

Workgroup: CFRG  
Internet-Draft: draft-irtf-cfrg-vrf-11  
Published: 6 February 2022  
Intended Status: Standards Track  
Expires: 10 August 2022  
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## **Verifiable Random Functions (VRFs)**

### **Abstract**

A Verifiable Random Function (VRF) is the public-key version of a keyed cryptographic hash. Only the holder of the private key can compute the hash, but anyone with the public key can verify the correctness of the hash. VRFs are useful for preventing enumeration of hash-based data structures. This document specifies several VRF constructions based on RSA and Elliptic Curves that are secure in the cryptographic random oracle model.

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## **1. Introduction**

### **1.1. Rationale**

A Verifiable Random Function (VRF) [[MRV99](#)] is the public-key version of a keyed cryptographic hash. Only the holder of the private VRF key can compute the hash, but anyone with the corresponding public key can verify the correctness of the hash.

A key application of the VRF is to provide privacy against offline dictionary attacks (also known as enumeration attacks) on data stored in a hash-based data structure. In this application, a Prover holds the VRF private key and uses the VRF hashing to construct a hash-based data structure on the input data. Due to the nature of the VRF, only the Prover can answer queries about whether or not some data is stored in the data structure. Anyone who knows the public VRF key can verify that the Prover has answered the queries correctly. However, no offline inferences (i.e. inferences without querying the Prover) can be made about the data stored in the data structure.

### **1.2. Requirements**

The key words "MUST", "MUST NOT", "REQUIRED", "SHALL", "SHALL NOT", "SHOULD", "SHOULD NOT", "RECOMMENDED", "MAY", and "OPTIONAL" in this document are to be interpreted as described in [[RFC2119](#)].

### **1.3. Terminology**

The following terminology is used through this document:

**SK:** The private key for the VRF.

**PK:** The public key for the VRF.

**alpha or alpha\_string:** The input to be hashed by the VRF.

**beta or beta\_string:** The VRF hash output.

**pi or pi\_string:**

The VRF proof.

**Prover:** The Prover holds the private VRF key SK and public VRF key PK.

**Verifier:** The Verifier holds the public VRF key PK.

## 2. VRF Algorithms

A VRF comes with a key generation algorithm that generates a public VRF key PK and private VRF key SK.

The prover hashes an input alpha using the private VRF key SK to obtain a VRF hash output beta

$$\text{beta} = \text{VRF\_hash}(\text{SK}, \text{alpha})$$

The VRF\_hash algorithm is deterministic, in the sense that it always produces the same output beta given the same pair of inputs (SK, alpha). The prover also uses the private key SK to construct a proof pi that beta is the correct hash output

$$\text{pi} = \text{VRF\_prove}(\text{SK}, \text{alpha})$$

The VRFs defined in this document allow anyone to deterministically obtain the VRF hash output beta directly from the proof value pi by using the function VRF\_proof\_to\_hash:

$$\text{beta} = \text{VRF\_proof\_to\_hash}(\text{pi})$$

Thus, for VRFs defined in this document, VRF\_hash is defined as

$$\text{VRF\_hash}(\text{SK}, \text{alpha}) = \text{VRF\_proof\_to\_hash}(\text{VRF\_prove}(\text{SK}, \text{alpha})),$$

and therefore this document will specify VRF\_prove and VRF\_proof\_to\_hash rather than VRF\_hash.

The proof pi allows a Verifier holding the public key PK to verify that beta is the correct VRF hash of input alpha under key PK. Thus, the VRFs defined in this document also come with an algorithm

$$\text{VRF\_verify}(\text{PK}, \text{alpha}, \text{pi})$$

that outputs (VALID, beta = VRF\_proof\_to\_hash(pi)) if pi is valid, and INVALID otherwise.

## 3. VRF Security Properties

VRFs are designed to ensure the following security properties.

### 3.1. Full Uniqueness or Trusted Uniqueness

Uniqueness means that, for any fixed public VRF key and for any input  $\alpha$ , there is a unique VRF output  $\beta$  that can be proved to be valid. Uniqueness must hold even for an adversarial Prover that knows the VRF private key  $SK$ .

More precisely, "full uniqueness" states that a computationally-bounded adversary cannot choose a VRF public key  $PK$ , a VRF input  $\alpha$ , and two proofs  $\pi_1$  and  $\pi_2$  such that  $VRF\_verify(PK, \alpha, \pi_1)$  outputs  $(VALID, \beta_1)$ ,  $VRF\_verify(PK, \alpha, \pi_2)$  outputs  $(VALID, \beta_2)$ , and  $\beta_1$  is not equal to  $\beta_2$ .

For many applications, a slightly weaker security property called "trusted uniqueness" suffices. Trusted uniqueness is the same as full uniqueness, but it is guaranteed to hold only if the VRF keys  $PK$  and  $SK$  were generated in a trustworthy manner.

As further discussed in [Section 7.1.1](#), some VRFs specified in this document satisfy only trusted uniqueness, while others satisfy full uniqueness. VRFs in this document that satisfy only trusted uniqueness but not full uniqueness MUST NOT be used if the key generation process cannot be trusted.

### 3.2. Full Collision Resistance or Trusted Collision Resistance

Like any cryptographic hash function, VRFs need to be collision resistant. Collision resistance must hold even for an adversarial Prover that knows the VRF private key  $SK$ .

More precisely, "full collision resistance" states that it should be computationally infeasible for an adversary to find two distinct VRF inputs  $\alpha_1$  and  $\alpha_2$  that have the same VRF hash  $\beta$ , even if that adversary knows the private VRF key  $SK$ .

For many applications, a slightly weaker security property called "trusted collision resistance" suffices. Trusted collision resistance is the same as collision resistance, but it is guaranteed to hold only if the VRF keys  $PK$  and  $SK$  were generated in a trustworthy manner.

As further discussed in [Section 7.1.1](#), some VRFs specified in this document satisfy only trusted collision resistance, while others satisfy full collision resistance. VRFs in this document that satisfy only trusted collision resistance but not full collision resistance MUST NOT be used if the key generation process cannot be trusted.

### 3.3. Full Pseudorandomness or Selective Pseudorandomness

Pseudorandomness ensures that when an adversarial Verifier sees a VRF hash output  $\beta$  without its corresponding VRF proof  $\pi$ , then  $\beta$  is indistinguishable from a random value.

More precisely, suppose the public and private VRF keys ( $PK$ ,  $SK$ ) were generated in a trustworthy manner. Pseudorandomness ensures that the VRF hash output  $\beta$  (without its corresponding VRF proof  $\pi$ ) on any adversarially-chosen "target" VRF input  $\alpha$  looks indistinguishable from random for any computationally bounded adversary who does not know the private VRF key  $SK$ . This holds even if the adversary also gets to choose other VRF inputs  $\alpha'$  and observe their corresponding VRF hash outputs  $\beta'$  and proofs  $\pi'$ .

With "full pseudorandomness", the adversary is allowed to choose the "target" VRF input  $\alpha$  at any time, even after it observes VRF outputs  $\beta'$  and proofs  $\pi'$  on a variety of chosen inputs  $\alpha'$ .

"Selective pseudorandomness" is a weaker security property which suffices in many applications. Here, the adversary must choose the target VRF input  $\alpha$  independently of the public VRF key  $PK$ , and before it observes VRF outputs  $\beta'$  and proofs  $\pi'$  on inputs  $\alpha'$  of its choice.

As further discussed in [Section 7.3](#), VRFs specified in this document satisfy both full pseudorandomness and selective pseudorandomness, but their quantitative security against the selective pseudorandomness attack is stronger.

It is important to remember that the VRF output  $\beta$  does not look random to the Prover, or to any other party that knows the private VRF key  $SK$ ! Such a party can easily distinguish  $\beta$  from a random value by comparing  $\beta$  to the result of  $\text{VRF\_hash}(SK, \alpha)$ .

Also, the VRF output  $\beta$  does not look random to any party that knows a valid VRF proof  $\pi$  corresponding to the VRF input  $\alpha$ , even if this party does not know the private VRF key  $SK$ . Such a party can easily distinguish  $\beta$  from a random value by checking whether  $\text{VRF\_verify}(PK, \alpha, \pi)$  returns (VALID,  $\beta$ ).

Also, the VRF output  $\beta$  may not look random if VRF key generation was not done in a trustworthy fashion. (For example, if VRF keys were generated with bad randomness.)

### 3.4. Some VRFs: Unpredictability Under Malicious Key Generation

As explained in [Section 3.3](#), pseudorandomness is guaranteed only if the VRF keys were generated in a trustworthy fashion. For instance, if an adversary outputs VRF keys that are deterministically

generated (or hard-coded and publicly known), then the outputs are easily derived by anyone and are therefore not pseudorandom.

There is, however, a different type of unpredictability that is desirable in certain VRF applications (such as leader selection in the consensus protocols of [\[GHMVZ17\]](#) and [\[DGKR18\]](#)), called "unpredictability under malicious key generation". This property is similar to the unpredictability achieved by an (ordinary, unkeyed) cryptographic hash function: if the input has enough entropy (i.e., cannot be predicted), then the correct output is indistinguishable from uniform, no matter how the VRF keys are generated.

A formal definition of this property appears in Section 3.2 of [\[DGKR18\]](#). The RSA-FDH-VRF presented in this document does not satisfy this property. The ECVRF presented in this document satisfies this property if `validate_key` parameter given to the `ECVRF_verify` is `TRUE`.

