs
    Yb: (length: 32 bytes)
      fc8e84ae4ab725909af05a56ef9714db6930e4a5589b3fee6cdd2662
      36676d63
    MSGb = lv_cat(Yb,ADb): (length: 37 bytes)
      20fc8e84ae4ab725909af05a56ef9714db6930e4a5589b3fee6cdd26
      6236676d6303414462
```

B.3.4. Test vector for secret points K

```
    scalar_mult_vfy(ya,Yb): (length: 32 bytes)
      3efef1706f42efa354020b087b37fbd9f81cf72a16f4947e4a042a7f
      1aaa2b6f
    scalar_mult_vfy(yb,Ya): (length: 32 bytes)
      3efef1706f42efa354020b087b37fbd9f81cf72a16f4947e4a042a7f
      1aaa2b6f
```

B.3.5. Test vector for ISK calculation initiator/responder

```
    unordered cat of transcript : (length: 74 bytes)
      20a8fc42c4d57b3c7346661011122a00563d0995fd72b62123ae2444
      00e86d7b1a0341446120fc8e84ae4ab725909af05a56ef9714db6930
      e4a5589b3fee6cdd266236676d6303414462
    DSI = G.DSI_ISK, b'CPaceRistretto255_ISK':
    (length: 21 bytes)
      435061636552697374726574746f3235355f49534b
    lv_cat(DSI,sid,K)||MSGa||MSGb: (length: 146 bytes)
      1543506163655269737472657474 6f3235355f49534b107e4b4791d6
      a8ef019b936c79fb7f2c57203efef1706f42efa354020b087b37fbd9
      f81cf72a16f4947e4a042a7f1aaa2b6f20a8fc42c4d57b3c73466610
      11122a00563d0995fd72b62123ae244400e86d7b1a0341446120fc8e
      84ae4ab725909af05a56ef9714db6930e4a5589b3fee6cdd26623667
      6d6303414462
    ISK result: (length: 64 bytes)
      0e33c5822bd495dea94ba7af161501f1b2d6a16d464b5d6e1a53dcbf
      b9244b9ba66c09c430fffdfe4fb4e99b4ea46f991a272de0431c132c
      2c79fd6de1a7e5e4
```

B.3.6. Test vector for ISK calculation parallel execution

```
ordered cat of transcript : (length: 74 bytes)
  20fc8e84ae4ab725909af05a56ef9714db6930e4a5589b3fee6cdd26
  6236676d630341446220a8fc42c4d57b3c7346661011122a00563d09
  95fd72b62123ae244400e86d7b1a03414461
DSI = G.DSI_ISK, b'CPaceRistretto255_ISK':
(length: 21 bytes)
  435061636552697374726574746f3235355f49534b
lv_cat(DSI,sid,K)||oCAT(MSGa,MSGb): (length: 146 bytes)
  154350616365526973747265747 46f3235355f49534b107e4b4791d6
  a8ef019b936c79fb7f2c57203efef1706f42efa354020b087b37fbd9
  f81cf72a16f4947e4a042a7f1aaa2b6f20fc8e84ae4ab725909af05a
  56ef9714db6930e4a5589b3fee6cdd266236676d630341446220a8fc
  42c4d57b3c7346661011122a00563d0995fd72b62123ae244400e86d
  7b1a03414461
ISK result: (length: 64 bytes)
  ca36335be682a480a9fc63977d044a10ff7adfcda0f2978fbcf8713d
  2a4e23e25c05a9a02edcfbff2ede65b752f8ea1f4454d764ad8ed860
  7c158ef662614567
```

B.3.7. Corresponding ANSI-C initializers

```
const uint8_t tc_PRS[] = {
 0x50,0x61,0x73,0x73,0x77,0x6f,0x72,0x64,
};
const uint8_t tc_CI[] = {
 0x0a,0x41,0x69,0x6e,0x69,0x74,0x69,0x61,0x74,0x6f,0x72,0x0a,
 0x42,0x72,0x65,0x73,0x70,0x6f,0x6e,0x64,0x65,0x72,
};
const uint8_t tc_sid[] = {
 0x7e,0x4b,0x47,0x91,0xd6,0xa8,0xef,0x01,0x9b,0x93,0x6c,0x79,
 0xfb,0x7f,0x2c,0x57,
};
const uint8_t tc_g[] = {
 0x9c,0x57,0x12,0x17,0x85,0x70,0x95,0x72,0x04,0xd8,0x9a,0xc1,
 0x1a,0xcb,0xef,0x78,0x9d,0xd0,0x76,0x99,0x2b,0xa3,0x61,0x42,
 0x9a,0xcb,0x2b,0xc3,0x8c,0x71,0xd1,0x4c,
};
const uint8_t tc_ya[] = {
 0x14,0x33,0xdd,0x19,0x35,0x99,0x92,0xd4,0xe0,0x6d,0x74,0x0d,
 0x39,0x93,0xd4,0x29,0xaf,0x63,0x38,0xff,0xb4,0x53,0x1c,0xe1,
 0x75,0xd2,0x24,0x49,0x85,0x3a,0x79,0x0b,
};
const uint8_t tc_ADa[] = {
 0x41,0x44,0x61,
};
const uint8_t tc_Ya[] = {
 0xa8,0xfc,0x42,0xc4,0xd5,0x7b,0x3c,0x73,0x46,0x66,0x10,0x11,
 0x12,0x2a,0x00,0x56,0x3d,0x09,0x95,0xfd,0x72,0xb6,0x21,0x23,
 0xae,0x24,0x44,0x00,0xe8,0x6d,0x7b,0x1a,
};
const uint8_t tc_yb[] = {
 0x0e,0x65,0x66,0xd3,0x2d,0x80,0xa5,0xa1,0x13,0x5f,0x99,0xc2,
 0x7f,0x2d,0x63,0x7a,0xa2,0x4d,0xa2,0x30,0x27,0xc3,0xfa,0x76,
 0xb9,0xd1,0xcf,0xd9,0x74,0x2f,0xdc,0x00,
};
const uint8_t tc_ADb[] = {
 0x41,0x44,0x62,
};
const uint8_t tc_Yb[] = {
 0xfc,0x8e,0x84,0xae,0x4a,0xb7,0x25,0x90,0x9a,0xf0,0x5a,0x56,
 0xef,0x97,0x14,0xdb,0x69,0x30,0xe4,0xa5,0x58,0x9b,0x3f,0xee,
 0x6c,0xdd,0x26,0x62,0x36,0x67,0x6d,0x63,
};
const uint8_t tc_K[] = {
 0x3e,0xfe,0xf1,0x70,0x6f,0x42,0xef,0xa3,0x54,0x02,0x0b,0x08,
 0x7b,0x37,0xfb,0xd9,0xf8,0x1c,0xf7,0x2a,0x16,0xf4,0x94,0x7e,
 0x4a,0x04,0x2a,0x7f,0x1a,0xaa,0x2b,0x6f,
};
const uint8_t tc_ISK_IR[] = {
 0x0e,0x33,0xc5,0x82,0x2b,0xd4,0x95,0xde,0xa9,0x4b,0xa7,0xaf,
```

```
 0x16,0x15,0x01,0xf1,0xb2,0xd6,0xa1,0x6d,0x46,0x4b,0x5d,0x6e,
 0x1a,0x53,0xdc,0xbf,0xb9,0x24,0x4b,0x9b,0xa6,0x6c,0x09,0xc4,
 0x30,0xff,0xfd,0xfe,0x4f,0xb4,0xe9,0x9b,0x4e,0xa4,0x6f,0x99,
 0x1a,0x27,0x2d,0xe0,0x43,0x1c,0x13,0x2c,0x2c,0x79,0xfd,0x6d,
 0xe1,0xa7,0xe5,0xe4,
};
const uint8_t tc_ISK_SY[] = {
 0xca,0x36,0x33,0x5b,0xe6,0x82,0xa4,0x80,0xa9,0xfc,0x63,0x97,
 0x7d,0x04,0x4a,0x10,0xff,0x7a,0xdf,0xcd,0xa0,0xf2,0x97,0x8f,
 0xbc,0xf8,0x71,0x3d,0x2a,0x4e,0x23,0xe2,0x5c,0x05,0xa9,0xa0,
 0x2e,0xdc,0xfb,0xff,0x2e,0xde,0x65,0xb7,0x52,0xf8,0xea,0x1f,
 0x44,0x54,0xd7,0x64,0xad,0x8e,0xd8,0x60,0x7c,0x15,0x8e,0xf6,
 0x62,0x61,0x45,0x67,
};
```

B.3.8. Test case for scalar_mult with valid inputs

```
s: (length: 32 bytes)
  7cd0e075fa7955ba52c02759a6c90dbbfc10e6d40aea8d283e407d88
  cf538a05
X: (length: 32 bytes)
  021ca069484e890c9e494d8ed6bb0f66cbd9a8f0ef67168f36c51e0e
  feb8f347
G.scalar_mult(s,decode(X)): (length: 32 bytes)
  62aaa018755dc881902097c2a993c0b7c0a4fe33bce2c0182b46a44c
  40b95119
G.scalar_mult_vfy(s,X): (length: 32 bytes)
  62aaa018755dc881902097c2a993c0b7c0a4fe33bce2c0182b46a44c
  40b95119
```

B.3.9. Invalid inputs for scalar_mult_vfy

For these test cases scalar_mult_vfy(y,.) MUST return the representation of the neutral element G.I. When points Y_i1 or Y_i2 are included in MSGa or MSGb the protocol MUST abort.

