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Verifiable Random Functions (VRFs)

Abstract

A Verifiable Random Function (VRF) is the public-key version of a keyed cryptographic hash. Only the holder of the private key can compute the hash, but anyone with the public key can verify the correctness of the hash. VRFs are useful for preventing enumeration of hash-based data structures. This document specifies several VRF constructions based on RSA and Elliptic Curves that are secure in the cryptographic random oracle model.

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1. Introduction

This document represents the consensus of the Crypto Forum Research Group (CFRG).

1.1. Rationale

A Verifiable Random Function (VRF) [[MRV99](#)] is the public-key version of a keyed cryptographic hash. Only the holder of the private VRF key can compute the hash, but anyone with the corresponding public key can verify the correctness of the hash.

A key application of the VRF is to provide privacy against offline dictionary attacks (also known as enumeration attacks) on data stored in a hash-based data structure. In this application, a Prover holds the VRF private key and uses the VRF hashing to construct a hash-based data structure on the input data. Due to the nature of the VRF, only the Prover can answer queries about whether or not some data is stored in the data structure. Anyone who knows the public VRF key can verify that the Prover has answered the queries correctly. However, no offline inferences (i.e. inferences without querying the Prover) can be made about the data stored in the data structure.

1.2. Requirements

The key words "MUST", "MUST NOT", "REQUIRED", "SHALL", "SHALL NOT", "SHOULD", "SHOULD NOT", "RECOMMENDED", "MAY", and "OPTIONAL" in this document are to be interpreted as described in [[RFC2119](#)].

1.3. Terminology

The following terminology is used through this document:

SK: The private key for the VRF.

PK:

The public key for the VRF.

alpha or alpha_string: The input to be hashed by the VRF.

beta or beta_string: The VRF hash output.

pi or pi_string: The VRF proof.

Prover: The Prover holds the private VRF key SK and public VRF key PK.

Verifier: The Verifier holds the public VRF key PK.

2. VRF Algorithms

A VRF comes with a key generation algorithm that generates a public VRF key PK and private VRF key SK.

The prover hashes an input alpha using the private VRF key SK to obtain a VRF hash output beta

$$\text{beta} = \text{VRF_hash}(\text{SK}, \text{alpha})$$

The VRF_hash algorithm is deterministic, in the sense that it always produces the same output beta given the same pair of inputs (SK, alpha). The prover also uses the private key SK to construct a proof pi that beta is the correct hash output

$$\text{pi} = \text{VRF_prove}(\text{SK}, \text{alpha})$$

The VRFs defined in this document allow anyone to deterministically obtain the VRF hash output beta directly from the proof value pi by using the function VRF_proof_to_hash:

$$\text{beta} = \text{VRF_proof_to_hash}(\text{pi})$$

Thus, for VRFs defined in this document, VRF_hash is defined as

$$\text{VRF_hash}(\text{SK}, \text{alpha}) = \text{VRF_proof_to_hash}(\text{VRF_prove}(\text{SK}, \text{alpha})),$$

and therefore this document will specify VRF_prove and VRF_proof_to_hash rather than VRF_hash.

The proof pi allows a Verifier holding the public key PK to verify that beta is the correct VRF hash of input alpha under key PK. Thus, the VRFs defined in this document also come with an algorithm

$$\text{VRF_verify}(\text{PK}, \text{alpha}, \text{pi})$$

that outputs (VALID, $\beta = \text{VRF_proof_to_hash}(\pi)$) if π is valid, and INVALID otherwise.

3. VRF Security Properties

VRFs are designed to ensure the following security properties.

3.1. Full Uniqueness or Trusted Uniqueness

Uniqueness means that, for any fixed public VRF key and for any input α , there is a unique VRF output β that can be proved to be valid. Uniqueness must hold even for an adversarial Prover that knows the VRF private key SK.

More precisely, "full uniqueness" states that a computationally-bounded adversary cannot choose a VRF public key PK, a VRF input α , and two proofs π_1 and π_2 such that $\text{VRF_verify}(\text{PK}, \alpha, \pi_1)$ outputs (VALID, β_1), $\text{VRF_verify}(\text{PK}, \alpha, \pi_2)$ outputs (VALID, β_2), and β_1 is not equal to β_2 .

For many applications, a slightly weaker security property called "trusted uniqueness" suffices. Trusted uniqueness is the same as full uniqueness, but it is guaranteed to hold only if the VRF keys PK and SK were generated in a trustworthy manner.

As further discussed in [Section 7.1.1](#), some VRFs specified in this document satisfy only trusted uniqueness, while others satisfy full uniqueness. VRFs in this document that satisfy only trusted uniqueness but not full uniqueness MUST NOT be used if the key generation process cannot be trusted.

3.2. Full Collision Resistance or Trusted Collision Resistance

Like any cryptographic hash function, VRFs need to be collision resistant. Collision resistance must hold even for an adversarial Prover that knows the VRF private key SK.

More precisely, "full collision resistance" states that it should be computationally infeasible for an adversary to find two distinct VRF inputs α_1 and α_2 that have the same VRF hash β , even if that adversary knows the private VRF key SK.

For many applications, a slightly weaker security property called "trusted collision resistance" suffices. Trusted collision resistance is the same as collision resistance, but it is guaranteed to hold only if the VRF keys PK and SK were generated in a trustworthy manner.