#### 4. RSA Full Domain Hash VRF (RSA-FDH-VRF)

The RSA Full Domain Hash VRF (RSA-FDH-VRF) is a VRF that, for suitable key lengths, satisfies the "trusted uniqueness", "trusted collision resistance", and "full pseudorandomness" properties defined in [Section 3](#), as further discussed in [Section 7](#). Its security follows from the standard RSA assumption in the random oracle model. Formal security proofs are in [\[PWHVNRG17\]](#).

The VRF computes the proof  $\pi$  as a deterministic RSA signature on input  $\alpha$  using the RSA Full Domain Hash Algorithm [\[RFC8017\]](#) parametrized with the selected hash algorithm. RSA signature verification is used to verify the correctness of the proof. The VRF hash output  $\beta$  is simply obtained by hashing the proof  $\pi$  with the selected hash algorithm.

The key pair for RSA-FDH-VRF MUST be generated in a way that it satisfies the conditions specified in Section 3 of [\[RFC8017\]](#).

In this section, the notation from [\[RFC8017\]](#) is used.

Parameters used:

$(n, e)$  - RSA public key

$K$  - RSA private key (its representation is implementation-dependent)

$k$  - length in octets of the RSA modulus  $n$  ( $k$  must be less than  $2^{32}$ )

Fixed options (specified in [Section 4.4](#)):

Hash - cryptographic hash function

hLen - output length in octets of hash function Hash

suite\_string - an octet string specifying the RSA-FDH-VRF  
ciphersuite, which determines the above options

Primitives used:

I2OSP - Conversion of a nonnegative integer to an octet string as defined in Section 4.1 of [\[RFC8017\]](#) (given an integer and a length in octets, produces a big-endian representation of the integer, zero-padded to the desired length)

OS2IP - Conversion of an octet string to a nonnegative integer as defined in Section 4.2 of [\[RFC8017\]](#) (given a big-endian encoding of an integer, produces the integer)

RSASP1 - RSA signature primitive as defined in Section 5.2.1 of [\[RFC8017\]](#) (given a private key and an input, raises the input to the private RSA exponent modulo n)

RSAPV1 - RSA verification primitive as defined in Section 5.2.2 of [\[RFC8017\]](#) (given a public key and an input, raises the input to the public RSA exponent modulo n)

MGF1 - Mask Generation Function based on the hash function Hash as defined in Section B.2.1 of [\[RFC8017\]](#) (given an input, produces a random-oracle-like output of desired length)

|| - octet string concatenation

#### 4.1. RSA-FDH-VRF Proving

RSAFDHVRF\_prove(K, alpha\_string[, MGF\_salt])

Input:

K - RSA private key

alpha\_string - VRF hash input, an octet string

Optional Input:

MGF\_salt - a public octet string used as a hash function salt; this input is not used when MGF\_salt is specified as part of the ciphersuite



Output:

pi\_string - proof, an octet string of length k

Steps:

1. mgf\_domain\_separator = 0x01
2. EM = MGF1(suite\_string || mgf\_domain\_separator || MGF\_salt || alpha\_string, k - 1)
3. m = OS2IP(EM)
4. s = RSASP1(K, m)
5. pi\_string = I2OSP(s, k)
6. Output pi\_string

#### 4.2. RSA-FDH-VRF Proof to Hash

RSAFDHVRF\_proof\_to\_hash(pi\_string)

Input:

pi\_string - proof, an octet string of length k

Output:

beta\_string - VRF hash output, an octet string of length hLen

Important note:

RSAFDHVRF\_proof\_to\_hash should be run only on pi\_string that is known to have been produced by RSAFDHVRF\_prove, or from within RSAFDHVRF\_verify as specified in [Section 4.3](#).

Steps:

1. proof\_to\_hash\_domain\_separator = 0x02
2. beta\_string = Hash(suite\_string || proof\_to\_hash\_domain\_separator || pi\_string)
3. Output beta\_string

#### 4.3. RSA-FDH-VRF Verifying

RSAFDHVRF\_verify((n, e), alpha\_string, pi\_string[, MGF\_salt])

Input:

(n, e) - RSA public key

alpha\_string - VRF hash input, an octet string

pi\_string - proof to be verified, an octet string of length k

Optional Input:

MGF\_salt - a public octet string used as a hash function salt;  
this input is not used when MGF\_salt is specified as part of the  
ciphersuite

Output:

Output:

("VALID", beta\_string), where beta\_string is the VRF hash output,  
an octet string of length hLen; or

"INVALID"

Steps:

1. s = OS2IP(pi\_string)
2. m = RSAVP1((n, e), s); if RSAVP1 returns "signature  
representative out of range", output "INVALID" and stop.
3. mgf\_domain\_separator = 0x01
4. EM' = MGF1(suite\_string || mgf\_domain\_separator || MGF\_salt ||  
alpha\_string, k - 1)
5. m' = OS2IP(EM')
6. If m and m' are equal, output ("VALID",  
RSAFDH\_VRF\_proof\_to\_hash(pi\_string)); else output "INVALID".

#### 4.4. RSA-FDH-VRF Ciphersuites

This document defines RSA-FDH-VRF-SHA256 as follows:

\*suite\_string = 0x01

\*The hash function Hash is SHA-256 as specified in [[RFC6234](#)], with  
hLen = 32

\*MGF\_salt = I2OSP(k, 4) || I2OSP(n, k)

This document defines RSA-FDH-VRF-SHA384 as follows:

\*suite\_string = 0x02

\*The hash function Hash is SHA-384 as specified in [[RFC6234](#)], with  
hLen = 48

\*MGF\_salt = I2OSP(k, 4) || I2OSP(n, k)

This document defines RSA-FDH-VRF-SHA512 as follows:

\*suite\_string = 0x03

\*The hash function Hash is SHA-512 as specified in [[RFC6234](#)], with  
hLen = 64

\*MGF\_salt = I2OSP(k, 4) || I2OSP(n, k)

## 5. Elliptic Curve VRF (ECVRF)

The Elliptic Curve Verifiable Random Function (ECVRF) is a VRF that, for suitable parameter choices, satisfies the "full uniqueness", "trusted collision resistance", and "full pseudorandomness properties" defined in [Section 3](#). If validate\_key parameter given to the ECVRF\_verify is TRUE, then the ECVRF additionally satisfies "full collision resistance" and "unpredictability under malicious key generation". See [Section 7](#) for further discussion. Formal security proofs are in [[PWHVNRG17](#)].

Notation used:

Elliptic curve operations are written in additive notation, with  
P+Q denoting point addition and x\*P denoting scalar  
multiplication of a point P by a scalar x

$x^y$  - x raised to the power y

$x*y$  - x multiplied by y

s || t - concatenation of octet strings s and t

0xMN (where M and N are hexadecimal digits) - a single octet with  
value  $M*16+N$ ; equivalently, `int_to_string(M*16+N, 1)`, where  
`int_to_string` is as defined below.

Fixed options (specified in [Section 5.5](#)):

F - finite field

fLen - length, in octets, of an element in F encoded as an octet string

E - elliptic curve (EC) defined over F

ptLen - length, in octets, of a point on E encoded as an octet string

G - subgroup of E of large prime order

q - prime order of group G

qLen - length of q in octets, i.e., smallest integer such that  $2^{(8qLen)} > q$

cLen - length, in octets, of a challenge value used by the VRF (note that in the typical case, cLen is qLen/2 or close to it)

cofactor - number of points on E divided by q

B - generator of group G

Hash - cryptographic hash function

hLen - output length in octets of Hash (hLen must be at least cLen; in the typical case, it is at least qLen)

ECVRF\_encode\_to\_curve - a function that hashes strings to points on E.

ECVRF\_nonce\_generation - a function that derives a pseudorandom nonce from SK and the input as part of ECVRF proving.

suite\_string - an octet string specifying the ECVRF ciphersuite, which determines the above options as well as type conversions and parameter generation

Type conversions (specified in [Section 5.5](#)):

int\_to\_string(a, len) - conversion of nonnegative integer a to octet string of length len

string\_to\_int(a\_string) - conversion of an octet string a\_string to a nonnegative integer

point\_to\_string - conversion of a point on E to an ptLen-octet string

`string_to_point` - conversion of an `ptLen`-octet string to a point on `E`. `string_to_point` returns `INVALID` if the octet string does not convert to a valid EC point on the curve `E`.

Note that with certain software libraries (for big integer and elliptic curve arithmetic), the `int_to_string` and `point_to_string` conversions are not needed, when the libraries encode integers and EC points in the same way as required by the ciphersuites. For example, in some implementations, EC point operations will take octet strings as inputs and produce octet strings as outputs, without introducing a separate elliptic curve point type.