```
s: (length: 32 bytes)
  7cd0e075fa7955ba52c02759a6c90dbbfc10e6d40aea8d283e407d88
  cf538a05
Y_i1: (length: 32 bytes)
  011ca069484e890c9e494d8ed6bb0f66cbd9a8f0ef67168f36c51e0e
  feb8f347
Y_i2 == G.I: (length: 32 bytes)
  0000000000000000000000000000000000000000000000000000000
  00000000
G.scalar_mult_vfy(s,Y_i1) = G.scalar_mult_vfy(s,Y_i2) = G.I
```

B.4. Test vector for CPace using group decaf448 and hash SHAKE-256

B.4.1. Test vectors for calculate_generator with group decaf448

```
  Inputs
    H   = SHAKE-256 with input block size 136 bytes.
    PRS = b'Password' ; ZPAD length: 112 ;
    DSI = b'CPaceDecaf448'
    CI = b'\nAinitiator\nBresponder'
    CI = 0a41696e69746961746f720a42726573706f6e646572
    sid = 5223e0cdc45d6575668d64c552004124
  Outputs
    generator_string(G.DSI,PRS,CI,sid,H.s_in_bytes):
    (length: 176 bytes)
      0d435061636544656361663434380850617373776f72647000000000
      0000000000000000000000000000000000000000000000000000000
      0000000000000000000000000000000000000000000000000000000
      0000000000000000000000000000000000000000000000000000000
      0000000000000000000000000000000000000000000160a4169
      6e69746961746f720a42726573706f6e646572105223e0cdc45d6575
      668d64c552004124
    hash result: (length: 112 bytes)
      8955b426ff1d3a22032d21c013cf94134cee9a4235e93261a4911edb
      f68f2945f0267c983954262c7f59badb9caf468ebe21b7e9885657af
      b8f1a3b783c2047ba519e113ecf81b2b580dd481f499beabd401cc77
      1d28915fb750011209040f5f03b2ceb5e5eb259c96b478382d5a5c57
    encoded generator g: (length: 56 bytes)
      c811b3f6b0d27b58a74d8274bf5f9ca6b7ada15b0bf57b79a6b45c13
      2eb0c28bdcc3abf4e5932cea97a80997ead1c146b98b1a1f1def30f3
```

B.4.2. Test vector for MSGa

```
  Inputs
    ADa = b'ADa'
    ya (little endian): (length: 56 bytes)
      d8d2e26c821a12d7f59a8dee023d3f6155976152e16c73cbf68c303d
      f0404399f0a7b614a65df50a9788f00b410586b443f738ad7ff03930
  Outputs
    Ya: (length: 56 bytes)
      223f95a5430a2f2a499431696d23ea2d0a90f432e5491e45e4005f3d
      d785e7be1235b79252670099bc993c2df5c261dfb7a8989f091e2be3
    MSGa = lv_cat(Ya,ADa): (length: 61 bytes)
      38223f95a5430a2f2a499431696d23ea2d0a90f432e5491e45e4005f
      3dd785e7be1235b79252670099bc993c2df5c261dfb7a8989f091e2b
      e303414461
```

B.4.3. Test vector for MSGb

```
  Inputs
    ADb = b'ADb'
    yb (little endian): (length: 56 bytes)
      91bae9793f4a8aceb1b5c54375a7ed1858a79a6e72dab959c8bdf3a7
      5ac9bb4de2a25af4d4a9a5c5bc5441d19b8e3f6fcce7196c6afc2236
  Outputs
    Yb: (length: 56 bytes)
      b6ba0a336c103c6c92019ae4cfbcb88d8f6bfc361e979c9e0d3a0967
      e630094ba3d1555821ac1f979996ef5ce79f012ffe279ac89b287bee
    MSGb = lv_cat(Yb,ADb): (length: 61 bytes)
      38b6ba0a336c103c6c92019ae4cfbcb88d8f6bfc361e979c9e0d3a09
      67e630094ba3d1555821ac1f979996ef5ce79f012ffe279ac89b287b
      ee03414462
```

B.4.4. Test vector for secret points K

```
    scalar_mult_vfy(ya,Yb): (length: 56 bytes)
      dc504938fb70eb13916697aa3e076e82537c171aa326121399c896fe
      ea0e198b41b6bae300bb86f8c61d4b170eee4717b5497016f34364a9
    scalar_mult_vfy(yb,Ya): (length: 56 bytes)
      dc504938fb70eb13916697aa3e076e82537c171aa326121399c896fe
      ea0e198b41b6bae300bb86f8c61d4b170eee4717b5497016f34364a9
```

B.4.5. Test vector for ISK calculation initiator/responder

```
    unordered cat of transcript : (length: 122 bytes)
      38223f95a5430a2f2a499431696d23ea2d0a90f432e5491e45e4005f
      3dd785e7be1235b79252670099bc993c2df5c261dfb7a8989f091e2b
      e30341446138b6ba0a336c103c6c92019ae4cfbcb88d8f6bfc361e97
      9c9e0d3a0967e630094ba3d1555821ac1f979996ef5ce79f012ffe27
      9ac89b287bee03414462
    DSI = G.DSI_ISK, b'CPaceDecaf448_ISK': (length: 17 bytes)
      435061636544656361663434385f49534b
    lv_cat(DSI,sid,K)||MSGa||MSGb: (length: 214 bytes)
      1143506163654465636166343438 5f49534b105223e0cdc45d657566
      8d64c55200412438dc504938fb70eb13916697aa3e076e82537c171a
      a326121399c896feea0e198b41b6bae300bb86f8c61d4b170eee4717
      b5497016f34364a938223f95a5430a2f2a499431696d23ea2d0a90f4
      32e5491e45e4005f3dd785e7be1235b79252670099bc993c2df5c261
      dfb7a8989f091e2be30341446138b6ba0a336c103c6c92019ae4cfbc
      b88d8f6bfc361e979c9e0d3a0967e630094ba3d1555821ac1f979996
      ef5ce79f012ffe279ac89b287bee03414462
    ISK result: (length: 64 bytes)
      ebe28369491f8899a5af3b339d4993881b69d22607c58719da6eaab3
      8f0d9025eae413ca2b072b156ce4a0d4778ff471a63c4d908cab70bc
      2081951d504cbb03
```

B.4.6. Test vector for ISK calculation parallel execution

```
ordered cat of transcript : (length: 122 bytes)
  38b6ba0a336c103c6c92019ae4cfbcb88d8f6bfc361e979c9e0d3a09
  67e630094ba3d1555821ac1f979996ef5ce79f012ffe279ac89b287b
  ee0341446238223f95a5430a2f2a499431696d23ea2d0a90f432e549
  1e45e4005f3dd785e7be1235b79252670099bc993c2df5c261dfb7a8
  989f091e2be303414461
DSI = G.DSI_ISK, b'CPaceDecaf448_ISK': (length: 17 bytes)
  435061636544656361663434385f49534b
lv_cat(DSI,sid,K)||oCAT(MSGa,MSGb): (length: 214 bytes)
  114350616365446563616634343 85f49534b105223e0cdc45d657566
  8d64c55200412438dc504938fb70eb13916697aa3e076e82537c171a
  a326121399c896feea0e198b41b6bae300bb86f8c61d4b170eee4717
  b5497016f34364a938b6ba0a336c103c6c92019ae4cfbcb88d8f6bfc
  361e979c9e0d3a0967e630094ba3d1555821ac1f979996ef5ce79f01
  2ffe279ac89b287bee0341446238223f95a5430a2f2a499431696d23
  ea2d0a90f432e5491e45e4005f3dd785e7be1235b79252670099bc99
  3c2df5c261dfb7a8989f091e2be303414461
ISK result: (length: 64 bytes)
  2996d1953320581b587f473cfd5c974c5a8597b22b37fefe49bdb7b8
  4073424f7f7a6e456498665a69530741398c6010bdb346f79944acc9
  0c5c537fa35cd29a
```

B.4.7. Corresponding ANSI-C initializers

```
const uint8_t tc_PRS[] = {
 0x50,0x61,0x73,0x73,0x77,0x6f,0x72,0x64,
};
const uint8_t tc_CI[] = {
 0x0a,0x41,0x69,0x6e,0x69,0x74,0x69,0x61,0x74,0x6f,0x72,0x0a,
 0x42,0x72,0x65,0x73,0x70,0x6f,0x6e,0x64,0x65,0x72,
};
const uint8_t tc_sid[] = {
 0x52,0x23,0xe0,0xcd,0xc4,0x5d,0x65,0x75,0x66,0x8d,0x64,0xc5,
 0x52,0x00,0x41,0x24,
};
const uint8_t tc_g[] = {
 0xc8,0x11,0xb3,0xf6,0xb0,0xd2,0x7b,0x58,0xa7,0x4d,0x82,0x74,
 0xbf,0x5f,0x9c,0xa6,0xb7,0xad,0xa1,0x5b,0x0b,0xf5,0x7b,0x79,
 0xa6,0xb4,0x5c,0x13,0x2e,0xb0,0xc2,0x8b,0xdc,0xc3,0xab,0xf4,
 0xe5,0x93,0x2c,0xea,0x97,0xa8,0x09,0x97,0xea,0xd1,0xc1,0x46,
 0xb9,0x8b,0x1a,0x1f,0x1d,0xef,0x30,0xf3,
};
const uint8_t tc_ya[] = {
 0xd8,0xd2,0xe2,0x6c,0x82,0x1a,0x12,0xd7,0xf5,0x9a,0x8d,0xee,
 0x02,0x3d,0x3f,0x61,0x55,0x97,0x61,0x52,0xe1,0x6c,0x73,0xcb,
 0xf6,0x8c,0x30,0x3d,0xf0,0x40,0x43,0x99,0xf0,0xa7,0xb6,0x14,
 0xa6,0x5d,0xf5,0x0a,0x97,0x88,0xf0,0x0b,0x41,0x05,0x86,0xb4,
 0x43,0xf7,0x38,0xad,0x7f,0xf0,0x39,0x30,
};
const uint8_t tc_ADa[] = {
 0x41,0x44,0x61,
};
const uint8_t tc_Ya[] = {
 0x22,0x3f,0x95,0xa5,0x43,0x0a,0x2f,0x2a,0x49,0x94,0x31,0x69,
 0x6d,0x23,0xea,0x2d,0x0a,0x90,0xf4,0x32,0xe5,0x49,0x1e,0x45,
 0xe4,0x00,0x5f,0x3d,0xd7,0x85,0xe7,0xbe,0x12,0x35,0xb7,0x92,
 0x52,0x67,0x00,0x99,0xbc,0x99,0x3c,0x2d,0xf5,0xc2,0x61,0xdf,
 0xb7,0xa8,0x98,0x9f,0x09,0x1e,0x2b,0xe3,
};
const uint8_t tc_yb[] = {
 0x91,0xba,0xe9,0x79,0x3f,0x4a,0x8a,0xce,0xb1,0xb5,0xc5,0x43,
 0x75,0xa7,0xed,0x18,0x58,0xa7,0x9a,0x6e,0x72,0xda,0xb9,0x59,
 0xc8,0xbd,0xf3,0xa7,0x5a,0xc9,0xbb,0x4d,0xe2,0xa2,0x5a,0xf4,
 0xd4,0xa9,0xa5,0xc5,0xbc,0x54,0x41,0xd1,0x9b,0x8e,0x3f,0x6f,
 0xcc,0xe7,0x19,0x6c,0x6a,0xfc,0x22,0x36,
};
const uint8_t tc_ADb[] = {
 0x41,0x44,0x62,
};
const uint8_t tc_Yb[] = {
 0xb6,0xba,0x0a,0x33,0x6c,0x10,0x3c,0x6c,0x92,0x01,0x9a,0xe4,
 0xcf,0xbc,0xb8,0x8d,0x8f,0x6b,0xfc,0x36,0x1e,0x97,0x9c,0x9e,
 0x0d,0x3a,0x09,0x67,0xe6,0x30,0x09,0x4b,0xa3,0xd1,0x55,0x58,
```

