Abstract

This document describes SPAKE2 which is a protocol for two parties that share a password to derive a strong shared key without disclosing the password. This method is compatible with any group, is computationally efficient, and SPAKE2 has a security proof. This document predated the CFRG PAKE competition and it was not selected, however, given existing use of variants in Kerberos and other applications it was felt publication was beneficial. Applications that need a symmetric PAKE (password authenticated key exchange) and where hashing onto an elliptic curve at execution time is not possible can use SPAKE2. This document is a product of the Crypto Forum Research Group (CFRG) in the IRTF.

Status of This Memo

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1. Introduction

This document describes SPAKE2, a means for two parties that share a password to derive a strong shared key without disclosing the password. This password-based key exchange protocol is compatible with any group (requiring only a scheme to map a random input of fixed length per group to a random group element), is computationally efficient, and has a security proof. Predetermined parameters for a selection of commonly used groups are also provided for use by other protocols.

SPAKE2 was not selected as the result of the CFRG PAKE selection competition. However, given existing use of variants in Kerberos and other applications it was felt publication was beneficial. This RFC represents the individual opinion(s) of one or more members of the Crypto Forum Research Group of the Internet Research Task Force (IRTF).

Many of these applications predated methods to hash to elliptic
curves being available or predated the publication of the PAKEs that were chosen as an outcome of the PAKE selection competition. In cases where a symmetric PAKE is needed, and hashing onto an elliptic curve at protocol execution time is not available, SPAKE2 is useful.

2. Requirements Notation

The key words "MUST", "MUST NOT", "REQUIRED", "SHALL", "SHALL NOT", "SHOULD", "SHOULD NOT", "RECOMMENDED", "NOT RECOMMENDED", "MAY", and "OPTIONAL" in this document are to be interpreted as described in BCP 14 [RFC2119] [RFC8174] when, and only when, they appear in all capitals, as shown here.

3. Definition of SPAKE2

3.1. Protocol Flow

SPAKE2 is a two round protocol, wherein the first round establishes a shared secret between A and B, and the second round serves as key confirmation. Prior to invocation, A and B are provisioned with information such as the input password needed to run the protocol. We assume that the roles of A and B are agreed upon by both sides: A goes first and uses M, and B goes second and uses N. If this assignment of roles is not possible a symmetric variant MUST be used as described later Section 5. For instance A may be the client when using TCP or TLS as an underlying protocol and B the server. Most protocols have such a distinction. During the first round, A sends a public value pA to B, and B responds with its own public value pB. Both A and B then derive a shared secret used to produce encryption and authentication keys. The latter are used during the second round for key confirmation. (Section 4 details the key derivation and confirmation steps.) In particular, A sends a key confirmation message cA to B, and B responds with its own key confirmation message cB. A MUST NOT consider the protocol complete until it receives and verifies cB. Likewise, B MUST NOT consider the protocol complete until it receives and verifies cA.

This sample flow is shown below.
3.2. Setup

Let $G$ be a group in which the gap Diffie-Hellman (GDH) problem is hard. Suppose $G$ has order $p \times h$ where $p$ is a large prime; $h$ will be called the cofactor. Let $I$ be the unit element in $G$, e.g., the point at infinity if $G$ is an elliptic curve group. We denote the operations in the group additively. We assume there is a representation of elements of $G$ as byte strings: common choices would be SEC1 [SEC1] uncompressed or compressed for elliptic curve groups or big endian integers of a fixed (per-group) length for prime field DH. Applications MUST specify this encoding, typically by referring to the document defining the group. We fix two elements $M$ and $N$ in
the prime-order subgroup of \( G \) as defined in the table in this document for common groups, as well as a generator \( P \) of the (large) prime-order subgroup of \( G \). In the case of a composite order group we will work in the quotient group. For common groups used in this document, \( P \) is specified in the document defining the group, and so we do not repeat it here.