Parameters used (the generation of these parameters is specified in [Section 5.5](#)):

`SK` - VRF private key

`x` - VRF secret scalar, an integer. Note: depending on the ciphersuite used, the VRF secret scalar may be equal to `SK`; else, it is derived from `SK`

`Y = x*B` - VRF public key, an point on `E`

`PK_string = point_to_string(Y)` - VRF public key represented as an octet string

`encode_to_curve_salt` - a public value used as a hash function salt

### 5.1. ECVRF Proving

`ECVRF_prove(SK, alpha_string[, encode_to_curve_salt])`

Input:

`SK` - VRF private key

`alpha_string` - input alpha, an octet string

Optional input:

`encode_to_curve_salt` - a public salt value, an octet string; this input is not used when `encode_to_curve_salt` is specified as part of the ciphersuite

Output:

`pi_string` - VRF proof, octet string of length `ptLen+cLen+qLen`

Steps:

1. Use SK to derive the VRF secret scalar  $x$  and the VRF public key  $Y = x*B$

(this derivation depends on the ciphersuite, as per [Section 5.5](#);

these values can be cached, for example, after key generation, and need not be rederived each time)

2.  $H = \text{ECVRF\_encode\_to\_curve}(\text{encode\_to\_curve\_salt}, \text{alpha\_string})$   
(see [Section 5.4.1](#))
3.  $\text{h\_string} = \text{point\_to\_string}(H)$
4.  $\text{Gamma} = x*H$
5.  $k = \text{ECVRF\_nonce\_generation}(\text{SK}, \text{h\_string})$  (see [Section 5.4.2](#))
6.  $c = \text{ECVRF\_challenge\_generation}(Y, H, \text{Gamma}, k*B, k*H)$  (see [Section 5.4.3](#))
7.  $s = (k + c*x) \bmod q$
8.  $\text{pi\_string} = \text{point\_to\_string}(\text{Gamma}) || \text{int\_to\_string}(c, \text{cLen}) || \text{int\_to\_string}(s, \text{qLen})$
9. Output  $\text{pi\_string}$

## 5.2. ECVRF Proof to Hash

$\text{ECVRF\_proof\_to\_hash}(\text{pi\_string})$

Input:

$\text{pi\_string}$  - VRF proof, octet string of length  $\text{ptLen} + \text{cLen} + \text{qLen}$

Output:

"INVALID", or

$\text{beta\_string}$  - VRF hash output, octet string of length  $\text{hLen}$

Important note:

$\text{ECVRF\_proof\_to\_hash}$  should be run only on  $\text{pi\_string}$  that is known to have been produced by  $\text{ECVRF\_prove}$ , or from within  $\text{ECVRF\_verify}$  as specified in [Section 5.3](#).

Steps:

1. `D = ECVRF_decode_proof(pi_string)` (see [Section 5.4.4](#))
2. If `D` is "INVALID", output "INVALID" and stop
3. `(Gamma, c, s) = D`
4. `proof_to_hash_domain_separator_front = 0x03`
5. `proof_to_hash_domain_separator_back = 0x00`
6. `beta_string = Hash(suite_string ||  
proof_to_hash_domain_separator_front ||  
point_to_string(cofactor * Gamma) ||  
proof_to_hash_domain_separator_back)`
7. Output `beta_string`

### 5.3. ECVRF Verifying

`ECVRF_verify(PK_string, alpha_string, pi_string[,  
encode_to_curve_salt, validate_key])`

Input:

`PK_string` - public key, an octet string

`alpha_string` - VRF input, octet string

`pi_string` - VRF proof, octet string of length `ptLen+cLen+qLen`

Optional input:

`encode_to_curve_salt` - a public salt value, an octet string; this input is not used when `encode_to_curve_salt` is specified as part of the ciphersuite

`validate_key` - a boolean. An implementation MAY support only the option of `validate_key = TRUE`, or only the option of `validate_key = FALSE`, in which case this input is not needed. If an implementation supports only one option, it MUST specify which option is supported.

Output:

`("VALID", beta_string)`, where `beta_string` is the VRF hash output, octet string of length `hLen`; or

`"INVALID"`

Steps:

1.  $Y = \text{string\_to\_point}(\text{PK\_string})$
2. If  $Y$  is "INVALID", output "INVALID" and stop
3. If `validate_key`, run `ECVRF_validate_key(Y)` ([Section 5.4.5](#)); if it outputs "INVALID", output "INVALID" and stop
4.  $D = \text{ECVRF\_decode\_proof}(\text{pi\_string})$  (see [Section 5.4.4](#))
5. If  $D$  is "INVALID", output "INVALID" and stop
6.  $(\text{Gamma}, c, s) = D$
7.  $H = \text{ECVRF\_encode\_to\_curve}(\text{encode\_to\_curve\_salt}, \text{alpha\_string})$  (see [Section 5.4.1](#))
8.  $U = s*B - c*Y$
9.  $V = s*H - c*\text{Gamma}$
10.  $c' = \text{ECVRF\_challenge\_generation}(Y, H, \text{Gamma}, U, V)$  (see [Section 5.4.3](#))
11. If  $c$  and  $c'$  are equal, output ("VALID", `ECVRF_proof_to_hash(pi_string)`); else output "INVALID"

Note that the first three steps need to be performed only once for a given public key.

## 5.4. ECVRF Auxiliary Functions

### 5.4.1. ECVRF Encode to Curve

The `ECVRF_encode_to_curve` algorithm takes a public salt (see [Section 7.9](#)) and the VRF input  $\alpha$  and converts it to  $H$ , an EC point in  $G$ . This algorithm is the only place the VRF input  $\alpha$  is used for proving and verifying. See [Section 7.7](#) for further discussion.

This section specifies a number of such algorithms, which are not compatible with each other and are intended to use with various ciphersuites specified in [Section 5.5](#).

Input:

`encode_to_curve_salt` - public salt value, an octet string

`alpha_string` - value to be hashed, an octet string



Output:

H - hashed value, a point in G

#### 5.4.1.1. ECVRF\_encode\_to\_curve\_try\_and\_increment

The following

ECVRF\_encode\_to\_curve\_try\_and\_increment(`encode_to_curve_salt`, `alpha_string`) algorithm implements ECVRF\_encode\_to\_curve in a simple and generic way that works for any elliptic curve. To use this algorithm, `hlen` MUST be at least `flen`.

The running time of this algorithm depends on `alpha_string`. For the ciphersuites specified in [Section 5.5](#), this algorithm is expected to find a valid curve point after approximately two attempts (i.e., when `ctr=1`) on average.

However, because the running time of algorithm depends on `alpha_string`, this algorithm SHOULD be avoided in applications where it is important that the VRF input `alpha` remain secret.

ECVRF\_encode\_to\_curve\_try\_and\_increment(`encode_to_curve_salt`, `alpha_string`)

Fixed option (specified in [Section 5.5](#)):

`interpret_hash_value_as_a_point` - a function that attempts to convert a cryptographic hash value to a point on E; may output `INVALID`.

Steps:

1. `ctr = 0`
2. `encode_to_curve_domain_separator_front = 0x01`
3. `encode_to_curve_domain_separator_back = 0x00`
4. `H = "INVALID"`
5. While H is "INVALID" or H is the identity element of the elliptic curve group:
  - a. `ctr_string = int_to_string(ctr, 1)`
  - b. `hash_string = Hash(suite_string || encode_to_curve_domain_separator_front || encode_to_curve_salt || alpha_string || ctr_string || encode_to_curve_domain_separator_back)`

- c. `H = interpret_hash_value_as_a_point(hash_string)`
- d. If `H` is not "INVALID" and `cofactor > 1`, set `H = cofactor * H`
- e. `ctr = ctr + 1`

#### 6. Output `H`

Note even though the loop is infinite as written, and `int_to_string(ctr,1)` may fail when `ctr` reaches 256, `interpret_hash_value_as_a_point` functions specified in [Section 5.5](#) will succeed on roughly half hash\_string values. Thus the loop is expected to stop after two iterations, and `ctr` is overwhelmingly unlikely (probability about  $2^{-256}$ ) to reach 256.

#### 5.4.1.2. **ECVRF\_encode\_to\_curve\_h2c\_suite**

The `ECVRF_encode_to_curve_h2c_suite(encode_to_curve_salt, alpha_string)` algorithm implements `ECVRF_encode_to_curve` using one of the several hash-to-curve options defined in [[I-D.irtf-cfrg-hash-to-curve](#)]. The specific choice of the hash-to-curve option (called Suite ID in [[I-D.irtf-cfrg-hash-to-curve](#)]) is given by the `h2c_suite_ID_string` parameter.

`ECVRF_encode_to_curve_h2c_suite(encode_to_curve_salt, alpha_string)`

Fixed option (specified in [Section 5.5](#)):

`h2c_suite_ID_string` - a hash-to-curve suite ID, encoded in ASCII (see discussion below)

Steps:

1. `string_to_be_hashed = encode_to_curve_salt || alpha_string`
2. `H = encode(string_to_be_hashed)`

(the encode function is discussed below)

#### 3. Output `H`

The encode function is provided by the hash-to-curve suite whose ID is `h2c_suite_ID_string`, as specified in [[I-D.irtf-cfrg-hash-to-curve](#)], Section 8. The domain separation tag DST, a parameter to the hash-to-curve suite, SHALL be set to

`"ECVRF_" || h2c_suite_ID_string || suite_string`

where "ECVRF\_" is represented as a 6-byte ASCII encoding (in hexadecimal, octets 45 43 56 52 46 5F).

#### 5.4.2. ECVRF Nonce Generation

The following algorithms generate the nonce value  $k$  in a deterministic pseudorandom fashion. This section specifies a number of such algorithms, which are not compatible with each other. The choice of a particular algorithm from the options specified in this section depends on the ciphersuite, as specified in [Section 5.5](#).