```
 0x21,0xac,0x1f,0x97,0x99,0x96,0xef,0x5c,0xe7,0x9f,0x01,0x2f,
 0xfe,0x27,0x9a,0xc8,0x9b,0x28,0x7b,0xee,
};
const uint8_t tc_K[] = {
 0xdc,0x50,0x49,0x38,0xfb,0x70,0xeb,0x13,0x91,0x66,0x97,0xaa,
 0x3e,0x07,0x6e,0x82,0x53,0x7c,0x17,0x1a,0xa3,0x26,0x12,0x13,
 0x99,0xc8,0x96,0xfe,0xea,0x0e,0x19,0x8b,0x41,0xb6,0xba,0xe3,
 0x00,0xbb,0x86,0xf8,0xc6,0x1d,0x4b,0x17,0x0e,0xee,0x47,0x17,
 0xb5,0x49,0x70,0x16,0xf3,0x43,0x64,0xa9,
};
const uint8_t tc_ISK_IR[] = {
 0xeb,0xe2,0x83,0x69,0x49,0x1f,0x88,0x99,0xa5,0xaf,0x3b,0x33,
 0x9d,0x49,0x93,0x88,0x1b,0x69,0xd2,0x26,0x07,0xc5,0x87,0x19,
 0xda,0x6e,0xaa,0xb3,0x8f,0x0d,0x90,0x25,0xea,0xe4,0x13,0xca,
 0x2b,0x07,0x2b,0x15,0x6c,0xe4,0xa0,0xd4,0x77,0x8f,0xf4,0x71,
 0xa6,0x3c,0x4d,0x90,0x8c,0xab,0x70,0xbc,0x20,0x81,0x95,0x1d,
 0x50,0x4c,0xbb,0x03,
};
const uint8_t tc_ISK_SY[] = {
 0x29,0x96,0xd1,0x95,0x33,0x20,0x58,0x1b,0x58,0x7f,0x47,0x3c,
 0xfd,0x5c,0x97,0x4c,0x5a,0x85,0x97,0xb2,0x2b,0x37,0xfe,0xfe,
 0x49,0xbd,0xb7,0xb8,0x40,0x73,0x42,0x4f,0x7f,0x7a,0x6e,0x45,
 0x64,0x98,0x66,0x5a,0x69,0x53,0x07,0x41,0x39,0x8c,0x60,0x10,
 0xbd,0xb3,0x46,0xf7,0x99,0x44,0xac,0xc9,0x0c,0x5c,0x53,0x7f,
 0xa3,0x5c,0xd2,0x9a,
};
```

B.4.8. Test case for scalar_mult with valid inputs

```
s: (length: 56 bytes)
  dd1bc7015daabb7672129cc35a3ba815486b139deff9bdeca7a4fc61
  34323d34658761e90ff079972a7ca8aa5606498f4f4f0ebc0933a819
X: (length: 56 bytes)
  c803a6c8171ac38b66c5306553f45a487a24eb8581414444715bd2e5
  cf4c749a3b56a550f3c9a6ea3efa6e11ae6a6da12b98ef2f51174b9a
G.scalar_mult(s,decode(X)): (length: 56 bytes)
  b831a1f804fd3c902ae82f731d298aebf9152ea855f5b5da5ee88584
  84c55a7f65cc3ccf5f678496dc4cb1c8d6bc7ed17d2fe535fdc8f60e
G.scalar_mult_vfy(s,X): (length: 56 bytes)
  b831a1f804fd3c902ae82f731d298aebf9152ea855f5b5da5ee88584
  84c55a7f65cc3ccf5f678496dc4cb1c8d6bc7ed17d2fe535fdc8f60e
```

B.4.9. Invalid inputs for scalar_mult_vfy

For these test cases scalar_mult_vfy(y,.) MUST return the representation of the neutral element G.I. When points Y_i1 or Y_i2 are included in MSGa or MSGb the protocol MUST abort.

```
s: (length: 56 bytes)
  dd1bc7015daabb7672129cc35a3ba815486b139deff9bdeca7a4fc61
  34323d34658761e90ff079972a7ca8aa5606498f4f4f0ebc0933a819
Y_i1: (length: 56 bytes)
  c703a6c8171ac38b66c5306553f45a487a24eb8581414444715bd2e5
  cf4c749a3b56a550f3c9a6ea3efa6e11ae6a6da12b98ef2f51174b9a
Y_i2 == G.I: (length: 56 bytes)
  00000000000000000000000000000000000000000000000000000000
  00000000000000000000000000000000000000000000000000000000
G.scalar_mult_vfy(s,Y_i1) = G.scalar_mult_vfy(s,Y_i2) = G.I
```

B.5. Test vector for CPace using group NIST P-256 and hash SHA-256

B.5.1. Test vectors for calculate_generator with group NIST P-256

```
  Inputs
    H   = SHA-256 with input block size 64 bytes.
    PRS = b'Password' ; ZPAD length: 23 ;
    DSI = b'CPaceP256_XMD:SHA-256_SSWU_NU_'
    CI = b'\nAinitiator\nBresponder'
    CI = 0a41696e69746961746f720a42726573706f6e646572
    sid = 34b36454cab2e7842c389f7d88ecb7df
  Outputs
    generator_string(PRS,G.DSI,CI,sid,H.s_in_bytes):
    (length: 104 bytes)
      1e435061636550323536 5f584d443a5348412d3235365f535357555f
      4e555f0850617373776f726417000000000000000000000000000000
      0000000000000000160a41696e69746961746f720a42726573706f6e
      6465721034b36454cab2e7842c389f7d88ecb7df
    generator g: (length: 65 bytes)
      04993b46e30ba9cfc3dc2d3ae2cf9733cf03994e74383c4e1b4a92e8
      d6d466b321c4a642979162fbde9e1c9a6180bd27a0594491e4c231f5
      1006d0bf7992d07127
```

B.5.2. Test vector for MSGa

```
  Inputs
    ADa = b'ADa'
    ya (big endian): (length: 32 bytes)
      c9e47ca5debd2285727af47e55f5b7763fa79719da428f800190cc66
      59b4eafb
  Outputs
    Ya: (length: 65 bytes)
      0478ac925a6e3447a537627a2163be005a422f55c08385c1ef7d051c
      a94593df5946314120faa87165cba131c1da3aac429dc3d99a9bac7d
      4c4cbb8570b4d5ea10
    Alternative correct value for Ya: g^(-ya):
    (length: 65 bytes)
      0478ac925a6e3447a537627a2163be005a422f55c08385c1ef7d051c
      a94593df59b9cebede05578e9b345ece3e25c553bd623c2666645382
      b3b3447a8f4b2a15ef
    MSGa = lv_cat(Ya,ADa): (length: 70 bytes)
      410478ac925a6e3447a537627a2163be005a422f55c08385c1ef7d05
      1ca94593df5946314120faa87165cba131c1da3aac429dc3d99a9bac
      7d4c4cbb8570b4d5ea1003414461
```

B.5.3. Test vector for MSGb

```
  Inputs
    ADb = b'ADb'
    yb (big endian): (length: 32 bytes)
      a0b768ba7555621d133012d1dee27a0013c1bcfddd675811df12771e
      44d77b10
  Outputs
    Yb: (length: 65 bytes)
      04df13ffa89b0ce3cc553b1495ff027886564d94b8d9165cd50e5f65
      4247959951bfac90839fca218bf8e2d1258eb7d7d9f733fe4cd558e6
      fa57bf1f801aae7d3a
    Alternative correct value for Yb: g^(-yb):
    (length: 65 bytes)
      04df13ffa89b0ce3cc553b1495ff027886564d94b8d9165cd50e5f65
      424795995140536f7b6035de75071d2eda7148282608cc01b42aa719
      05a840e07fe55182c5
    MSGb = lv_cat(Yb,ADb): (length: 70 bytes)
      4104df13ffa89b0ce3cc553b1495ff027886564d94b8d9165cd50e5f
      654247959951bfac90839fca218bf8e2d1258eb7d7d9f733fe4cd558
      e6fa57bf1f801aae7d3a03414462
```

B.5.4. Test vector for secret points K

```
    scalar_mult_vfy(ya,Yb): (length: 32 bytes)
      27f7059d88f02007dc18c911c9b4034d3c0f13f8f7ed9603b0927f23
      fbab1037
    scalar_mult_vfy(yb,Ya): (length: 32 bytes)
      27f7059d88f02007dc18c911c9b4034d3c0f13f8f7ed9603b0927f23
      fbab1037
```

B.5.5. Test vector for ISK calculation initiator/responder

```
unordered cat of transcript : (length: 140 bytes)
  410478ac925a6e3447a537627a2163be005a422f55c08385c1ef7d05
  1ca94593df5946314120faa87165cba131c1da3aac429dc3d99a9bac
  7d4c4cbb8570b4d5ea10034144614104df13ffa89b0ce3cc553b1495
  ff027886564d94b8d9165cd50e5f654247959951bfac90839fca218b
  f8e2d1258eb7d7d9f733fe4cd558e6fa57bf1f801aae7d3a03414462
DSI = G.DSI_ISK, b'CPaceP256_XMD:SHA-256_SSWU_NU__ISK':
(length: 34 bytes)
  4350616365503235365f584d443a5348412d3235365f535357555f4e
  555f5f49534b
lv_cat(DSI,sid,K)||MSGa||MSGb: (length: 225 bytes)
  2243506163655032353 65f584d443a5348412d3235365f535357555f
  4e555f5f49534b1034b36454cab2e7842c389f7d88ecb7df2027f705
  9d88f02007dc18c911c9b4034d3c0f13f8f7ed9603b0927f23fbab10
  37410478ac925a6e3447a537627a2163be005a422f55c08385c1ef7d
  051ca94593df5946314120faa87165cba131c1da3aac429dc3d99a9b
  ac7d4c4cbb8570b4d5ea10034144614104df13ffa89b0ce3cc553b14
  95ff027886564d94b8d9165cd50e5f654247959951bfac90839fca21
  8bf8e2d1258eb7d7d9f733fe4cd558e6fa57bf1f801aae7d3a034144
  62
ISK result: (length: 32 bytes)
  ddc1b133c387ecf344c0b496bc1223656cd6e7d99a5def8b3b026796
  50811fc9
```

B.5.6. Test vector for ISK calculation parallel execution