For elliptic curves other than the ones in this document the methods of [I-D.irtf-cfrg-hash-to-curve] SHOULD be used to generate \( M \) and \( N \), e.g. via 
\[
M = \text{hash\_to\_curve}("M \text{ SPAKE2 seed OID } x") \\
N = \text{hash\_to\_curve}("N \text{ SPAKE2 seed OID } x")
\]
where \( x \) is an OID for the curve. Applications MAY include a DST tag in this step, as specified in [I-D.irtf-cfrg-hash-to-curve], though this is not required.

\( || \) denotes concatenation of byte strings. We also let \( \text{len}(S) \) denote the length of a string in bytes, represented as an eight-byte little-endian number. Finally, let nil represent an empty string, i.e., \( \text{len}(\text{nil}) = 0 \). Text strings in double quotes are treated as their ASCII encodings throughout this document.

KDF(ikm, salt, info, L) is a key-derivation function that takes as input a salt, intermediate keying material (IKM), info string, and derived key length L to derive a cryptographic key of length L. MAC(key, message) is a Message Authentication Code algorithm that takes a secret key and message as input to produce an output. Let Hash be a hash function from arbitrary strings to bit strings of a fixed length, at least 256 bits long. Common choices for Hash are SHA256 or SHA512 [RFC6234]. Let MHF be a memory-hard hash function designed to slow down brute-force attackers. Scrypt [RFC7914] is a common example of this function. The output length of MHF matches that of Hash. Parameter selection for MHF is out of scope for this document. Section 6 specifies variants of KDF, MAC, and Hash suitable for use with the protocols contained herein.

Let A and B be two parties. A and B may also have digital representations of the parties' identities such as Media Access Control addresses or other names (hostnames, usernames, etc). A and B may share Additional Authenticated Data (AAD) of length at most \( 2^{16} - 128 \) bits that is separate from their identities which they may want to include in the protocol execution. One example of AAD is a list of supported protocol versions if SPAKE2 were used in a higher-level protocol which negotiates use of a particular PAKE. Including
this list would ensure that both parties agree upon the same set of supported protocols and therefore prevent downgrade attacks. We also assume A and B share an integer w; typically w = MHF(pw) mod p, for a user-supplied password pw. Standards such as NIST.SP.800-56Ar3 suggest taking mod p of a hash value that is 64 bits longer than that needed to represent p to remove statistical bias introduced by the modulation. Protocols using this specification MUST define the method used to compute w. In some cases, it may be necessary to carry out various forms of normalization of the password before hashing [RFC8265]. The hashing algorithm SHOULD be a MHF so as to slow down brute-force attackers.

3.3. SPAKE2

To begin, A picks x randomly and uniformly from the integers in [0,p), and calculates X=x*P and pA=w*M+X, then transmits pA to B.

B selects y randomly and uniformly from the integers in [0,p), and calculates Y=y*P, pB=w*N+Y, then transmits pB to A.

Both A and B calculate a group element K. A calculates it as h*x*(pB-w*N), while B calculates it as h*y*(pA-w*M). A knows pB because it has received it, and likewise B knows pA. The multiplication by h prevents small subgroup confinement attacks by computing a unique value in the quotient group.

K is a shared value, though it MUST NOT be used or output as a shared secret from the protocol. Both A and B must derive two additional shared secrets from the protocol transcript, which includes K. This prevents man-in-the-middle attackers from inserting themselves into the exchange. The transcript TT is encoded as follows:

\[ TT = \text{len}(A) \ || \ A \ || \ \text{len}(B) \ || \ B \ || \ \text{len}(pA) \ || \ pA \ || \ \text{len}(pB) \ || \ pB \ || \ \text{len}(K) \ || \ K \ || \ \text{len}(w) \ || \ w \]

Here w is encoded as a big endian number padded to the length of p. This representation prevents timing attacks that otherwise would
reveal the length of \( w \). \( \text{len}(w) \) is thus a constant. We include it for consistency.

If an identity is absent, it is encoded as a zero-length string. This MUST only be done for applications in which identities are implicit. Otherwise, the protocol risks unknown key share attacks, where both sides of a connection disagree over who is authenticated.