##### 5.4.2.1. ECVRF Nonce Generation from RFC 6979

ECVRF\_nonce\_generation\_RFC6979(SK, h\_string)

Input:

SK - an ECVRF secret key

h\_string - an octet string

Output:

$k$  - an integer nonce between 1 and  $q-1$

The ECVRF\_nonce\_generation function is as specified in [\[RFC6979\]](#) Section 3.2 where

Input  $m$  is set equal to h\_string

The "suitable for DSA or ECDSA" check in step h.3 is omitted

The hash function  $H$  is Hash and its output length  $hlen$  (in bits) is set as  $hlen*8$

The secret key  $x$  is set equal to the VRF secret scalar  $x$

The prime  $q$  is the same as in this specification

$qlen$  is the binary length of  $q$ , i.e., the smallest integer such that  $2^{qlen} > q$  (this  $qlen$  is not to be confused with  $qLen$  in this document, which is the length of  $q$  in octets)

All the other values and primitives as defined in [\[RFC6979\]](#)

##### 5.4.2.2. ECVRF Nonce Generation from RFC 8032

The following is from Steps 2-3 of Section 5.1.6 in [\[RFC8032\]](#). To use this algorithm,  $hlen$  MUST be at least 64.

ECVRF\_nonce\_generation\_RFC8032(SK, h\_string)

Input:

SK - an ECVRF secret key

h\_string - an octet string

Output:

k - an integer nonce between 0 and q-1

Steps:

1. hashed\_sk\_string = Hash(SK)
2. truncated\_hashed\_sk\_string =  
hashed\_sk\_string[32]...hashed\_sk\_string[63]
3. k\_string = Hash(truncated\_hashed\_sk\_string || h\_string)
4. k = string\_to\_int(k\_string) mod q

#### 5.4.3. ECVRF Challenge Generation

ECVRF\_challenge\_generation(P1, P2, P3, P4, P5)

Input:

P1, P2, P3, P4, P5 - EC points

Output:

c - challenge value, integer between 0 and  $2^{(8 \cdot cLen)} - 1$

Steps:

1. challenge\_generation\_domain\_separator\_front = 0x02
2. Initialize str = suite\_string ||  
challenge\_generation\_domain\_separator\_front
3. for PJ in [P1, P2, P3, P4, P5]:  
  
str = str || point\_to\_string(PJ)
4. challenge\_generation\_domain\_separator\_back = 0x00
5. str = str || challenge\_generation\_domain\_separator\_back
6. c\_string = Hash(str)
7. truncated\_c\_string = c\_string[0]...c\_string[cLen-1]

8. `c = string_to_int(truncated_c_string)`

9. Output `c`

#### 5.4.4. ECVRF Decode Proof

`ECVRF_decode_proof(pi_string)`

Input:

`pi_string` - VRF proof, octet string (`ptLen+cLen+qLen` octets)

Output:

"INVALID", or

`Gamma` - a point on `E`

`c` - integer between 0 and  $2^{(8*cLen)}-1$

`s` - integer between 0 and `q-1`

Steps:

1. `gamma_string = pi_string[0]...pi_string[ptLen-1]`

2. `c_string = pi_string[ptLen]...pi_string[ptLen+cLen-1]`

3. `s_string = pi_string[ptLen+cLen]...pi_string[ptLen+cLen+qLen-1]`

4. `Gamma = string_to_point(gamma_string)`

5. if `Gamma = "INVALID"` output "INVALID" and stop

6. `c = string_to_int(c_string)`

7. `s = string_to_int(s_string)`

8. if `s >= q` output "INVALID" and stop

9. Output `Gamma`, `c`, and `s`

#### 5.4.5. ECVRF Validate Key

`ECVRF_validate_key(Y)`

Input:

`Y` - public key, a point on `E`

Output:

"VALID" or "INVALID"

Important note: the public key  $Y$  given to this procedure MUST be a valid point on  $E$ .

Steps:

1. Let  $Y' = \text{cofactor} * Y$
2. If  $Y'$  is the identity element of the elliptic curve group, output "INVALID" and stop
3. Output "VALID"

Note that if the cofactor = 1, then Step 1 simply sets  $Y' = Y$ . In particular, for the P-256 curve, ECVRF\_validate\_key simply ensures that  $Y$  is not the point at infinity.

Also note that if the cofactor is small, the total number of  $Y$  values that could cause Step 2 to output "INVALID" may be small, and it may be more efficient to simply check  $Y$  against a fixed list of such points. For example, the following algorithm can be used for the edwards25519 curve:

1.  $\text{PK\_string} = \text{point\_to\_string}(Y)$
2.  $\text{oneTwentySeven\_string} = 0x7F$
3.  $\text{y\_string}[31] = \text{y\_string}[31] \& \text{oneTwentySeven\_string}$   
(this step clears the high-order bit of octet 31)
4.  $\text{bad\_pk}[0] = \text{int\_to\_string}(0, 32)$
5.  $\text{bad\_pk}[1] = \text{int\_to\_string}(1, 32)$
6.  $\text{bad\_y2} =$   
2707385501144840649318225287225658788936804267575313519463743609750303402022
7.  $\text{bad\_pk}[2] = \text{int\_to\_string}(\text{bad\_y2}, 32)$
8.  $\text{bad\_pk}[3] = \text{int\_to\_string}(p - \text{bad\_y2}, 32)$
9.  $\text{bad\_pk}[4] = \text{int\_to\_string}(p - 1, 32)$
10.  $\text{bad\_pk}[5] = \text{int\_to\_string}(p, 32)$
11.  $\text{bad\_pk}[6] = \text{int\_to\_string}(p + 1, 32)$

12. If `y_string` is in the list `[bad_pk[0], ..., bad_pk[6]]`, output "INVALID" and stop

13. Output `Y`

(`bad_pk[0]`, `bad_pk[2]`, `bad_pk[3]` each match two bad public keys, depending on the sign of the x-coordinate, which was cleared in step 5, in order to make sure that it does not affect the comparison. `bad_pk[1]` and `bad_pk[4]` each match one bad public key, because x-coordinate is 0 for these two public keys. `bad_pk[5]` and `bad_pk[6]` are simply `bad_pk[0]` and `bad_pk[1]` shifted by `p`, in case the y-coordinate had not been modular reduced by `p`. There is no need to shift the other `bad_pk` values by `p`, because they will exceed  $2^{255}$ . These bad keys, which represent all points of order 1, 2, 4, and 8, have been obtained by converting the points specified in [\[X25519\]](#) to Edwards coordinates.)

### 5.5. ECVRF Ciphersuites

This document defines ECVRF-P256-SHA256-TAI as follows:

\*`suite_string` = 0x01.

\*The EC group `G` is the NIST P-256 elliptic curve, with curve parameters as specified in [\[FIPS-186-4\]](#) (Section D.1.2.3) and [\[RFC5114\]](#) (Section 2.6). For this group, `fLen` = `qLen` = 32 and `cofactor` = 1.

\*`cLen` = 16.

\*The key pair generation primitive is specified in Section 3.2.1 of [\[SECG1\]](#) (`q`, `B`, `SK`, and `Y` in this document correspond to `n`, `G`, `d`, and `Q` in Section 3.2.1 of [\[SECG1\]](#)). In this ciphersuite, the secret scalar `x` is equal to the private key `SK`.

\*`encode_to_curve_salt` = `PK_string`

\*The `ECVRF_nonce_generation` function is as specified in [Section 5.4.2.1](#).

\*The `int_to_string` function is the `I2OSP` function specified in Section 4.1 of [\[RFC8017\]](#). (This is big-endian representation.)

\*The `string_to_int` function is the `OS2IP` function specified in Section 4.2 of [\[RFC8017\]](#). (This is big-endian representation.)

\*The `point_to_string` function converts a point on `E` to an octet string according to the encoding specified in Section 2.3.3 of [\[SECG1\]](#) with point compression on. This implies `ptLen` = `fLen` + 1 = 33. (Note that certain software implementations do not

introduce a separate elliptic curve point type and instead directly treat the EC point as an octet string per above encoding. When using such an implementation, the point\_to\_string function can be treated as the identity function.)

\*The string\_to\_point function converts an octet string to an a point on E according to the encoding specified in Section 2.3.4 of [[SECG1](#)]. This function MUST output INVALID if the octet string does not decode to a point on the curve E.

\*The hash function Hash is SHA-256 as specified in [[RFC6234](#)], with hLen = 32.

\*The ECVRF\_encode\_to\_curve function is as specified in [Section 5.4.1.1](#), with interpret\_hash\_value\_as\_a\_point(s) = string\_to\_point(0x02 || s).

This document defines ECVRF-P256-SHA256-SSWU as identical to ECVRF-P256-SHA256-TAI, except that:

\*suite\_string = 0x02.

\*the ECVRF\_encode\_to\_curve function is as specified in [Section 5.4.1.2](#) with h2c\_suite\_ID\_string = P256\_XMD:SHA-256\_SSWU\_NU\_ (the suite is defined in [[I-D.irtf-cfrg-hash-to-curve](#)] Section 8.2)

This document defines ECVRF-EDWARDS25519-SHA512-TAI as follows:

\*suite\_string = 0x03.

\*The EC group G is the edwards25519 elliptic curve with parameters defined in Table 1 of [[RFC8032](#)]. For this group, fLen = qLen = 32 and cofactor = 8.

\*cLen = 16.

\*The private key and generation of the secret scalar and the public key are specified in Section 5.1.5 of [[RFC8032](#)].

\*encode\_to\_curve\_salt = PK\_string

\*The ECVRF\_nonce\_generation function is as specified in [Section 5.4.2.2](#).

\*The int\_to\_string function as specified in the first paragraph of Section 5.1.2 of [[RFC8032](#)]. (This is little-endian representation.)