```
ordered cat of transcript : (length: 140 bytes)
  4104df13ffa89b0ce3cc553b1495ff027886564d94b8d9165cd50e5f
  654247959951bfac90839fca218bf8e2d1258eb7d7d9f733fe4cd558
  e6fa57bf1f801aae7d3a03414462410478ac925a6e3447a537627a21
  63be005a422f55c08385c1ef7d051ca94593df5946314120faa87165
  cba131c1da3aac429dc3d99a9bac7d4c4cbb8570b4d5ea1003414461
DSI = G.DSI_ISK, b'CPaceP256_XMD:SHA-256_SSWU_NU__ISK':
(length: 34 bytes)
  435061636550323536 5f584d443a5348412d3235365f535357555f4e
  555f5f49534b
lv_cat(DSI,sid,K)||oCAT(MSGa,MSGb): (length: 225 bytes)
  224350616365503235365f584d443a5348412d3235365f535357555f
  4e555f5f49534b1034b36454cab2e7842c389f7d88ecb7df2027f705
  9d88f02007dc18c911c9b4034d3c0f13f8f7ed9603b0927f23fbab10
  374104df13ffa89b0ce3cc553b1495ff027886564d94b8d9165cd50e
  5f654247959951bfac90839fca218bf8e2d1258eb7d7d9f733fe4cd5
  58e6fa57bf1f801aae7d3a03414462410478ac925a6e3447a537627a
  2163be005a422f55c08385c1ef7d051ca94593df5946314120faa871
  65cba131c1da3aac429dc3d99a9bac7d4c4cbb8570b4d5ea10034144
  61
ISK result: (length: 32 bytes)
  6ea775b0fb3c31502687565a52150fc595c63fe901a11d5fc1995cd5
  089a17ae
```

B.5.7. Corresponding ANSI-C initializers

```
const uint8_t tc_PRS[] = {
 0x50,0x61,0x73,0x73,0x77,0x6f,0x72,0x64,
};
const uint8_t tc_CI[] = {
 0x0a,0x41,0x69,0x6e,0x69,0x74,0x69,0x61,0x74,0x6f,0x72,0x0a,
 0x42,0x72,0x65,0x73,0x70,0x6f,0x6e,0x64,0x65,0x72,
};
const uint8_t tc_sid[] = {
 0x34,0xb3,0x64,0x54,0xca,0xb2,0xe7,0x84,0x2c,0x38,0x9f,0x7d,
 0x88,0xec,0xb7,0xdf,
};
const uint8_t tc_g[] = {
 0x04,0x99,0x3b,0x46,0xe3,0x0b,0xa9,0xcf,0xc3,0xdc,0x2d,0x3a,
 0xe2,0xcf,0x97,0x33,0xcf,0x03,0x99,0x4e,0x74,0x38,0x3c,0x4e,
 0x1b,0x4a,0x92,0xe8,0xd6,0xd4,0x66,0xb3,0x21,0xc4,0xa6,0x42,
 0x97,0x91,0x62,0xfb,0xde,0x9e,0x1c,0x9a,0x61,0x80,0xbd,0x27,
 0xa0,0x59,0x44,0x91,0xe4,0xc2,0x31,0xf5,0x10,0x06,0xd0,0xbf,
 0x79,0x92,0xd0,0x71,0x27,
};
const uint8_t tc_ya[] = {
 0xc9,0xe4,0x7c,0xa5,0xde,0xbd,0x22,0x85,0x72,0x7a,0xf4,0x7e,
 0x55,0xf5,0xb7,0x76,0x3f,0xa7,0x97,0x19,0xda,0x42,0x8f,0x80,
 0x01,0x90,0xcc,0x66,0x59,0xb4,0xea,0xfb,
};
const uint8_t tc_ADa[] = {
 0x41,0x44,0x61,
};
const uint8_t tc_Ya[] = {
 0x04,0x78,0xac,0x92,0x5a,0x6e,0x34,0x47,0xa5,0x37,0x62,0x7a,
 0x21,0x63,0xbe,0x00,0x5a,0x42,0x2f,0x55,0xc0,0x83,0x85,0xc1,
 0xef,0x7d,0x05,0x1c,0xa9,0x45,0x93,0xdf,0x59,0x46,0x31,0x41,
 0x20,0xfa,0xa8,0x71,0x65,0xcb,0xa1,0x31,0xc1,0xda,0x3a,0xac,
 0x42,0x9d,0xc3,0xd9,0x9a,0x9b,0xac,0x7d,0x4c,0x4c,0xbb,0x85,
 0x70,0xb4,0xd5,0xea,0x10,
};
const uint8_t tc_yb[] = {
 0xa0,0xb7,0x68,0xba,0x75,0x55,0x62,0x1d,0x13,0x30,0x12,0xd1,
 0xde,0xe2,0x7a,0x00,0x13,0xc1,0xbc,0xfd,0xdd,0x67,0x58,0x11,
 0xdf,0x12,0x77,0x1e,0x44,0xd7,0x7b,0x10,
};
const uint8_t tc_ADb[] = {
 0x41,0x44,0x62,
};
const uint8_t tc_Yb[] = {
 0x04,0xdf,0x13,0xff,0xa8,0x9b,0x0c,0xe3,0xcc,0x55,0x3b,0x14,
 0x95,0xff,0x02,0x78,0x86,0x56,0x4d,0x94,0xb8,0xd9,0x16,0x5c,
 0xd5,0x0e,0x5f,0x65,0x42,0x47,0x95,0x99,0x51,0xbf,0xac,0x90,
 0x83,0x9f,0xca,0x21,0x8b,0xf8,0xe2,0xd1,0x25,0x8e,0xb7,0xd7,
 0xd9,0xf7,0x33,0xfe,0x4c,0xd5,0x58,0xe6,0xfa,0x57,0xbf,0x1f,
```

```
 0x80,0x1a,0xae,0x7d,0x3a,
};
const uint8_t tc_K[] = {
 0x27,0xf7,0x05,0x9d,0x88,0xf0,0x20,0x07,0xdc,0x18,0xc9,0x11,
 0xc9,0xb4,0x03,0x4d,0x3c,0x0f,0x13,0xf8,0xf7,0xed,0x96,0x03,
 0xb0,0x92,0x7f,0x23,0xfb,0xab,0x10,0x37,
};
const uint8_t tc_ISK_IR[] = {
 0xdd,0xc1,0xb1,0x33,0xc3,0x87,0xec,0xf3,0x44,0xc0,0xb4,0x96,
 0xbc,0x12,0x23,0x65,0x6c,0xd6,0xe7,0xd9,0x9a,0x5d,0xef,0x8b,
 0x3b,0x02,0x67,0x96,0x50,0x81,0x1f,0xc9,
};
const uint8_t tc_ISK_SY[] = {
 0x6e,0xa7,0x75,0xb0,0xfb,0x3c,0x31,0x50,0x26,0x87,0x56,0x5a,
 0x52,0x15,0x0f,0xc5,0x95,0xc6,0x3f,0xe9,0x01,0xa1,0x1d,0x5f,
 0xc1,0x99,0x5c,0xd5,0x08,0x9a,0x17,0xae,
};
```

B.5.8. Test case for scalar_mult_vfy with correct inputs

```
s: (length: 32 bytes)
  f012501c091ff9b99a123fffe571d8bc01e8077ee581362e1bd21399
  0835643b
X: (length: 65 bytes)
  0476ab88669dc640ca098b3d19ed87084d22d7e7c86b3b87451554d6
  93a7d98fb6bf0a6938fe0cec7be7563499ba3792909c8b9f4c936ef5
  2828b78a8d6254f49c
G.scalar_mult(s,X) (full coordinates): (length: 65 bytes)
  0492b0eb1fe6a988797a85e6de8ec5de7ec685c83164570d79f0d568
  b918bfe7718b049dac20ea4631d8c4f321ddb48d70416f4929eb9a85
  2528114d3a560537c7
G.scalar_mult_vfy(s,X) (only X-coordinate):
(length: 32 bytes)
  92b0eb1fe6a988797a85e6de8ec5de7ec685c83164570d79f0d568b9
  18bfe771
```

B.5.9. Invalid inputs for scalar_mult_vfy

For these test cases scalar_mult_vfy(y,.) MUST return the representation of the neutral element G.I. When including Y_i1 or Y_i2 in MSGa or MSGb the protocol MUST abort.

```
s: (length: 32 bytes)
  f012501c091ff9b99a123fffe571d8bc01e8077ee581362e1bd21399
  0835643b
Y_i1: (length: 65 bytes)
  0476ab88669dc640ca098b3d19ed87084d22d7e7c86b3b87451554d6
  93a7d98fb6bf0a6938fe0cec7be7563499ba3792909c8b9f4c936ef5
  2828b78a8d6254f4f3
Y_i2: (length: 1 bytes)
  00
G.scalar_mult_vfy(s,Y_i1) = G.scalar_mult_vfy(s,Y_i2) = G.I
```

B.6. Test vector for CPace using group NIST P-384 and hash SHA-384

B.6.1. Test vectors for calculate_generator with group NIST P-384

```
  Inputs
    H   = SHA-384 with input block size 128 bytes.
    PRS = b'Password' ; ZPAD length: 87 ;
    DSI = b'CPaceP384_XMD:SHA-384_SSWU_NU_'
    CI = b'\nAinitiator\nBresponder'
    CI = 0a41696e69746961746f720a42726573706f6e646572
    sid = 5b3773aa90e8f23c61563a4b645b276c
  Outputs
    generator_string(PRS,G.DSI,CI,sid,H.s_in_bytes):
    (length: 168 bytes)
      1e435061636550333834 5f584d443a5348412d3338345f535357555f
      4e555f0850617373776f726457000000000000000000000000000000
      0000000000000000000000000000000000000000000000000000000000
      0000000000000000000000000000000000000000000000000000000000
      0000000000000000000000000000160a41696e69746961746f72
      0a42726573706f6e646572105b3773aa90e8f23c61563a4b645b276c
    generator g: (length: 97 bytes)
      04bb6f046a601d0a0b134c6221e20e83c3f9ac0390be56c5a95b68eb
      f41c82ade6f4977ea21341239d194c38dabd1a7eb5887d9fed2550a1
      d5e6789327f2a039cd9c41239b240f775f5f2bef8744561b3a7e98f3
      2234cb1b318f66616de777aeef
```

B.6.2. Test vector for MSGa