Upon completion of this protocol, A and B compute shared secrets \( K_e, K_{cA}, \) and \( K_{cB} \) as specified in Section 4. A MUST send B a key confirmation message so both parties agree upon these shared secrets. This confirmation message \( c_A \) is computed as a MAC over the protocol transcript \( TT \) using \( K_{cA} \), as follows: \( c_A = \text{MAC}(K_{cA}, TT) \). Similarly, B MUST send A a confirmation message using a MAC computed equivalently except with the use of \( K_{cB} \). Key confirmation verification requires computing \( c_B \) and checking for equality against that which was received.

4. Key Schedule and Key Confirmation

The protocol transcript \( TT \), as defined in Section 3.3, is unique and secret to A and B. Both parties use \( TT \) to derive shared symmetric secrets \( K_e \) and \( K_a \) as \( K_e \| K_a = \text{Hash}(TT) \), with \( |K_e| = |K_a| \). The length of each key is equal to half of the digest output, e.g., 128 bits for SHA-256. Keys MUST be at least 128 bits in length.

Both endpoints use \( K_a \) to derive subsequent MAC keys for key confirmation messages. Specifically, let \( K_{cA} \) and \( K_{cB} \) be the MAC keys used by A and B, respectively. A and B compute them as \( K_{cA} \| K_{cB} = \text{KDF}(K_a, \text{nil}, "\text{ConfirmationKeys}" \| \text{AAD}, L) \), where \( \text{AAD} \) is the associated data each given to each endpoint, or \( \text{nil} \) if none was provided. The length of each of \( K_{cA} \) and \( K_{cB} \) is equal to half of the underlying hash output length, e.g., \( |K_{cA}| = |K_{cB}| = 128 \) bits for \( \text{HKDF}(\text{SHA256}) \), with \( L=256 \) bits.
The resulting key schedule for this protocol, given transcript TT and additional associated data AAD, is as follows.

\[
\begin{align*}
TT \rightarrow \text{Hash}(TT) &= Ke || Ka \\
AAD \rightarrow \text{KDF}(Ka, \text{nil}, "\text{ConfirmationKeys}" || AAD) &= KcA || KcB
\end{align*}
\]

A and B output Ke as the shared secret from the protocol. Ka and its derived keys are not used for anything except key confirmation.

5. Per-User M and N and M=N

To avoid concerns that an attacker needs to solve a single ECDH instance to break the authentication of SPAKE2, it is possible to vary M and N using \[\text{I-D.irtf-cfrg-hash-to-curve}\] as follows:

\[
\begin{align*}
M &= \text{hash_to_curve}(\text{Hash}("M \text{ SPAKE2}" || \text{len}(A) \ || \ A \ || \ \text{len}(B) \ || \ B)) \\
N &= \text{hash_to_curve}(\text{Hash}("N \text{ SPAKE2}" || \text{len}(A) \ || \ A \ || \ \text{len}(B) \ || \ B))
\end{align*}
\]

There is also a symmetric variant where M=N. For this variant we set

\[
M = \text{hash_to_curve}(\text{Hash}("M \text{ AND N SPAKE2}"))
\]

This variant MUST be used when it is not possible to determine which of A and B should use M or N, due to asymmetries in the protocol flows or the desire to use only a single shared secret with nil identities for authentication. The security of these variants is examined in \[\text{MNVAR}\]. The variant with per-user M and N may not be suitable for protocols that require the initial messages to be generated by each party at the same time and do not know the exact identity of the parties before the flow begins.

6. Ciphersuites

This section documents SPAKE2 ciphersuite configurations. A ciphersuite indicates a group, cryptographic hash function, and pair of KDF and MAC functions, e.g., SPAKE2-P256-SHA256-HKDF-HMAC. This ciphersuite indicates a SPAKE2 protocol instance over P-256 that uses SHA256 along with HKDF \[\text{RFC5869}\] and HMAC \[\text{RFC2104}\] for G, Hash, KDF, and MAC functions, respectively. For Ed25519 the compressed encoding is used \[\text{RFC8032}\], all others use the uncompressed SEC1 encoding.
| G       | Hash      | KDF   | MAC       |
|---------+-----------+-------+-----------|

Table 1: SPAKE2 Ciphersuites

The following points represent permissible point generation seeds for the groups listed in the Table 1 using the algorithm presented in Appendix A. These bytestrings are compressed points as in [SEC1] for curves from [SEC1].