\*The string\_to\_int function interprets the string as an integer in little-endian representation.



\*The `point_to_string` function converts an point on E to an octet string according to the encoding specified in Section 5.1.2 of [RFC8032]. This implies `ptLen = fLen = 32`. (Note that certain software implementations do not introduce a separate elliptic curve point type and instead directly treat the EC point as an octet string per above encoding. When using such an implementation, the `point_to_string` function can be treated as the identity function.)

\*The `string_to_point` function converts an octet string to a point on E according to the encoding specified in Section 5.1.3 of [RFC8032]. This function MUST output INVALID if the octet string does not decode to a point on the curve E.

\*The hash function Hash is SHA-512 as specified in [RFC6234], with `hLen = 64`.

\*The `ECVRF_encode_to_curve` function is as specified in Section 5.4.1.1, with `interpret_hash_value_as_a_point(s) = string_to_point(s[0]...s[31])`.

This document defines ECVRF-EDWARDS25519-SHA512-ELL2 as identical to ECVRF-EDWARDS25519-SHA512-TAI, except:

\*`suite_string = 0x04`.

\*the `ECVRF_encode_to_curve` function is as specified in Section 5.4.1.2 with `h2c_suite_ID_string = edwards25519_XMD:SHA-512_ELL2_NU_` (the suite is defined in [I-D.irtf-cfrg-hash-to-curve] Section 8.5).

## 6. Implementation Status

Note to RFC editor: Remove before publication

A reference C++ implementation of ECVRF-P256-SHA256-TAI, ECVRF-P256-SHA256-SSWU, ECVRF-EDWARDS25519-SHA512-TAI, and ECVRF-EDWARDS25519-SHA512-ELL2 is available at <https://github.com/reyzin/ecvrf>. This implementation is neither secure nor especially efficient, but can be used to generate test vectors.

A Python implementation of an older version of ECVRF-EDWARDS25519-SHA512-ELL2 from the -05 version of this draft is available at <https://github.com/integritychain/draft-irtf-cfrg-vrf-05>.

A C implementation of an older version of ECVRF-EDWARDS25519-SHA512-ELL2 from the -03 version of this draft is available at [https://github.com/algorand/libsodium/tree/draft-irtf-cfrg-vrf-03/src/libsodium/crypto\\_vrf/ietf-draft03](https://github.com/algorand/libsodium/tree/draft-irtf-cfrg-vrf-03/src/libsodium/crypto_vrf/ietf-draft03).

A Rust implementation of an older version of ECVRF-P256-SHA256-TAI from the -05 version of this draft, as well as variants for the sect163k1 and secp256k1 curves, is available at <https://crates.io/crates/vrf>.

A C implementation of a variant of ECVRF-P256-SHA256-TAI from the -05 version of this draft adapted for the secp256k1 curve is available at <https://github.com/aergoio/secp256k1-vrf>.

An implementation of an earlier version of RSA-FDH-VRF (SHA-256) and ECVRF-P256-SHA256-TAI was first developed as a part of the NSEC5 project [I-D.vcelak-nsec5] and is available at <http://github.com/fcelda/nsec5-crypto>.

The Key Transparency project at Google uses a VRF implementation that is similar to the ECVRF-P256-SHA256-TAI, with a few changes including the use of SHA-512 instead of SHA-256. Its implementation is available at <https://github.com/google/keytransparency/blob/master/core/crypto/vrf/>

An implementation by Ryuji Ishiguro following an older version of ECVRF-EDWARDS25519-SHA512-TAI from the -00 version of this draft is available at <https://github.com/r2ishiguro/vrf>.

An implementation similar to ECVRF-EDWARDS25519-SHA512-ELL2 (with some changes, including the use of SHA-3) is available as part of the CONIKS implementation in Golang at <https://github.com/coniks-sys/coniks-go/tree/master/crypto/vrf>.

Open Whisper Systems also uses a VRF similar to ECVRF-EDWARDS25519-SHA512-ELL2, called VXEdDSA, and specified here <https://whispersystems.org/docs/specifications/xeddsa/> and here <https://moderncrypto.org/mail-archive/curves/2017/000925.html>. Implementations in C and Java are available at <https://github.com/signalapp/curve25519-java> and <https://github.com/wavesplatform/curve25519-java>.

## **7. Security Considerations**

### **7.1. Key Generation**

Applications that use the VRFs defined in this document MUST ensure that the VRF key is generated correctly, using good randomness.

#### **7.1.1. Uniqueness and collision resistance with untrusted keys**

The RSA-FDH-VRF satisfies the "trusted uniqueness" (see [Section 3.1](#)) and "trusted collision resistance" (see [Section 3.2](#)) properties as long as the VRF keys are generated correctly. Uniqueness and collision resistance may not hold if the keys are generated

adversarially (specifically, if the RSA function specified in the public key is not bijective because the modulus  $n$  or the exponent  $e$  are chosen not in compliance with the standard); thus, RSA-FDH-VRF defined in this document does not have "full uniqueness" and "full collision resistance". Therefore, if adversarial key generation is a concern, the RSA-FDH-VRF has to be enhanced by additional cryptographic checks that its public key has the right form. These enhancements are left for future specification.

For the ECVRF, the Verifier MUST obtain  $E$  and  $B$  from a trusted source, such as a ciphersuite specification, rather than from the prover. If the verifier does so, then the ECVRF satisfies the "full uniqueness" (see [Section 3.1](#)) and "trusted collision resistance" (see [Section 3.2](#)) properties. It additionally satisfies "full collision resistance" if `validate_key` parameter given to the `ECVRF_verify` is `TRUE`.

#### 7.1.2. Pseudorandomness with untrusted keys

Without good randomness, the "pseudorandomness" properties of the VRF may not hold. Note that it is not possible to guarantee pseudorandomness in the face of adversarially generated VRF keys. This is because an adversary can always use bad randomness to generate the VRF keys, and thus, the VRF output may not be pseudorandom.

### 7.2. Security Levels

As shown in [\[PWHVNRG17\]](#), RSA-FDH-VRF satisfies the trusted uniqueness property unconditionally. The security level of the RSA-FDH-VRF, measured in bits, for the other two properties is as follows (in the random oracle model for the functions MGF1 and Hash):

- \*For trusted collision resistance: approximately  $8 \cdot \min(k/2, hLen/2)$  (as shown in [\[PWHVNRG17\]](#)).

- \*For selective pseudorandomness: approximately as strong as the security, in bits, of the RSA problem for the key  $(n, e)$  (as shown in [\[GNPRVZ15\]](#)).

As shown in [\[PWHVNRG17\]](#), the security level of the ECVRF, measured in bits, is as follows (in the random oracle model for the functions Hash and `ECVRF_encode_to_curve`):

- \*For trusted uniqueness: approximately  $8 \cdot \min(qLen, cLen)$ .

- \*For collision resistance (trusted or full, depending on whether validation is performed as explained in [Section 7.1.1](#)): approximately  $8 \cdot \min(qLen/2, hLen/2)$ .

\*For the selective pseudorandomness property: approximately as strong as the security, in bits, of the decisional Diffie-Hellman problem in the group  $G$  (which is at most  $8 \cdot q_{\text{Len}}/2$ ).

See [Section 3](#) for the definitions of these security properties. See [Section 7.3](#) for the discussion of full pseudorandomness.

### 7.3. Selective vs. Full Pseudorandomness

[[PWHVNRG17](#)] presents cryptographic reductions to an underlying hard problem (namely, the RSA problem for RSA-FDH-VRF and the Decisional Diffie-Hellman problem for the ECVRF) to prove that the VRFs specified in this document possess not only selective pseudorandomness, but also full pseudorandomness (see [Section 3.3](#) for an explanation of these notions). However, the cryptographic reductions are tighter for selective pseudorandomness than for full pseudorandomness. Specifically, the approximate provable security level, measured in bits, for full pseudorandomness may be obtained from the provable security level for selective pseudorandomness (given in [Section 7.2](#)) by subtracting the binary logarithm of the number of proofs produced for a given secret key. This holds for both the RSA-FDH-VRF and the ECVRF.

While no known attacks against full pseudorandomness are stronger than similar attacks against selective pseudorandomness, some applications may be concerned about tightness of cryptographic reductions. Such applications may consider the following two options:

\*They may choose to ensure that selective pseudorandomness is sufficient for the application. That is, that pseudorandomness of outputs matters only for inputs that are chosen independently of the VRF key.

\*They may increase security parameters to make up for the loose security reduction. For RSA-FDH-VRF, this means increasing the RSA key length. For ECVRF, this means increasing the cryptographic strength of the EC group  $G$  by specifying a new ciphersuite.

### 7.4. Proper pseudorandom nonce for ECVRF

The security of the ECVRF defined in this document relies on the fact that the nonce  $k$  used in the ECVRF\_prove algorithm is chosen uniformly and pseudorandomly modulo  $q$ , and is unknown to the adversary. Otherwise, an adversary may be able to recover the private VRF key  $x$  (and thus break pseudorandomness of the VRF) after observing several valid VRF proofs  $\pi_i$ . The nonce generation methods specified in the ECVRF ciphersuites of [Section 5.5](#) are designed with this requirement in mind.

## 7.5. Side-channel attacks

Side channel attacks on cryptographic primitives are an important issue. Implementers should take care to avoid side-channel attacks that leak information about the VRF private key SK (and the nonce  $k$  used in the ECVRF), which is used in VRF\_prove. In most applications, VRF\_proof\_to\_hash and VRF\_verify algorithms take only inputs that are public, and thus side channel attacks are typically not a concern for these algorithms.