```
Inputs
  ADa = b'ADa'
  ya (big endian): (length: 48 bytes)
    ef433dd5ad142c860e7cb6400dd315d388d5ec5420c550e9d6f0907f
    375d988bc4d704837e43561c497e7dd93edcdb9d
Outputs
  Ya: (length: 97 bytes)
    047214fc512921b3fa0b555b41d841c9c20227fa1ab0dda5bfc051f6
    de9be7983e6df11d4e8da738b739adfbd85d8f5e80b2b4bbc66f3dff
    c02136ee19773d05f9c0242c0dd51857763de98a2fdfec73a4b1010c
    bc419c7b23b50adedbb3ff6644
  Alternative correct value for Ya: g^(-ya):
  (length: 97 bytes)
    047214fc512921b3fa0b555b41d841c9c20227fa1ab0dda5bfc051f6
    de9be7983e6df11d4e8da738b739adfbd85d8f5e804d4b443990c200
    3fdec911e688c2fa063fdbd3f22ae7a889c21675d020138c5a4efef3
    42be6384dc4af521254c0099bb
  MSGa = lv_cat(Ya,ADa): (length: 102 bytes)
    61047214fc512921b3fa0b555b41d841c9c20227fa1ab0dda5bfc051
    f6de9be7983e6df11d4e8da738b739adfbd85d8f5e80b2b4bbc66f3d
    ffc02136ee19773d05f9c0242c0dd51857763de98a2fdfec73a4b101
    0cbc419c7b23b50adedbb3ff664403414461
```

B.6.3. Test vector for MSGb

```
Inputs
  ADb = b'ADb'
  yb (big endian): (length: 48 bytes)
    50b0e36b95a2edfaa8342b843dddc90b175330f2399c1b36586dedda
    3c255975f30be6a750f9404fccc62a6323b5e471
Outputs
  Yb: (length: 97 bytes)
    04e34cbd45b13ad11552ea7100b19899fa52662e268f2086e21262f7
    46efcb18e4b51ecfaf2e8ebab82addb6245f9bb1ff8138317c8045c4
    d2550e1566832b94acb91b670c4c4c00e59f5c15c74d4260e490caca
    aa860c11b8f369b72d5871bd94
  Alternative correct value for Yb: g^(-yb):
  (length: 97 bytes)
    04e34cbd45b13ad11552ea7100b19899fa52662e268f2086e21262f7
    46efcb18e4b51ecfaf2e8ebab82addb6245f9bb1ff7ec7ce837fba3b
    2daaf1ea997cd46b5346e498f3b3b3ff1a60a3ea38b2bd9f1a6f3535
    5479f3ee470c9648d3a78e426b
  MSGb = lv_cat(Yb,ADb): (length: 102 bytes)
    6104e34cbd45b13ad11552ea7100b19899fa52662e268f2086e21262
    f746efcb18e4b51ecfaf2e8ebab82addb6245f9bb1ff8138317c8045
    c4d2550e1566832b94acb91b670c4c4c00e59f5c15c74d4260e490ca
    caaa860c11b8f369b72d5871bd9403414462
```

B.6.4. Test vector for secret points K

```
scalar_mult_vfy(ya,Yb): (length: 48 bytes)
  e5ef578c410effb4ec114998a59fa5832f6101be479f1a97b021f224
  e378c3fb1f77f87a92e39fb415edf5458b3815bf
scalar_mult_vfy(yb,Ya): (length: 48 bytes)
  e5ef578c410effb4ec114998a59fa5832f6101be479f1a97b021f224
  e378c3fb1f77f87a92e39fb415edf5458b3815bf
```

B.6.5. Test vector for ISK calculation initiator/responder

```
unordered cat of transcript : (length: 204 bytes)
  61047214fc512921b3fa0b555b41d841c9c20227fa1ab0dda5bfc051
  f6de9be7983e6df11d4e8da738b739adfbd85d8f5e80b2b4bbc66f3d
  ffc02136ee19773d05f9c0242c0dd51857763de98a2fdfec73a4b101
  0cbc419c7b23b50adedbb3ff6644034144616104e34cbd45b13ad115
  52ea7100b19899fa52662e268f2086e21262f746efcb18e4b51ecfaf
  2e8ebab82addb6245f9bb1ff8138317c8045c4d2550e1566832b94ac
  b91b670c4c4c00e59f5c15c74d4260e490cacaaa860c11b8f369b72d
  5871bd9403414462
DSI = G.DSI_ISK, b'CPaceP384_XMD:SHA-384_SSWU_NU__ISK':
(length: 34 bytes)
  4350616365503338345f584d443a5348412d3338345f535357555f4e
  555f5f49534b
lv_cat(DSI,sid,K)||MSGa||MSGb: (length: 305 bytes)
  2243506163655033383 45f584d443a5348412d3338345f535357555f
  4e555f5f49534b105b3773aa90e8f23c61563a4b645b276c30e5ef57
  8c410effb4ec114998a59fa5832f6101be479f1a97b021f224e378c3
  fb1f77f87a92e39fb415edf5458b3815bf61047214fc512921b3fa0b
  555b41d841c9c20227fa1ab0dda5bfc051f6de9be7983e6df11d4e8d
  a738b739adfbd85d8f5e80b2b4bbc66f3dffc02136ee19773d05f9c0
  242c0dd51857763de98a2fdfec73a4b1010cbc419c7b23b50adedbb3
  ff6644034144616104e34cbd45b13ad11552ea7100b19899fa52662e
  268f2086e21262f746efcb18e4b51ecfaf2e8ebab82addb6245f9bb1
  ff8138317c8045c4d2550e1566832b94acb91b670c4c4c00e59f5c15
  c74d4260e490cacaaa860c11b8f369b72d5871bd9403414462
ISK result: (length: 48 bytes)
  401601de4a9f25bd57fc85985c9abf1de75191d68306b584547e6ac9
  e959cf2df49a9bf2205c3617ce99a169971bdbf8
```

B.6.6. Test vector for ISK calculation parallel execution

```
ordered cat of transcript : (length: 204 bytes)
  6104e34cbd45b13ad11552ea7100b19899fa52662e268f2086e21262
  f746efcb18e4b51ecfaf2e8ebab82addb6245f9bb1ff8138317c8045
  c4d2550e1566832b94acb91b670c4c4c00e59f5c15c74d4260e490ca
  caaa860c11b8f369b72d5871bd940341446261047214fc512921b3fa
  0b555b41d841c9c20227fa1ab0dda5bfc051f6de9be7983e6df11d4e
  8da738b739adfbd85d8f5e80b2b4bbc66f3dffc02136ee19773d05f9
  c0242c0dd51857763de98a2fdfec73a4b1010cbc419c7b23b50adedb
  b3ff664403414461
DSI = G.DSI_ISK, b'CPaceP384_XMD:SHA-384_SSWU_NU__ISK':
(length: 34 bytes)
  4350616365503338345f584d443a5348412d3338345f535357555f4e
  555f5f49534b
lv_cat(DSI,sid,K)||oCAT(MSGa,MSGb): (length: 305 bytes)
  2243506163655033383 45f584d443a5348412d3338345f535357555f
  4e555f5f49534b105b3773aa90e8f23c61563a4b645b276c30e5ef57
  8c410effb4ec114998a59fa5832f6101be479f1a97b021f224e378c3
  fb1f77f87a92e39fb415edf5458b3815bf6104e34cbd45b13ad11552
  ea7100b19899fa52662e268f2086e21262f746efcb18e4b51ecfaf2e
  8ebab82addb6245f9bb1ff8138317c8045c4d2550e1566832b94acb9
  1b670c4c4c00e59f5c15c74d4260e490cacaaa860c11b8f369b72d58
  71bd940341446261047214fc512921b3fa0b555b41d841c9c20227fa
  1ab0dda5bfc051f6de9be7983e6df11d4e8da738b739adfbd85d8f5e
  80b2b4bbc66f3dffc02136ee19773d05f9c0242c0dd51857763de98a
  2fdfec73a4b1010cbc419c7b23b50adedbb3ff664403414461
ISK result: (length: 48 bytes)
  1eb17f7f7126a07acd510e9d60c84f63dc0113ac34f8d359e8f692a9
  06f828bde926d9ff65202c9801e9884aa05a43b6
```

B.6.7. Corresponding ANSI-C initializers

```
const uint8_t tc_PRS[] = {
 0x50,0x61,0x73,0x73,0x77,0x6f,0x72,0x64,
};
const uint8_t tc_CI[] = {
 0x0a,0x41,0x69,0x6e,0x69,0x74,0x69,0x61,0x74,0x6f,0x72,0x0a,
 0x42,0x72,0x65,0x73,0x70,0x6f,0x6e,0x64,0x65,0x72,
};
const uint8_t tc_sid[] = {
 0x5b,0x37,0x73,0xaa,0x90,0xe8,0xf2,0x3c,0x61,0x56,0x3a,0x4b,
 0x64,0x5b,0x27,0x6c,
};
const uint8_t tc_g[] = {
 0x04,0xbb,0x6f,0x04,0x6a,0x60,0x1d,0x0a,0x0b,0x13,0x4c,0x62,
 0x21,0xe2,0x0e,0x83,0xc3,0xf9,0xac,0x03,0x90,0xbe,0x56,0xc5,
 0xa9,0x5b,0x68,0xeb,0xf4,0x1c,0x82,0xad,0xe6,0xf4,0x97,0x7e,
 0xa2,0x13,0x41,0x23,0x9d,0x19,0x4c,0x38,0xda,0xbd,0x1a,0x7e,
 0xb5,0x88,0x7d,0x9f,0xed,0x25,0x50,0xa1,0xd5,0xe6,0x78,0x93,
 0x27,0xf2,0xa0,0x39,0xcd,0x9c,0x41,0x23,0x9b,0x24,0x0f,0x77,
 0x5f,0x5f,0x2b,0xef,0x87,0x44,0x56,0x1b,0x3a,0x7e,0x98,0xf3,
 0x22,0x34,0xcb,0x1b,0x31,0x8f,0x66,0x61,0x6d,0xe7,0x77,0xae,
 0xef,
};
const uint8_t tc_ya[] = {
 0xef,0x43,0x3d,0xd5,0xad,0x14,0x2c,0x86,0x0e,0x7c,0xb6,0x40,
 0x0d,0xd3,0x15,0xd3,0x88,0xd5,0xec,0x54,0x20,0xc5,0x50,0xe9,
 0xd6,0xf0,0x90,0x7f,0x37,0x5d,0x98,0x8b,0xc4,0xd7,0x04,0x83,
 0x7e,0x43,0x56,0x1c,0x49,0x7e,0x7d,0xd9,0x3e,0xdc,0xdb,0x9d,
};
const uint8_t tc_ADa[] = {
 0x41,0x44,0x61,
};
const uint8_t tc_Ya[] = {
 0x04,0x72,0x14,0xfc,0x51,0x29,0x21,0xb3,0xfa,0x0b,0x55,0x5b,
 0x41,0xd8,0x41,0xc9,0xc2,0x02,0x27,0xfa,0x1a,0xb0,0xdd,0xa5,
 0xbf,0xc0,0x51,0xf6,0xde,0x9b,0xe7,0x98,0x3e,0x6d,0xf1,0x1d,
 0x4e,0x8d,0xa7,0x38,0xb7,0x39,0xad,0xfb,0xd8,0x5d,0x8f,0x5e,
 0x80,0xb2,0xb4,0xbb,0xc6,0x6f,0x3d,0xff,0xc0,0x21,0x36,0xee,
 0x19,0x77,0x3d,0x05,0xf9,0xc0,0x24,0x2c,0x0d,0xd5,0x18,0x57,
 0x76,0x3d,0xe9,0x8a,0x2f,0xdf,0xec,0x73,0xa4,0xb1,0x01,0x0c,
 0xbc,0x41,0x9c,0x7b,0x23,0xb5,0x0a,0xde,0xdb,0xb3,0xff,0x66,
 0x44,
};
const uint8_t tc_yb[] = {
 0x50,0xb0,0xe3,0x6b,0x95,0xa2,0xed,0xfa,0xa8,0x34,0x2b,0x84,
 0x3d,0xdd,0xc9,0x0b,0x17,0x53,0x30,0xf2,0x39,0x9c,0x1b,0x36,
 0x58,0x6d,0xed,0xda,0x3c,0x25,0x59,0x75,0xf3,0x0b,0xe6,0xa7,
 0x50,0xf9,0x40,0x4f,0xcc,0xc6,0x2a,0x63,0x23,0xb5,0xe4,0x71,
};
const uint8_t tc_ADb[] = {
```