For P256:
M =
02886e2f97ace46e55ba9dd7242579f2993b64e16ef3dcab95afd497333d8fa12f
seed: 1.2.840.10045.3.1.7 point generation seed (M)

N =
03d8bbd6c639c62937b04d997f38c3770719c629d7014d49a24b4f98baa1292b49
seed: 1.2.840.10045.3.1.7 point generation seed (N)

For P384:

M =
030ff0895ae5ebf6187080a82d82b42e2765e3b2f8749c7e05eba366434b363d3dc
36f15314739074d2eb8613fceee2853
seed: 1.3.132.0.34 point generation seed (M)

N =
02c72cf2e390853a1c1c4ad816a62fd15824f56078918f43f922ca21518f9c543bb
252c5490214cf9a3f0baab4b665c10
seed: 1.3.132.0.34 point generation seed (N)

For P521:

M =
02003f06f38131b2ba2600791e82488e8d20ab889af753a41806c5db18d37d85608
cfae06b82e4a72cd744c719193562a653e1f119eecf9356907edc9b56979962d7aa
seed: 1.3.132.0.35 point generation seed (M)

N =
0200c7924b9ec017f3094562894336a53c50167ba8c5963876880542bc669e494b25
32d76c5b53dfb349fdf69154b9e0048c58a42e8ed04cef052a3bc349d95575cd25
seed: 1.3.132.0.35 point generation seed (N)

For edwards25519:

M =
d048032c6ea0b6d697ddc2e86bda85a33adac920f1bf18e1b0c6d166a5cecdaf
seed: edwards25519 point generation seed (M)

N =
d3bf518f44f3430f29d0c92af503865a1ed3281dc69b35dd868ba85f886c4ab
seed: edwards25519 point generation seed (N)

For edwards448:
M =
b6221038a775ecd007a4e4dde39fd76ae91d3cf0cc92be8f0c2fa6d6b66f9a12
942f5a92646109152292464f3e63d354701c7848d9fc3b8880
seed: edwards448 point generation seed (M)

N =
6034c65b66e4cd7a49b0edec3e3c9ccc4588af8cf324e29f0a84a072531c4db
f97ff9af195ed714a689251f08f8e06e2d1f24a0fffc0146600
seed: edwards448 point generation seed (N)

7. Security Considerations

A security proof of SPAKE2 for prime order groups is found in [REF], reducing the security of SPAKE2 to the gap Diffie-Hellman assumption. Note that the choice of M and N is critical for the security proof. The generation methods specified in this document are designed to eliminate concerns related to knowing discrete logs of M and N.

Elements received from a peer MUST be checked for group membership: failure to properly deserialize and validate group elements can lead to attacks. An endpoint MUST abort the protocol if any received public value is not a member of G.

The choices of random numbers MUST BE uniform. Randomly generated values, e.g., x and y, MUST NOT be reused; such reuse violates the security assumptions of the protocol and results in significant insecurity. It is RECOMMENDED to generate these uniform numbers using rejection sampling.

Some implementations of elliptic curve multiplication may leak information about the length of the scalar. These MUST NOT be used. All operations on elliptic curve points must take time independent of the inputs. Hashing of the transcript may take time depending only on the length of the transcript, but not the contents.

SPAKE2 does not support augmentation. As a result, the server has to store a password equivalent. This is considered a significant drawback in some use cases. Applications that need augmented PAKEs should use [I-D.irtf-cfrg-opaque].
The HMAC keys in this document are shorter than recommended in [RFC8032]. This is appropriate as the difficulty of the discrete logarithm problem is comparable with the difficulty of brute forcing the keys.

8. IANA Considerations

No IANA action is required.