The VRF input alpha may be also a sensitive input to VRF\_prove and may need to be protected against side channel attacks. Below we discuss one particular class of such attacks: timing attacks that can be used to leak information about the VRF input alpha.

The ECVRF\_encode\_to\_curve\_try\_and\_increment algorithm defined in [Section 5.4.1.1](#) SHOULD NOT be used in applications where the VRF input alpha is secret and is hashed by the VRF on-the-fly. This is because the algorithm's running time depends on the VRF input alpha, and thus creates a timing channel that can be used to learn information about alpha. That said, for most inputs the amount of information obtained from such a timing attack is likely to be small (1 bit, on average), since the algorithm is expected to find a valid curve point after only two attempts. However, there might be inputs which cause the algorithm to make many attempts before it finds a valid curve point; for such inputs, the information leaked in a timing attack will be more than 1 bit.

ECVRF-P256-SHA256-SSWU and ECVRF-EDWARDS25519-SHA512-ELL2 can be made to run in time independent of alpha, following recommendations in [\[I-D.irtf-cfrg-hash-to-curve\]](#).

## 7.6. Proofs provide no secrecy for the VRF input

The VRF proof pi is not designed to provide secrecy and, in general, may reveal the VRF input alpha. Anyone who knows PK and pi is able to perform an offline dictionary attack to search for alpha, by verifying guesses for alpha using VRF\_verify. This is in contrast to the VRF hash output beta which, without the proof, is pseudorandom and thus is designed to reveal no information about alpha.

## 7.7. Prehashing

The VRFs specified in this document allow for read-once access to the input alpha for both signing and verifying. Thus, additional prehashing of alpha (as specified, for example, in [\[RFC8032\]](#) for EdDSA signatures) is not needed, even for applications that need to handle long alpha or to support the Initialize-Update-Finalize (IUF) interface (in such an interface, alpha is not supplied all at once, but rather in pieces by a sequence of calls to Update). The ECVRF,

in particular, uses alpha only in ECVRF\_encode\_to\_curve. The curve point H becomes the representative of alpha thereafter.

### 7.8. Hash function domain separation

Hashing is used for different purposes in the two VRFs (namely, in the RSA-FDH-VRF, in MGF1 and in proof\_to\_hash; in the ECVRF, in encode\_to\_curve, nonce\_generation, challenge\_generation, and proof\_to\_hash). The theoretical analysis treats each of these functions as a separate hash function, modeled as a random oracle. This analysis still holds even if the same hash function is used, as long as the four queries made to the hash function for a given SK and alpha are overwhelmingly unlikely to equal each other or to any queries made to the hash function for the same SK and different alpha. This is indeed the case for the RSA-FDH-VRF defined in this document, because the second octets of the input to the hash function used in MGF1 and in proof\_to\_hash are different.

This is also the case for the ECVRF ciphersuites defined in this document, because:

- \*inputs to the hash function used during nonce\_generation are unlikely to equal inputs used in encode\_to\_curve, proof\_to\_hash, and challenge\_generation. This follows since nonce\_generation inputs a secret to the hash function that is not used by honest parties as input to any other hash function, and is not available to the adversary.

- \*the second octets of the inputs to the hash function used in proof\_to\_hash, challenge\_generation, and encode\_to\_curve\_try\_and\_increment are all different.

- \*the last octet of the input to the hash function used in proof\_to\_hash, challenge\_generation, and encode\_to\_curve\_try\_and\_increment is always zero, and therefore different from the last octet of the input to the hash function used in ECVRF\_encode\_to\_curve\_h2c\_suite, which is set equal to the nonzero length of the domain separation tag by [[I-D.irtf-cfrg-hash-to-curve](#)].

### 7.9. Hash function salting

In case a hash collision is found, in order to make it more difficult for the adversary to exploit such a collision, the MGF1 function for the RSA-FDH-VRF and ECVRF\_encode\_to\_curve function for the ECVRF use a public value in addition to alpha (as a so-called salt). This value is determined by the ciphersuite. For the ciphersuites defined in this document, it is set equal to the string representation of the RSA modulus and EC public key, respectively. Implementations that do not use one of the ciphersuites (see [Section](#)

[7.10](#)) MAY use a different salt. For example, if a group of public keys to share the same salt, then the hash of the VRF input alpha will be the same for the entire group of public keys, which may aid in some protocol that uses the VRF.

## 7.10. Futureproofing

if future designs need to specify variants (e.g., additional ciphersuites) of the RSA-FDH-VRF or the ECVRF in this document, then, to avoid the possibility that an adversary can obtain a VRF output under one variant, and then claim it was obtained under another variant, they should specify a different suite\_string constant. The suite\_string constants in this document are all single octets; if a future suite\_string constant is longer than one octet, then it should start with a different octet than the suite\_string constants in this document. Then, for the RSA-FDH-VRF, the inputs to the hash function used in MGF1 and proof\_to\_hash will be different from other ciphersuites. For the ECVRF, the inputs ECVRF\_encode\_to\_curve hash function used in producing H are then guaranteed to be different from other ciphersuites; since all the other hashing done by the prover depends on H, inputs to all the hash functions used by the prover will also be different from other ciphersuites as long as ECVRF\_encode\_to\_curve is collision resistant.

## 8. Change Log

Note to RFC Editor: if this document does not obsolete an existing RFC, please remove this appendix before publication as an RFC.

00 - Forked this document from draft-goldbe-vrf-01.

01 - Minor updates, mostly highlighting TODO items.

02 - Added specification of elligator2 for Curve25519, along with ciphersuites for ECVRF-ED25519-SHA512-Elligator. Changed ECVRF-ED25519-SHA256 suite\_string to ECVRF-ED25519-SHA512. (This change made because Ed25519 in [[RFC8032](#)] signatures use SHA512 and not SHA256.) Made ECVRF nonce generation a separate component, so that nonces are deterministic. In ECVRF proving, changed + to - (and made corresponding verification changes) in order to be consistent with EdDSA and ECDSA. Highlighted that ECVRF\_hash\_to\_curve acts like a prehash. Added "suites" variable to ECVRF for futureproofing. Ensured domain separation for hash functions by modifying hash\_points and added discussion about domain separation. Updated todos in the "additional pseudorandomness property" section. Added a discussion of secrecy into security considerations. Removed B and PK=Y from ECVRF\_hash\_points because they are already present via H, which

is computed via `hash_to_curve` using the `suite_string` (which identifies B) and Y.

03 - Changed Ed25519 conversions to little-endian, to match RFC 8032; added simple key validation for Ed25519; added Simple SWU cipher suite; clarified Elligator and removed the extra x0 bit, to make Montgomery and Edwards Elligator the same; added domain separation for RSA VRF; improved notation throughout; added nonce generation as a section; changed counter in try-and-increment from four bytes to one, to avoid endian issues; renamed try-and-increment ciphersuites to -TAI; added `qLen` as a separate parameter; changed output length to `hLen` for ECVRF, to match RSAVRF; made Verify return beta so unverified proofs don't end up in `proof_to_hash`; added test vectors.

04 - Clarified handling of optional arguments `x` and `PK` in `ECVRF_prove`. Edited implementation status to bring it up to date.

05 - Renamed `ed25519` into the more commonly used `edwards25519`. Corrected `ECVRF_nonce_generation_RFC6979` (thanks to Gorka Irazoqui Apecechea and Mario Cao Cueto for finding the problem) and corresponding test vectors for the P256 suites. Added a reference to the Rust implementation.

06 - Made some variable names more descriptive. Added a few implementation references.

07 - Incorporated hash-to-curve draft by reference to replace our own Elligator2 and Simple SWU. Clarified discussion of EC parameters and functions. Added a 0 octet to all hashing to enforce domain separation from hashing done inside hash-to-curve.

08 - Incorporated suggestions from crypto panel review by Chloe Martindale. Changed Reyzin's affiliation. Updated references.

09 - Added a note to remove the implementation page before publication.

10 - Added a check in `ECVRF_decode_proof` to ensure that `s` is reduced mod `q`. Connected security properties (Section 3) and security considerations (Section 7) with more cross-references.

11 - Processed last call comments. Clarified various notation, including lengths of various parameters for ECVRF; added error handling to RSA-FDH-VRF; added security levels section; clarified full vs trusted uniqueness and full vs selective pseudorandomness; added RSA ciphersuites; made key validation clearer; renamed `hash_to_curve` to `encode_to_curve` to be consistent with the `hash_to_curve` draft; allowed a more general salt in hashing, added the public key as input to



ECVRF\_challenge\_generation, and added an explanation about the salt.

## 9. Contributors

This document also would not be possible without the work of Moni Naor, Sachin Vasant, and Asaf Ziv. Chloe Martindale provided a thorough cryptographer's review. Liliya Akhmetzyanova, Tony Arcieri, Gary Belvin, Mario Cao Cueto, Brian Chen, Sergey Gorbunov, Shumon Huque, Gorka Irazoqui Apecechea, Marek Jankowski, Burt Kaliski, David C. Lawrence, Derek Ting-Haye Leung, Antonio Marcedone, Piotr Nojszewski, Chris Peikert, Trevor Perrin, Sam Scott, Stanislav Smyshlyaev, Adam Suhl, Nick Sullivan, Christopher Wood, Jiayu Xu, and Annie Yousar provided valuable input to this draft. Riad Wahby helped this document align with draft-irtf-cfrg-hash-to-curve.