```
 0x41,0x44,0x62,
};
const uint8_t tc_Yb[] = {
 0x04,0xe3,0x4c,0xbd,0x45,0xb1,0x3a,0xd1,0x15,0x52,0xea,0x71,
 0x00,0xb1,0x98,0x99,0xfa,0x52,0x66,0x2e,0x26,0x8f,0x20,0x86,
 0xe2,0x12,0x62,0xf7,0x46,0xef,0xcb,0x18,0xe4,0xb5,0x1e,0xcf,
 0xaf,0x2e,0x8e,0xba,0xb8,0x2a,0xdd,0xb6,0x24,0x5f,0x9b,0xb1,
 0xff,0x81,0x38,0x31,0x7c,0x80,0x45,0xc4,0xd2,0x55,0x0e,0x15,
 0x66,0x83,0x2b,0x94,0xac,0xb9,0x1b,0x67,0x0c,0x4c,0x4c,0x00,
 0xe5,0x9f,0x5c,0x15,0xc7,0x4d,0x42,0x60,0xe4,0x90,0xca,0xca,
 0xaa,0x86,0x0c,0x11,0xb8,0xf3,0x69,0xb7,0x2d,0x58,0x71,0xbd,
 0x94,
};
const uint8_t tc_K[] = {
 0xe5,0xef,0x57,0x8c,0x41,0x0e,0xff,0xb4,0xec,0x11,0x49,0x98,
 0xa5,0x9f,0xa5,0x83,0x2f,0x61,0x01,0xbe,0x47,0x9f,0x1a,0x97,
 0xb0,0x21,0xf2,0x24,0xe3,0x78,0xc3,0xfb,0x1f,0x77,0xf8,0x7a,
 0x92,0xe3,0x9f,0xb4,0x15,0xed,0xf5,0x45,0x8b,0x38,0x15,0xbf,
};
const uint8_t tc_ISK_IR[] = {
 0x40,0x16,0x01,0xde,0x4a,0x9f,0x25,0xbd,0x57,0xfc,0x85,0x98,
 0x5c,0x9a,0xbf,0x1d,0xe7,0x51,0x91,0xd6,0x83,0x06,0xb5,0x84,
 0x54,0x7e,0x6a,0xc9,0xe9,0x59,0xcf,0x2d,0xf4,0x9a,0x9b,0xf2,
 0x20,0x5c,0x36,0x17,0xce,0x99,0xa1,0x69,0x97,0x1b,0xdb,0xf8,
};
const uint8_t tc_ISK_SY[] = {
 0x1e,0xb1,0x7f,0x7f,0x71,0x26,0xa0,0x7a,0xcd,0x51,0x0e,0x9d,
 0x60,0xc8,0x4f,0x63,0xdc,0x01,0x13,0xac,0x34,0xf8,0xd3,0x59,
 0xe8,0xf6,0x92,0xa9,0x06,0xf8,0x28,0xbd,0xe9,0x26,0xd9,0xff,
 0x65,0x20,0x2c,0x98,0x01,0xe9,0x88,0x4a,0xa0,0x5a,0x43,0xb6,
};
```

B.6.8. Test case for scalar_mult_vfy with correct inputs

```
s: (length: 48 bytes)
  6e8a99a5cdd408eae98e1b8aed286e7b12adbbdac7f2c628d9060ce9
  2ae0d90bd57a564fd3500fbcce3425dc94ba0ade
X: (length: 97 bytes)
  04a32d8d8e1057d37b090d92f46d0bac1874e6cd7c13774774385c30
  39fa8fa3539884b436e49743d2d6279f5bd69dda5fe79fc6ecfb8547
  bf32d8c64ac51f177a70041a1300944f255eea38ca7e964c9d02c5e7
  e28d744e7cdc0bd80437363999
G.scalar_mult(s,X) (full coordinates): (length: 97 bytes)
  045eb8202664ec20fed23ed6005c7be398174946a0f6a8a2e5fd2fed
  9ca159f22652899f820a2d472f926f57de30035a9d11e8006fb66e79
  f3db5d58bd5688954c7284d1e4a616a935dfb761955be13d29de5745
  074a863140dcc9a5c0056ced3b
G.scalar_mult_vfy(s,X) (only X-coordinate):
(length: 48 bytes)
  5eb8202664ec20fed23ed6005c7be398174946a0f6a8a2e5fd2fed9c
  a159f22652899f820a2d472f926f57de30035a9d
```

B.6.9. Invalid inputs for scalar_mult_vfy

For these test cases scalar_mult_vfy(y,.) MUST return the representation of the neutral element G.I. When including Y_i1 or Y_i2 in MSGa or MSGb the protocol MUST abort.

```
s: (length: 48 bytes)
  6e8a99a5cdd408eae98e1b8aed286e7b12adbbdac7f2c628d9060ce9
  2ae0d90bd57a564fd3500fbcce3425dc94ba0ade
Y_i1: (length: 97 bytes)
  04a32d8d8e1057d37b090d92f46d0bac1874e6cd7c13774774385c30
  39fa8fa3539884b436e49743d2d6279f5bd69dda5fe79fc6ecfb8547
  bf32d8c64ac51f177a70041a1300944f255eea38ca7e964c9d02c5e7
  e28d744e7cdc0bd80437363938
Y_i2: (length: 1 bytes)
  00
G.scalar_mult_vfy(s,Y_i1) = G.scalar_mult_vfy(s,Y_i2) = G.I
```

B.7. Test vector for CPace using group NIST P-521 and hash SHA-512

B.7.1. Test vectors for calculate_generator with group NIST P-521

```
  Inputs
    H   = SHA-512 with input block size 128 bytes.
    PRS = b'Password' ; ZPAD length: 87 ;
    DSI = b'CPaceP521_XMD:SHA-512_SSWU_NU_'
    CI = b'\nAinitiator\nBresponder'
    CI = 0a41696e69746961746f720a42726573706f6e646572
    sid = 7e4b4791d6a8ef019b936c79fb7f2c57
  Outputs
    generator_string(PRS,G.DSI,CI,sid,H.s_in_bytes):
    (length: 168 bytes)
      1e435061636550353231 5f584d443a5348412d3531325f535357555f
      4e555f0850617373776f726457000000000000000000000000000000
      0000000000000000000000000000000000000000000000000000000000
      0000000000000000000000000000000000000000000000000000000000
      0000000000000000000000000000160a41696e69746961746f72
      0a42726573706f6e646572107e4b4791d6a8ef019b936c79fb7f2c57
    generator g: (length: 133 bytes)
      0400523c2be75a6fdb50e33d917597f182810ea6afe04b7297fccdfc
      f8c1c9f0f1a0c794056c729c275a654d1f9f52cd3d1d0ecc8f2f6a1b
      ab958d36cc539c558496a901bbe4fd573f2a6e6cc0c9afee3ee25c4b
      6f0474dd012eff5af0cbf55c4ec3c0ab4f1187353f815eb2a01ebc52
      d076d45a77a9b86d14fb21066df1d09f10b0a97546
```

B.7.2. Test vector for MSGa

```
  Inputs
    ADa = b'ADa'
    ya (big endian): (length: 66 bytes)
      016fac7bb757452e7b788d68a1510eda90113c65db1213fa08927d50
      bcf2635fd66ca254e82927071001353e265082fd609af47ad06fab42
      0c2295df4056ee9ff997
  Outputs
    Ya: (length: 133 bytes)
      0400484dcee6d54cb356830cd764079360a03b06a7db1a82188e09c9
      2e02d7e78a1e3710da9554db11697d242893e2114d6cbee89f5999b7
      e545d9fdf59f4c9acd408901ad73e01ec22ae6ecc122cf257e81826e
      348cd410ddb9245c61889fe97b2bbb98b2038eb2ed23e989ec7013a6
      10fb2f3b4fb958cc860dd10c98745b9d89e37f2bf9
    Alternative correct value for Ya: g^(-ya):
    (length: 133 bytes)
      0400484dcee6d54cb356830cd764079360a03b06a7db1a82188e09c9
      2e02d7e78a1e3710da9554db11697d242893e2114d6cbee89f5999b7
      e545d9fdf59f4c9acd408900528c1fe13dd519133edd30da817e7d91
      cb732bef2246dba39e77601684d444674dfc714d12dc1676138fec59
      ef04d0c4b046a73379f22ef3678ba462761c80d406
    MSGa = lv_cat(Ya,ADa): (length: 139 bytes)
      85010400484dcee6d54cb356830cd764079360a03b06a7db1a82188e
      09c92e02d7e78a1e3710da9554db11697d242893e2114d6cbee89f59
      99b7e545d9fdf59f4c9acd408901ad73e01ec22ae6ecc122cf257e81
      826e348cd410ddb9245c61889fe97b2bbb98b2038eb2ed23e989ec70
      13a610fb2f3b4fb958cc860dd10c98745b9d89e37f2bf903414461
```

B.7.3. Test vector for MSGb