9. Acknowledgments

Special thanks to Nathaniel McCallum and Greg Hudson for generation of M and N, and Chris Wood for test vectors. Thanks to Mike Hamburg for advice on how to deal with cofactors. Greg Hudson also suggested the addition of warnings on the reuse of x and y. Thanks to Fedor Brunner, Adam Langley, Liliya Akhmetzyanova, and the members of the CFRG for comments and advice. Thanks to Scott Fluhrer and those Crypto Panel experts involved in the PAKE selection process (https://github.com/cfrg/pake-selection) who have provided valuable comments. Chris Wood contributed substantial text and reformatting to address the excellent review comments from Kenny Paterson.

10. References

10.1. Normative References

[I-D.irtf-cfrg-hash-to-curve]

10.2. Informative References


Appendix A. Algorithm used for Point Generation

This section describes the algorithm that was used to generate the points M and N in the table in Section 6.
For each curve in the table below, we construct a string using the curve OID from [RFC5480] (as an ASCII string) or its name, combined with the needed constant, e.g., "1.3.132.0.35 point generation seed (M)" for P-521. This string is turned into a series of blocks by hashing with SHA256, and hashing that output again to generate the next 32 bytes, and so on. This pattern is repeated for each group and value, with the string modified appropriately.

A byte string of length equal to that of an encoded group element is constructed by concatenating as many blocks as are required, starting from the first block, and truncating to the desired length. The byte string is then formatted as required for the group. In the case of Weierstrass curves, we take the desired length as the length for representing a compressed point (section 2.3.4 of [SEC1]), and use the low-order bit of the first byte as the sign bit. In order to obtain the correct format, the value of the first byte is set to 0x02 or 0x03 (clearing the first six bits and setting the seventh bit), leaving the sign bit as it was in the byte string constructed by concatenating hash blocks. For the [RFC8032] curves a different procedure is used. For edwards448 the 57-byte input has the least-significant 7 bits of the last byte set to zero, and for edwards25519 the 32-byte input is not modified. For both the [RFC8032] curves the (modified) input is then interpreted as the representation of the group element. If this interpretation yields a valid group element with the correct order (p), the (modified) byte string is the output. Otherwise, the initial hash block is discarded and a new byte string constructed from the remaining hash blocks. The procedure of constructing a byte string of the appropriate length, formatting it as required for the curve, and checking if it is a valid point of the correct order, is repeated until a valid element is found.

The following python snippet generates the above points, assuming an elliptic curve implementation following the interface of Edwards25519Point.stdbase() and Edwards448Point.stdbase() in Appendix A of [RFC8032]:

```python
def iterated_hash(seed, n):
    h = seed
    for i in range(n):
        h = hashlib.sha256(h).digest()
    return h
```
def bighash(seed, start, sz):
    n = -(-sz // 32)
    hashes = [iterated_hash(seed, i) for i in range(start, start + n)]
    return b''.join(hashes)[:sz]

def canon_pointstr(ecname, s):
    if ecname == 'edwards25519':
        return s
    elif ecname == 'edwards448':
        return s[:-1] + bytes([s[-1] & 0x80])
    else:
        return bytes(([s[0] & 1] | 2) + s[1:])

def gen_point(seed, ecname, ec):
    for i in range(1, 1000):
        hval = bighash(seed, i, len(ec.encode()))
        pointstr = canon_pointstr(ecname, hval)
        try:
            p = ec.decode(pointstr)
            if p != ec.zero_elem() and p * p.l() == ec.zero_elem():
                return pointstr, i
        except Exception:
            pass

Appendix B. Test Vectors

This section contains test vectors for SPAKE2 using the P256-SHA256-HKDF-HMAC ciphersuite. (Choice of MHF is omitted and values for w, x, and y are provided directly.) All points are encoded using the uncompressed format, i.e., with a 0x04 octet prefix, specified in [SEC1]. A and B identity strings are provided in the protocol invocation.