## 10. References

### 10.1. Normative References

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**[SECG1]** Standards for Efficient Cryptography Group (SECG), "SEC 1: Elliptic Curve Cryptography", Version 2.0, May 2009, <<http://www.secg.org/sec1-v2.pdf>>.

## **10.2. Informative References**

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Bernstein, D.J., "How do I validate Curve25519 public keys?", 2006, <<https://cr.yp.to/ecdh.html#validate>>.

## **Appendix A. Test Vectors for the ECVRFs**

The test vectors in this section were generated using the reference implementation at <https://github.com/reyzin/ecvrf>.

### **A.1. ECVRF-P256-SHA256-TAI**

The example secret keys and messages in Examples 1 and 2 are taken from Appendix A.2.5 of [RFC6979].

Example 1:

SK = x =

c9afa9d845ba75166b5c215767b1d6934e50c3db36e89b127b8a622b120f6721

PK =

0360fed4ba255a9d31c961eb74c6356d68c049b8923b61fa6ce669622e60f29fb6

alpha = 73616d706c65 (ASCII "sample")

try\_and\_increment succeeded on ctr = 1

H =

0272a877532e9ac193aff4401234266f59900a4a9e3fc3cfc6a4b7e467a15d06d4

k =

0d90591273453d2dc67312d39914e3a93e194ab47a58cd598886897076986f77

U = k\*B =

02bb6a034f67643c6183c10f8b41dc4babf88bff154b674e377d90bde009c21672

V = k\*H =

02893ebee7af9a0faa6da810da8a91f9d50e1dc071240c9706726820ff919e8394

pi =

035b5c726e8c0e2c488a107c600578ee75cb702343c153cb1eb8dec77f4b5071b4a53f0a46f018bc2c56e5

beta =

a3ad7b0ef73d8fc6655053ea22f9bede8c743f08bbbed3d38821f0e16474b505e

Example 2:

SK = x =

c9afa9d845ba75166b5c215767b1d6934e50c3db36e89b127b8a622b120f6721

PK =  
0360fed4ba255a9d31c961eb74c6356d68c049b8923b61fa6ce669622e60f29fb6

alpha = 74657374 (ASCII "test")  
try\_and\_increment succeeded on ctr = 3  
H =  
02173119b4fff5e6f8afed4868a29fe8920f1b54c2cf89cc7b301d0d473de6b974

k =  
5852353a868bdce26938cde1826723e58bf8cb06dd2fed475213ea6f3b12e961  
U = k\*B =  
022779a2cafcfb65414c4a04a4b4d2adf4c50395f57995e89e6de823250d91bc48e

V = k\*H =  
033b4a14731672e82339f03b45ff6b5b13dee7ada38c9bf1d6f8f61e2ce5921119

pi =  
034dac60aba508ba0c01aa9be80377ebd7562c4a52d74722e0abae7dc3080ddb56c19e067b15a8a8174905f

beta =  
a284f94ceec2ff4b3794629da7cbafa49121972671b466cab4ce170aa365f26d

The example secret key in Example 3 is taken from Appendix L.4.2 of  
[[ANSI.X9-62-2005](#)].

Example 3:

SK = x =  
2ca1411a41b17b24cc8c3b089cfd033f1920202a6c0de8abb97df1498d50d2c8  
PK =  
03596375e6ce57e0f20294fc46bdfcfd19a39f8161b58695b3ec5b3d16427c274d

alpha =  
4578616d706c65207573696e67204543445341206b65792066726f6d20417070656e646978204c2e342e32  
(ASCII "Example using ECDSA key from Appendix L.4.2 of  
ANSI.X9-62-2005")  
try\_and\_increment succeeded on ctr = 1  
H =  
0258055c26c4b01d01c00fb57567955f7d39cd6f6e85fd37c58f696cc6b7aa761d

k =  
5689e2e08e1110b4dda293ac21667eac6db5de4a46a519c73d533f69be2f4da3  
U = k\*B =  
020f465cd0ec74d2e23af0abde4c07e866ae4e5138bde5dd1196b8843f380db84

V = k\*H =  
036cb6f811428fc4904370b86c488f60c280fa5b496d2f34ff8772f60ed24b2d1d

```
pi =
03d03398bf53aa23831d7d1b2937e005fb0062cbefa06796579f2a1fc7e7b8c667d091c00b0f5c3619d10e0

beta =
90871e06da5caa39a3c61578ebb844de8635e27ac0b13e829997d0d95dd98c19
```

## A.2. ECVRF-P256-SHA256-SSWU

The example secret keys and messages in Examples 4 and 5 are taken from Appendix A.2.5 of [[RFC6979](#)].

Example 4:

```
SK = x =
c9afa9d845ba75166b5c215767b1d6934e50c3db36e89b127b8a622b120f6721
PK =
0360fed4ba255a9d31c961eb74c6356d68c049b8923b61fa6ce669622e60f29fb6

alpha = 73616d706c65 (ASCII "sample")
In SSWU: uniform_bytes =
5024e98d6067dec313af09ff0cbe78218324a645c2a4b0aae2453f6fe91aa3bd9471f7b4a5fbf128e4b53f0

In SSWU: u =
df565615a2372e8b31b8771f7503bafc144e48b05688b97958cc27ce29a8d810
In SSWU: x1 =
e7e39eb8a4c982426fcff629e55a3e13516cf62c02c369b1e750316f5e94eb
In SSWU: gx1 is a nonsquare
H =
02b31973e872d4a097e2cfae9f37af9f9d73428fde74ac537dda93b5f18dbc5842

k =
e92820035a0a8afe132826c6312662b6ea733fc1a0d33737945016de54d02dd8
U = k*B =
031490f49d0355ffcdf66e40df788bee93861917ee713acff79be40d20cc91a30a

V = k*H =
03701df0228138fa3d16612c0d720389326b3265151bc7ac696ea4d0591cd053e3

pi =
0331d984ca8fece9cbb9a144c0d53df3c4c7a33080c1e02ddb1a96a365394c7888782fffd7b842c38c20c0

beta =
21e66dc9747430f17ed9efeda054cf4a264b097b9e8956a1787526ed00dc664b
```

Example 5:

```
SK = x =
c9afa9d845ba75166b5c215767b1d6934e50c3db36e89b127b8a622b120f6721
```

PK =  
0360fed4ba255a9d31c961eb74c6356d68c049b8923b61fa6ce669622e60f29fb6

alpha = 74657374 (ASCII "test")

In SSWU: uniform\_bytes =

910cc66d84a57985a1d15843dad83fd9138a109afb243b7fa5d64d766ec9ca3894fdcf46eb21a3972eb4

In SSWU: u =

d8b0107f7e7aa36390240d834852f8703a6dc407019d6196bda5861b8fc00181

In SSWU: x1 =

ccc747fa7318b9486ce4044adbbecaa084c27be6eda88eb7b7f3d688fd0968c7

In SSWU: gx1 is a square

H =

03ccc747fa7318b9486ce4044adbbecaa084c27be6eda88eb7b7f3d688fd0968c7

k =

febc3451ea7639fde2cf41ffd03f463124ecb3b5a79913db1ed069147c8a7dea

U = k\*B =

031200f9900e96f811d1247d353573f47e0d9da601fc992566234fc1a5b37749ae

V = k\*H =

02d3715dcfee136c7ae50e95ffca76f4ca6c29ddfb92a39c31a0d48e75c6605cd1

pi =

03f814c0455d32dbc75ad3aea08c7e2db31748e12802db23640203aebf1fa8db2743aad348a3006dc1caad

beta =

8e7185d2b420e4f4681f44ce313a26d05613323837da09a69f00491a83ad25dd

The example secret key in Example 6 is taken from Appendix L.4.2 of  
[[ANSI.X9-62-2005](#)].

Example 6:

SK = x =

2ca1411a41b17b24cc8c3b089cfd033f1920202a6c0de8abb97df1498d50d2c8

PK =

03596375e6ce57e0f20294fc46bdfcfd19a39f8161b58695b3ec5b3d16427c274d

alpha =

4578616d706c65207573696e67204543445341206b65792066726f6d20417070656e646978204c2e342e32

(ASCII "Example using ECDSA key from Appendix L.4.2 of

ANSI.X9-62-2005")

In SSWU: uniform\_bytes =

9b81d55a242d3e8438d3bcfb1bee985a87fd144802c9268cf9adeee160e6e9ff765569797a0f701cb43160

In SSWU: u =

e43c98c2ae06d13839fedb0303e5ee815896beda39be83fb11325b97976efdce

```

In SSWU: x1 =
be9e195a50f175d3563aed8dc2d9f513a5536c1e9aee1757d86c08d32d582a86
In SSWU: gx1 is a nonsquare
H =
022dd5150e5a2a24c66feab2f68532be1486e28e07f1b9a055cf38ccc16f6595ff

k =
8e29221f33564f3f66f858ba2b0c14766e1057adbd422c3e7d0d99d5e142b613
U = k*B =
03a8823ff9fd16bf879261c740b9c7792b77fee0830f21314117e441784667958d

V = k*H =
02d48fbb45921c755b73b25be2f23379e3ce69294f6cee9279815f57f4b422659d

pi =
039f8d9cdc162c89be2871cbcb1435144739431db7fab437ab7bc4e2651a9e99d5488405a11a6c7fc8defd

beta =
4fbadf33b42a5f42f23a6f89952d2e634a6e3810f15878b46ef1bb85a04fe95a

```

### A.3. ECVRF-EDWARDS25519-SHA512-TAI

The example secret keys and messages in Examples 7, 8, and 9 are taken from Section 7.1 of [[RFC8032](#)].