```
  Inputs
    ADb = b'ADb'
    yb (big endian): (length: 66 bytes)
      011a946e2d0f48dc440ae3f4fd9126198237042fd1d41d037068c284
      6d43ec130cbc55ef1208496be068f8682bcaf6156e51598e27c1fb24
      d77b43957bbc129bab80
  Outputs
    Yb: (length: 133 bytes)
      0401edf767bd7d9e67ff137b8f3210c55e9192e9ac8a10f32a2f0eef
      9ce34524a543e0d4eb9b3328ca114b02ab23b291f61b5bc814639a9e
      caff07e870733131747637004c2df1bec8abe6b252e7fe91bdb6f724
      2e65c36e7b960646c89aaf0262a4803ee4c90d1b58775a409a135bd1
      8fedbf4ba0eae172b4fe8a0fada83d699e44f2f861
    Alternative correct value for Yb: g^(-yb):
    (length: 133 bytes)
      0401edf767bd7d9e67ff137b8f3210c55e9192e9ac8a10f32a2f0eef
      9ce34524a543e0d4eb9b3328ca114b02ab23b291f61b5bc814639a9e
      caff07e8707331317476370 1b3d20e413754194dad18016e424908db
      d19a3c918469f9b9376550fd9d5b7fc11b36f2e4a788a5bf65eca42e
      701240b45f151e8d4b0175f05257c29661bb0d079e
    MSGb = lv_cat(Yb,ADb): (length: 139 bytes)
      85010401edf767bd7d9e67ff137b8f3210c55e9192e9ac8a10f32a2f
      0eef9ce34524a543e0d4eb9b3328ca114b02ab23b291f61b5bc81463
      9a9ecaff07e870733131747637004c2df1bec8abe6b252e7fe91bdb6
      f7242e65c36e7b960646c89aaf0262a4803ee4c90d1b58775a409a13
      5bd18fedbf4ba0eae172b4fe8a0fada83d699e44f2f86103414462
```

B.7.4. Test vector for secret points K

```
    scalar_mult_vfy(ya,Yb): (length: 66 bytes)
      0070a7460122c65d86bf9dd012ab45fc94be362619d1a1f0e75f1433
      3ed8b873b5724616b88dadaaba5f28bb783aeb01f60df5fdb8c0a237
      45900f462f405debfd51
    scalar_mult_vfy(yb,Ya): (length: 66 bytes)
      0070a7460122c65d86bf9dd012ab45fc94be362619d1a1f0e75f1433
      3ed8b873b5724616b88dadaaba5f28bb783aeb01f60df5fdb8c0a237
      45900f462f405debfd51
```

B.7.5. Test vector for ISK calculation initiator/responder

```
unordered cat of transcript : (length: 278 bytes)
  85010400484dcee6d54cb356830cd764079360a03b06a7db1a82188e
  09c92e02d7e78a1e3710da9554db11697d242893e2114d6cbee89f59
  99b7e545d9fdf59f4c9acd408901ad73e01ec22ae6ecc122cf257e81
  826e348cd410ddb9245c61889fe97b2bbb98b2038eb2ed23e989ec70
  13a610fb2f3b4fb958cc860dd10c98745b9d89e37f2bf90341446185
  010401edf767bd7d9e67ff137b8f3210c55e9192e9ac8a10f32a2f0e
  ef9ce34524a543e0d4eb9b3328ca114b02ab23b291f61b5bc814639a
  9ecaff07e870733131747637004c2df1bec8abe6b252e7fe91bdb6f7
  242e65c36e7b960646c89aaf0262a4803ee4c90d1b58775a409a135b
  d18fedbf4ba0eae172b4fe8a0fada83d699e44f2f86103414462
DSI = G.DSI_ISK, b'CPaceP521_XMD:SHA-512_SSWU_NU__ISK':
(length: 34 bytes)
  43506163655035323 15f584d443a5348412d3531325f535357555f4e
  555f5f49534b
lv_cat(DSI,sid,K)||MSGa||MSGb: (length: 397 bytes)
  22435061636550353231 5f584d443a5348412d3531325f535357555f
  4e555f5f49534b107e4b4791d6a8ef019b936c79fb7f2c57420070a7
  460122c65d86bf9dd012ab45fc94be362619d1a1f0e75f14333ed8b8
  73b5724616b88dadaaba5f28bb783aeb01f60df5fdb8c0a23745900f
  462f405debfd5185010400484dcee6d54cb356830cd764079360a03b
  06a7db1a82188e09c92e02d7e78a1e3710da9554db11697d242893e2
  114d6cbee89f5999b7e545d9fdf59f4c9acd408901ad73e01ec22ae6
  ecc122cf257e81826e348cd410ddb9245c61889fe97b2bbb98b2038e
  b2ed23e989ec7013a610fb2f3b4fb958cc860dd10c98745b9d89e37f
  2bf90341446185010401edf767bd7d9e67ff137b8f3210c55e9192e9
  ac8a10f32a2f0eef9ce34524a543e0d4eb9b3328ca114b02ab23b291
  f61b5bc814639a9ecaff07e870733131747637004c2df1bec8abe6b2
  52e7fe91bdb6f7242e65c36e7b960646c89aaf0262a4803ee4c90d1b
  58775a409a135bd18fedbf4ba0eae172b4fe8a0fada83d699e44f2f8
  6103414462
ISK result: (length: 64 bytes)
  2b2c534c352c446773bd334fac2f2c50ef8cd7991bd4e070f85b0367
  a2f7ffca445066cf20b756773687e1038b170896ec2677fe722acb0f
  9e6c2f11830e808a
```

B.7.6. Test vector for ISK calculation parallel execution

```
ordered cat of transcript : (length: 278 bytes)
  85010401edf767bd7d9e67ff137b8f3210c55e9192e9ac8a10f32a2f
  0eef9ce34524a543e0d4eb9b3328ca114b02ab23b291f61b5bc81463
  9a9ecaff07e870733131747637004c2df1bec8abe6b252e7fe91bdb6
  f7242e65c36e7b960646c89aaf0262a4803ee4c90d1b58775a409a13
  5bd18fedbf4ba0eae172b4fe8a0fada83d699e44f2f8610341446285
  010400484dcee6d54cb356830cd764079360a03b06a7db1a82188e09
  c92e02d7e78a1e3710da9554db11697d242893e2114d6cbee89f5999
  b7e545d9fdf59f4c9acd408901ad73e01ec22ae6ecc122cf257e8182
  6e348cd410ddb9245c61889fe97b2bbb98b2038eb2ed23e989ec7013
  a610fb2f3b4fb958cc860dd10c98745b9d89e37f2bf903414461
DSI = G.DSI_ISK, b'CPaceP521_XMD:SHA-512_SSWU_NU__ISK':
(length: 34 bytes)
  435061636550353231 5f584d443a5348412d3531325f535357555f4e
  555f5f49534b
lv_cat(DSI,sid,K)||oCAT(MSGa,MSGb): (length: 397 bytes)
  224350616365503532315f584d443a5348412d3531325f535357555f
  4e555f5f49534b107e4b4791d6a8ef019b936c79fb7f2c57420070a7
  460122c65d86bf9dd012ab45fc94be362619d1a1f0e75f14333ed8b8
  73b5724616b88dadaaba5f28bb783aeb01f60df5fdb8c0a23745900f
  462f405debfd5185010401edf767bd7d9e67ff137b8f3210c55e9192
  e9ac8a10f32a2f0eef9ce34524a543e0d4eb9b3328ca114b02ab23b2
  91f61b5bc814639a9ecaff07e870733131747637004c2df1bec8abe6
  b252e7fe91bdb6f7242e65c36e7b960646c89aaf0262a4803ee4c90d
  1b58775a409a135bd18fedbf4ba0eae172b4fe8a0fada83d699e44f2
  f861034144628501040 0484dcee6d54cb356830cd764079360a03b06
  a7db1a82188e09c92e02d7e78a1e3710da9554db11697d242893e211
  4d6cbee89f5999b7e545d9fdf59f4c9acd408901ad73e01ec22ae6ec
  c122cf257e81826e348cd410ddb9245c61889fe97b2bbb98b2038eb2
  ed23e989ec7013a610fb2f3b4fb958cc860dd10c98745b9d89e37f2b
  f903414461
ISK result: (length: 64 bytes)
  78c4dd7136309a2bbe1fdef3cf24a08690006b0c9de253b770c147dd
  0800681c82e4e67a388ed1cd9182e595b8e9e3f2976a0e6dab48b2cd
  205b19489e20f571
```

B.7.7. Corresponding ANSI-C initializers