B.1. SPAKE2 Test Vectors

spake2: A='server', B='client'
w = 0x2ee57912099d31560b3a44b1184b9b4866e904c49d12ac5042c97dca461b1a5f
x = 0x43dd0fd7215bdcb482879fca3220c6a968e66d70b1356cac18bb26c84a78d729
pA = 0x04a56fa807caaa53a4d28dbb9853b9815c61a411118a6fe516a8798434751470
\( f9010153ac33d0d5f2047ffdb1a3e42c9b4e6be62766e1eeb4116988ede5f912c \)
\( y = 0xdc60106f276b02606d8e0f0a328c9e4b629f84f89786af5bfe0bcb75b6e66be \)
\( pB = 0x0406557e482bd03097ad0cbba5d8e0ba2aaf8ed6e4a1155cf8f1543ccaca52abf591a0eb7 \)
\( K = 0x0412af7e89717850671913e6b469ace6bd790a4df8ce45c2af19010175e37eed69f75897996539356e2fa6406d52805f19097e04d97515fbe83db277b715d3325 \)
\( TT = 0x060000000000000073657276657260000000000000636c69656e744100000000000000009a4a807caaa53ad28dbb9853b915c61a411118a6fe516a8798434751470f9010153ac33d0d5f2047ffdb1a3e42c9b4e6be62766e1eeb44116988ede5f912c4000000000000046557e482bd03097ad0cbba5d8f2115460d951e3451962f1e4af367af20676d09857ccbc522686c83d1852abfa8ed6e4a1155cf8f1543ccaca52abf591a1e0b74110000000000000000412af7e89717850671913e6b469ace6bd790a4df8ce45c2af19010175e37eed69f75897996539356e2fa6406d52805f19097e04d97515fbe83db277b715d3325 \)
\( Hash(TT) = 0x0e0672dc86f8e455565d338b0540abeb69 \)
\( Ke = 0x0e0672dc86f8e455565d338b0540abeb69 \)
\( Ka = 0x00c12546835755c86d8c0db7851ae86f \)
\( KcA = 0x403abbe3b1b4b9ba17e3032849759723939027a27b9d921c500edde1e8ed654b \)
\( pA = 0x48f83ec9f6e4f87cc6f9dc740bc769725f923364f01c84148c049a39a735 \)
\( bda82ee03e09112f6a5710682767cc53f16f78e19e59d3dca2922e0d837aan0000000000000000548d872f730589e579b0475a582c1608183d7df054b73b5381c7e883e2efae \)

spake2: A='', B='client'
\( w = 0x0548d8729f730589e579b0475a582c1608183d7df054b73b5381c7e883e2efae \)
\( x = 0x403abbe3b1b4b9ba17e3032849759723939027a27b9d921c500edde1e8ed654b \)
\( pA = 0x48f83ec9f6e4f87cc6f9dc740bc769725f923364f01c84148c049a39a735 \)
\( bda82ee03e09112f6a5710682767cc53f16f78e19e59d3dca2922e0d837aan0000000000000000548d872f730589e579b0475a582c1608183d7df054b73b5381c7e883e2efae \)

Hash(TT) = 0x0e0672dc86f8e455565d338b0540abeb69
Ke = 0x642f05c473c2cd7909f9a841e2f30a7
Ka = 0x8bf89b18180af9735ba198789c2b963
KcA = 0xc6be376fc7cd1301fd0a13adf3e7bffd
KcB = 0xb7243f4ae6044a49b3f8cab3c1ba07
A conf = 0x47d29e666a1fbd450d5712330857a9866e4d49d2645e2df975489521232b
B conf = 0x3313c5cefc36127f1b6879a1c2a73b766ff2a904839122a9b70a2f6bd1d6df

spake2: A='server', B=''
Hash(TT) = 0xfc6374762ba5cf11f4b2caa08b2cd1b9907ae0e26e8d6234318d91583cd74c86
Ke = 0xfc6374762ba5cf11f4b2caa08b2cd1b9
Ka = 0x907ae0e26e8d6234318d91583cd74c86
KcA = 0x5dbd2f477166b7fb6d1feb2d77a5563
KcB = 0x7689b4654407a5faefcdc8f18359d8a3
A conf = 0xd0f0609d1613138d354f7e95f19fb556bf52d751947241e8c7118df5ef0ae175
B conf = 0xd0f0609d1613138d354f7e95f19fb556bf52d751947241e8c7118df5ef0ae175

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