Example 7:

```

SK =
9d61b19deffd5a60ba844af492ec2cc44449c5697b326919703bac031cae7f60
PK =
d75a980182b10ab7d54bfed3c964073a0ee172f3daa62325af021a68f707511a
alpha = (the empty string)
x =
307c83864f2833cb427a2ef1c00a013cfdff2768d980c0a3a520f006904de94f
try_and_increment succeeded on ctr = 0
H =
91bbbed02a99461df1ad4c6564a5f5d829d0b90cfc7903e7a5797bd658abf3318
k =
7100f3d9eadb6dc4743b029736ff283f5be494128df128df2817106f345b8594b6d6da2d6fb0b4c0257eb3

U = k*B =
aef27c725be964c6a9bf4c45ca8e35df258c1878b838f37d9975523f09034071
V = k*H =
5016572f71466c646c119443455d6cb9b952f07d060ec8286d678615d55f954f
pi =
8657106690b5526245a92b003bb079ccd1a92130477671f6fc01ad16f26f723f26f8a57ccaed74ee1b190b

```

beta =  
90cf1df3b703cce59e2a35b925d411164068269d7b2d29f3301c03dd757876ff66b71dda49d2de59d03450

Example 8:

SK =  
4ccd089b28ff96da9db6c346ec114e0f5b8a319f35aba624da8cf6ed4fb8a6fb  
PK =  
3d4017c3e843895a92b70aa74d1b7ebc9c982ccf2ec4968cc0cd55f12af4660c  
alpha = 72 (1 byte)  
x =  
68bd9ed75882d52815a97585caf4790a7f6c6b3b7f821c5e259a24b02e502e51  
try\_and\_increment succeeded on ctr = 1  
H =  
5b659fc3d4e9263fd9a4ed1d022d75eaacc20df5e09f9ea937502396598dc551  
k =  
42589bbf0c485c3c91c1621bb4bfe04aed7be76ee48f9b00793b2342acb9c167cab856f9f9d4febc311330  
  
U = k\*B =  
1dcb0a4821a2c48bf53548228b7f170962988f6d12f5439f31987ef41f034ab3  
V = k\*H =  
fd03c0bf498c752161bae4719105a074630a2aa5f200ff7b3995f7bfb1513423  
pi =  
f3141cd382dc42909d19ec5110469e4feae18300e94f304590abdc48aed5933bf0864a62558b3ed7f2fe  
  
beta =  
eb4440665d3891d668e7e0fc587f1b4bd7fbfe99d0eb2211ccec90496310eb5e33821bc613efb94db5e5

Example 9:

SK =  
c5aa8df43f9f837bedb7442f31dcb7b166d38535076f094b85ce3a2e0b4458f7  
PK =  
fc51cd8e6218a1a38da47ed00230f0580816ed13ba3303ac5deb911548908025  
alpha = af82 (2 bytes)  
x =  
909a8b755ed902849023a55b15c23d11ba4d7f4ec5c2f51b1325a181991ea95c  
try\_and\_increment succeeded on ctr = 0  
H =  
bf4339376f5542811de615e3313d2b36f6f53c0acfebb482159711201192576a  
k =  
38b868c335ccda94a088428cbf3ec8bc7955bfaffe1f3bd2aa2c59fc31a0febc59d0e1af3715773ce11b3b  
  
U = k\*B =  
2bae73e15a64042fceb062abe7e432b2eca6744f3e8265bc38e009cd577ecd5  
V = k\*H =  
88cba1cb0d4f9b649d9a86026b69de076724a93a65c349c988954f0961c5d506



```
pi =
9bc0f79119cc5604bf02d23b4caede71393cedfbb191434dd016d30177ccbf8096bb474e53895c362d86280

beta =
645427e5d00c62a23fb703732fa5d892940935942101e456ecca7bb217c61c452118fec1219202a0edcf030
```

#### A.4. ECVRF-EDWARDS25519-SHA512-ELL2

The example secret keys and messages in Examples 10, 11, and 12 are taken from Section 7.1 of [[RFC8032](#)].

Example 10:

```
SK =
9d61b19deffd5a60ba844af492ec2cc44449c5697b326919703bac031cae7f60
PK =
d75a980182b10ab7d54bfed3c964073a0ee172f3daa62325af021a68f707511a
alpha = (the empty string)
x =
307c83864f2833cb427a2ef1c00a013cfdff2768d980c0a3a520f006904de94f
In Elligator2: uniform_bytes =
d620782a206d9de584b74e23ae5ee1db5ca5298b3fc527c4867f049dee6dd419b3674967bd614890f621c10

In Elligator2: u =
30f037b9745a57a9a2b8a68da81f397c39d46dee9d047f86c427c53f8b29a55c
In Elligator2: gx1 =
8cb66318fb2cea01672d6c27a5ab662ae33220961607f69276080a56477b4a08
In Elligator2: gx1 is a square
H =
b8066ebbb706c72b64390324e4a3276f129569eab100c26b9f05011200c1bad9
k =
b5682049fee54fe2d519c9afff73bbfad724e69a82d5051496a42458f817bed7a386f96b1a78e5736756190

U = k*B =
762f5c178b68f0cddcc1157918edf45ec334ac8e8286601a3256c3bbf858edd9
V = k*H =
4652eba1c4612e6fce762977a59420b451e12964adbe4fbecd58a7aef5860af
pi =
7d9c633ffeee27349264cf5c667579fc583b4bda63ab71d001f89c10003ab46f14adf9a3cd8b8412d903850

beta =
9d574bf9b8302ec0fc1e21c3ec5368269527b87b462ce36dab2d14ccf80c53cccf6758f058c5b1c856b1160
```

Example 11:

```
SK =
4ccd089b28ff96da9db6c346ec114e0f5b8a319f35aba624da8cf6ed4fb8a6fb
```

```
PK =
3d4017c3e843895a92b70aa74d1b7ebc9c982ccf2ec4968cc0cd55f12af4660c
alpha = 72 (1 byte)
x =
68bd9ed75882d52815a97585caf4790a7f6c6b3b7f821c5e259a24b02e502e51
In Elligator2: uniform_bytes =
04ae20a9ad2a2330fb33318e376a2448bd77bb99e81d126f47952b156590444a9225b84128b66a2f15b4129

In Elligator2: u =
3092f033b16d4d5f74a3f7dc7091fe434b449065152b95476f121de899bb773d
In Elligator2: gx1 =
25d7fe7f82456e7078e99fdb24ef2582b4608357cdba9c39a8d535a3fd98464d
In Elligator2: gx1 is a nonsquare
H =
76ac3ccb86158a9104dff819b1ca293426d305fd76b39b13c9356d9b58c08e57
k =
88bf479281fd29a6cbdfdd67e2c5ec0024d92f14eaed58f43f22f37c4c37f1d41e65c036fbf01f9fba11d50

U = k*B =
8ec26e77b8cb3114dd2265fe1564a4efb40d109aa3312536d93dfe3d8d80a061
V = k*H =
fe799eb5770b4e3a5a27d22518bb631db183c8316bb552155f442c62a47d1c8b
pi =
47b327393ff2dd81336f8a2ef10339112401253b3c714eeda879f12c509072ef055b48372bb82efbdce8e10

beta =
38561d6b77b71d30eb97a062168ae12b667ce5c28cacddf76bc88e093e4635987cd96814ce55b4689b3dd20
```

Example 12:

```
SK =
c5aa8df43f9f837bedb7442f31dcb7b166d38535076f094b85ce3a2e0b4458f7
PK =
fc51cd8e6218a1a38da47ed00230f0580816ed13ba3303ac5deb911548908025
alpha = af82 (2 bytes)
x =
909a8b755ed902849023a55b15c23d11ba4d7f4ec5c2f51b1325a181991ea95c
In Elligator2: uniform_bytes =
be0aed556e36cdfddf8f1eeddbb7356a24fad64cf95a922a098038f215588b216beabbfe6acf20256188e80

In Elligator2: u =
f6675dc6d17fc790d4b3f1c6acf689a13d8b5815f23880092a925af94cd6fa24
In Elligator2: gx1 =
a63d48e3247c903e22dfdb88fd9295e396712a5fe576af335dbe16f99f0af26c
In Elligator2: gx1 is a square
H =
13d2a8b5ca32db7e98094a61f656a08c6c964344e058879a386a947a4e189ed1
```

k =  
a7ddd74a3a7d165d511b02fa268710ddbb3b939282d276fa2efcfa5aaf79cf576087299ca9234aacd7cd674  
U = k\*B =  
a012f35433df219a88ab0f9481f4e0065d00422c3285f3d34a8b0202f20bac60  
V = k\*H =  
fb613986d171b3e98319c7ca4dc44c5dd8314a6e5616c1a4f16ce72bd7a0c25a  
pi =  
926e895d308f5e328e7aa159c06eddb56d06846abf5d98c2512235eaa57fdce35b46edfc655bc828d44ad  
beta =  
121b7f9b9aaaa29099fc04a94ba52784d44eac976dd1a3cca458733be5cd090a7b5fbd148444f17f8daf1f

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