```
const uint8_t tc_PRS[] = {
 0x50,0x61,0x73,0x73,0x77,0x6f,0x72,0x64,
};
const uint8_t tc_CI[] = {
 0x0a,0x41,0x69,0x6e,0x69,0x74,0x69,0x61,0x74,0x6f,0x72,0x0a,
 0x42,0x72,0x65,0x73,0x70,0x6f,0x6e,0x64,0x65,0x72,
};
const uint8_t tc_sid[] = {
 0x7e,0x4b,0x47,0x91,0xd6,0xa8,0xef,0x01,0x9b,0x93,0x6c,0x79,
 0xfb,0x7f,0x2c,0x57,
};
const uint8_t tc_g[] = {
 0x04,0x00,0x52,0x3c,0x2b,0xe7,0x5a,0x6f,0xdb,0x50,0xe3,0x3d,
 0x91,0x75,0x97,0xf1,0x82,0x81,0x0e,0xa6,0xaf,0xe0,0x4b,0x72,
 0x97,0xfc,0xcd,0xfc,0xf8,0xc1,0xc9,0xf0,0xf1,0xa0,0xc7,0x94,
 0x05,0x6c,0x72,0x9c,0x27,0x5a,0x65,0x4d,0x1f,0x9f,0x52,0xcd,
 0x3d,0x1d,0x0e,0xcc,0x8f,0x2f,0x6a,0x1b,0xab,0x95,0x8d,0x36,
 0xcc,0x53,0x9c,0x55,0x84,0x96,0xa9,0x01,0xbb,0xe4,0xfd,0x57,
 0x3f,0x2a,0x6e,0x6c,0xc0,0xc9,0xaf,0xee,0x3e,0xe2,0x5c,0x4b,
 0x6f,0x04,0x74,0xdd,0x01,0x2e,0xff,0x5a,0xf0,0xcb,0xf5,0x5c,
 0x4e,0xc3,0xc0,0xab,0x4f,0x11,0x87,0x35,0x3f,0x81,0x5e,0xb2,
 0xa0,0x1e,0xbc,0x52,0xd0,0x76,0xd4,0x5a,0x77,0xa9,0xb8,0x6d,
 0x14,0xfb,0x21,0x06,0x6d,0xf1,0xd0,0x9f,0x10,0xb0,0xa9,0x75,
 0x46,
};
const uint8_t tc_ya[] = {
 0x01,0x6f,0xac,0x7b,0xb7,0x57,0x45,0x2e,0x7b,0x78,0x8d,0x68,
 0xa1,0x51,0x0e,0xda,0x90,0x11,0x3c,0x65,0xdb,0x12,0x13,0xfa,
 0x08,0x92,0x7d,0x50,0xbc,0xf2,0x63,0x5f,0xd6,0x6c,0xa2,0x54,
 0xe8,0x29,0x27,0x07,0x10,0x01,0x35,0x3e,0x26,0x50,0x82,0xfd,
 0x60,0x9a,0xf4,0x7a,0xd0,0x6f,0xab,0x42,0x0c,0x22,0x95,0xdf,
 0x40,0x56,0xee,0x9f,0xf9,0x97,
};
const uint8_t tc_ADa[] = {
 0x41,0x44,0x61,
};
const uint8_t tc_Ya[] = {
 0x04,0x00,0x48,0x4d,0xce,0xe6,0xd5,0x4c,0xb3,0x56,0x83,0x0c,
 0xd7,0x64,0x07,0x93,0x60,0xa0,0x3b,0x06,0xa7,0xdb,0x1a,0x82,
 0x18,0x8e,0x09,0xc9,0x2e,0x02,0xd7,0xe7,0x8a,0x1e,0x37,0x10,
 0xda,0x95,0x54,0xdb,0x11,0x69,0x7d,0x24,0x28,0x93,0xe2,0x11,
 0x4d,0x6c,0xbe,0xe8,0x9f,0x59,0x99,0xb7,0xe5,0x45,0xd9,0xfd,
 0xf5,0x9f,0x4c,0x9a,0xcd,0x40,0x89,0x01,0xad,0x73,0xe0,0x1e,
 0xc2,0x2a,0xe6,0xec,0xc1,0x22,0xcf,0x25,0x7e,0x81,0x82,0x6e,
 0x34,0x8c,0xd4,0x10,0xdd,0xb9,0x24,0x5c,0x61,0x88,0x9f,0xe9,
 0x7b,0x2b,0xbb,0x98,0xb2,0x03,0x8e,0xb2,0xed,0x23,0xe9,0x89,
 0xec,0x70,0x13,0xa6,0x10,0xfb,0x2f,0x3b,0x4f,0xb9,0x58,0xcc,
 0x86,0x0d,0xd1,0x0c,0x98,0x74,0x5b,0x9d,0x89,0xe3,0x7f,0x2b,
 0xf9,
```

```
};
const uint8_t tc_yb[] = {
 0x01,0x1a,0x94,0x6e,0x2d,0x0f,0x48,0xdc,0x44,0x0a,0xe3,0xf4,
 0xfd,0x91,0x26,0x19,0x82,0x37,0x04,0x2f,0xd1,0xd4,0x1d,0x03,
 0x70,0x68,0xc2,0x84,0x6d,0x43,0xec,0x13,0x0c,0xbc,0x55,0xef,
 0x12,0x08,0x49,0x6b,0xe0,0x68,0xf8,0x68,0x2b,0xca,0xf6,0x15,
 0x6e,0x51,0x59,0x8e,0x27,0xc1,0xfb,0x24,0xd7,0x7b,0x43,0x95,
 0x7b,0xbc,0x12,0x9b,0xab,0x80,
};
const uint8_t tc_ADb[] = {
 0x41,0x44,0x62,
};
const uint8_t tc_Yb[] = {
 0x04,0x01,0xed,0xf7,0x67,0xbd,0x7d,0x9e,0x67,0xff,0x13,0x7b,
 0x8f,0x32,0x10,0xc5,0x5e,0x91,0x92,0xe9,0xac,0x8a,0x10,0xf3,
 0x2a,0x2f,0x0e,0xef,0x9c,0xe3,0x45,0x24,0xa5,0x43,0xe0,0xd4,
 0xeb,0x9b,0x33,0x28,0xca,0x11,0x4b,0x02,0xab,0x23,0xb2,0x91,
 0xf6,0x1b,0x5b,0xc8,0x14,0x63,0x9a,0x9e,0xca,0xff,0x07,0xe8,
 0x70,0x73,0x31,0x31,0x74,0x76,0x37,0x00,0x4c,0x2d,0xf1,0xbe,
 0xc8,0xab,0xe6,0xb2,0x52,0xe7,0xfe,0x91,0xbd,0xb6,0xf7,0x24,
 0x2e,0x65,0xc3,0x6e,0x7b,0x96,0x06,0x46,0xc8,0x9a,0xaf,0x02,
 0x62,0xa4,0x80,0x3e,0xe4,0xc9,0x0d,0x1b,0x58,0x77,0x5a,0x40,
 0x9a,0x13,0x5b,0xd1,0x8f,0xed,0xbf,0x4b,0xa0,0xea,0xe1,0x72,
 0xb4,0xfe,0x8a,0x0f,0xad,0xa8,0x3d,0x69,0x9e,0x44,0xf2,0xf8,
 0x61,
};
const uint8_t tc_K[] = {
 0x00,0x70,0xa7,0x46,0x01,0x22,0xc6,0x5d,0x86,0xbf,0x9d,0xd0,
 0x12,0xab,0x45,0xfc,0x94,0xbe,0x36,0x26,0x19,0xd1,0xa1,0xf0,
 0xe7,0x5f,0x14,0x33,0x3e,0xd8,0xb8,0x73,0xb5,0x72,0x46,0x16,
 0xb8,0x8d,0xad,0xaa,0xba,0x5f,0x28,0xbb,0x78,0x3a,0xeb,0x01,
 0xf6,0x0d,0xf5,0xfd,0xb8,0xc0,0xa2,0x37,0x45,0x90,0x0f,0x46,
 0x2f,0x40,0x5d,0xeb,0xfd,0x51,
};
const uint8_t tc_ISK_IR[] = {
 0x2b,0x2c,0x53,0x4c,0x35,0x2c,0x44,0x67,0x73,0xbd,0x33,0x4f,
 0xac,0x2f,0x2c,0x50,0xef,0x8c,0xd7,0x99,0x1b,0xd4,0xe0,0x70,
 0xf8,0x5b,0x03,0x67,0xa2,0xf7,0xff,0xca,0x44,0x50,0x66,0xcf,
 0x20,0xb7,0x56,0x77,0x36,0x87,0xe1,0x03,0x8b,0x17,0x08,0x96,
 0xec,0x26,0x77,0xfe,0x72,0x2a,0xcb,0x0f,0x9e,0x6c,0x2f,0x11,
 0x83,0x0e,0x80,0x8a,
};
const uint8_t tc_ISK_SY[] = {
 0x78,0xc4,0xdd,0x71,0x36,0x30,0x9a,0x2b,0xbe,0x1f,0xde,0xf3,
 0xcf,0x24,0xa0,0x86,0x90,0x00,0x6b,0x0c,0x9d,0xe2,0x53,0xb7,
 0x70,0xc1,0x47,0xdd,0x08,0x00,0x68,0x1c,0x82,0xe4,0xe6,0x7a,
 0x38,0x8e,0xd1,0xcd,0x91,0x82,0xe5,0x95,0xb8,0xe9,0xe3,0xf2,
 0x97,0x6a,0x0e,0x6d,0xab,0x48,0xb2,0xcd,0x20,0x5b,0x19,0x48,
```

```
 0x9e,0x20,0xf5,0x71,
};
```

B.7.8. Test case for scalar_mult_vfy with correct inputs

```
s: (length: 66 bytes)
  0182dd7925f1753419e4bf83429763acd37d64000cd5a175edf53a15
  87dd986bc95acc1506991702b6ba1a9ee2458fee8efc00198cf0088c
  480965ef65ff2048b856
X: (length: 133 bytes)
  0400bf0a2632f954515e65c55553e25cde4c8bf3a48e5df86a3ef845
  fcf15c8d9a4640171188ff835df48b8f934070d225daa591e270a9cc
  539b82e8dc145caf38aeb900c30b83a1c9792e95c4a25f75b58001d3
  6331c2b71a86591e1b510a1740335bc9947da1f6bab91b86900c9258
  b28ee7b5ea33af2a8138a75cde4287613ab6673bcc
G.scalar_mult(s,X) (full coordinates): (length: 133 bytes)
  040100763e7ebe6a051e2195b1980686a2a5d7edbc1d9284e38d1e9e
  13673b65b6b3b5cb1b1ab146a315c32425edee8fdca06a07cf72d26d
  31e38ec6a38481b4f18d8600b2a7df9cc7db6cbf75b2eee98f9f14e5
  e24a789d45b9709278e8b74b30eb32d55fb8cfea4258dcf9de7fb36a
  67326584d5c8121c4802801115b908b937361c9828
G.scalar_mult_vfy(s,X) (only X-coordinate):
(length: 66 bytes)
  0100763e7ebe6a051e2195b1980686a2a5d7edbc1d9284e38d1e9e13
  673b65b6b3b5cb1b1ab146a315c32425edee8fdca06a07cf72d26d31
  e38ec6a38481b4f18d86
```

B.7.9. Invalid inputs for scalar_mult_vfy

For these test cases scalar_mult_vfy(y,.) MUST return the representation of the neutral element G.I. When including Y_i1 or Y_i2 in MSGa or MSGb the protocol MUST abort.

```
s: (length: 66 bytes)
  0182dd7925f1753419e4bf83429763acd37d64000cd5a175edf53a15
  87dd986bc95acc1506991702b6ba1a9ee2458fee8efc00198cf0088c
  480965ef65ff2048b856
Y_i1: (length: 133 bytes)
  0400bf0a2632f954515e65c55553e25cde4c8bf3a48e5df86a3ef845
  fcf15c8d9a4640171188ff835df48b8f934070d225daa591e270a9cc
  539b82e8dc145caf38aeb900c30b83a1c9792e95c4a25f75b58001d3
  6331c2b71a86591e1b510a1740335bc9947da1f6bab91b86900c9258
  b28ee7b5ea33af2a8138a75cde4287613ab6673b3a
Y_i2: (length: 1 bytes)
  00
G.scalar_mult_vfy(s,Y_i1) = G.scalar_mult_vfy(s,Y_i2) = G.I
```

Authors' Addresses

Michel Abdalla
DFINITY - Zurich

Email: michel.abdalla@gmail.com

Bjoern Haase
Endress + Hauser Liquid Analysis - Gerlingen

Email: bjoern.m.haase@web.de

Julia Hesse
IBM Research Europe - Zurich

Email: JHS@zurich.ibm.com