As further discussed in [Section 7.1.1](#), some VRFs specified in this document satisfy only trusted collision resistance, while others

satisfy full collision resistance. VRFs in this document that satisfy only trusted collision resistance but not full collision resistance MUST NOT be used if the key generation process cannot be trusted.

3.3. Full Pseudorandomness or Selective Pseudorandomness

Pseudorandomness ensures that when an adversarial Verifier sees a VRF hash output β without its corresponding VRF proof π , then β is indistinguishable from a random value.

More precisely, suppose the public and private VRF keys (PK , SK) were generated in a trustworthy manner. Pseudorandomness ensures that the VRF hash output β (without its corresponding VRF proof π) on any adversarially-chosen "target" VRF input α looks indistinguishable from random for any computationally bounded adversary who does not know the private VRF key SK . This holds even if the adversary also gets to choose other VRF inputs α' and observe their corresponding VRF hash outputs β' and proofs π' .

With "full pseudorandomness", the adversary is allowed to choose the "target" VRF input α at any time, even after it observes VRF outputs β' and proofs π' on a variety of chosen inputs α' .

"Selective pseudorandomness" is a weaker security property which suffices in many applications. Here, the adversary must choose the target VRF input α independently of the public VRF key PK , and before it observes VRF outputs β' and proofs π' on inputs α' of its choice.

As further discussed in [Section 7.3](#), VRFs specified in this document satisfy both full pseudorandomness and selective pseudorandomness, but their quantitative security against the selective pseudorandomness attack is stronger.

It is important to remember that the VRF output β does not look random to the Prover, or to any other party that knows the private VRF key SK ! Such a party can easily distinguish β from a random value by comparing β to the result of $VRF_hash(SK, \alpha)$.

Also, the VRF output β does not look random to any party that knows a valid VRF proof π corresponding to the VRF input α , even if this party does not know the private VRF key SK . Such a party can easily distinguish β from a random value by checking whether $VRF_verify(PK, \alpha, \pi)$ returns $(VALID, \beta)$.

Also, the VRF output β may not look random if VRF key generation was not done in a trustworthy fashion. (For example, if VRF keys were generated with bad randomness.)

3.4. Some VRFs: Unpredictability Under Malicious Key Generation

As explained in [Section 3.3](#), pseudorandomness is guaranteed only if the VRF keys were generated in a trustworthy fashion. For instance, if an adversary outputs VRF keys that are deterministically generated (or hard-coded and publicly known), then the outputs are easily derived by anyone and are therefore not pseudorandom.

There is, however, a different type of unpredictability that is desirable in certain VRF applications (such as leader selection in the consensus protocols of [\[GHMVZ17\]](#) and [\[DGKR18\]](#)), called "unpredictability under malicious key generation". This property is similar to the unpredictability achieved by an (ordinary, unkeyed) cryptographic hash function: if the input has enough entropy (i.e., cannot be predicted), then the correct output is indistinguishable from uniform, no matter how the VRF keys are generated.

A formal definition of this property appears in Section 3.2 of [\[DGKR18\]](#). The RSA-FDH-VRF presented in this document does not satisfy this property. The ECVRF presented in this document satisfies this property if `validate_key` parameter given to the `ECVRF_verify` is `TRUE`.

4. RSA Full Domain Hash VRF (RSA-FDH-VRF)

The RSA Full Domain Hash VRF (RSA-FDH-VRF) is a VRF that, for suitable key lengths, satisfies the "trusted uniqueness", "trusted collision resistance", and "full pseudorandomness" properties defined in [Section 3](#), as further discussed in [Section 7](#). Its security follows from the standard RSA assumption in the random oracle model. Formal security proofs are in [\[PWHVNRG17\]](#).

The VRF computes the proof π as a deterministic RSA signature on input α using the RSA Full Domain Hash Algorithm [\[RFC8017\]](#) parametrized with the selected hash algorithm. RSA signature verification is used to verify the correctness of the proof. The VRF hash output β is simply obtained by hashing the proof π with the selected hash algorithm.

The key pair for RSA-FDH-VRF MUST be generated in a way that it satisfies the conditions specified in Section 3 of [\[RFC8017\]](#).

In this section, the notation from [\[RFC8017\]](#) is used.

Parameters used:

(n, e) - RSA public key

K - RSA private key (its representation is implementation-dependent)

k - length in octets of the RSA modulus n (k must be less than 2^{32})

Fixed options (specified in [Section 4.4](#)):

Hash - cryptographic hash function

hLen - output length in octets of hash function Hash

suite_string - an octet string specifying the RSA-FDH-VRF ciphersuite, which determines the above options

Primitives used:

I2OSP - Conversion of a nonnegative integer to an octet string as defined in Section 4.1 of [\[RFC8017\]](#) (given an integer and a length in octets, produces a big-endian representation of the integer, zero-padded to the desired length)

OS2IP - Conversion of an octet string to a nonnegative integer as defined in Section 4.2 of [\[RFC8017\]](#) (given a big-endian encoding of an integer, produces the integer)

RSASP1 - RSA signature primitive as defined in Section 5.2.1 of [\[RFC8017\]](#) (given a private key and an input, raises the input to the private RSA exponent modulo n)

RSAPV1 - RSA verification primitive as defined in Section 5.2.2 of [\[RFC8017\]](#) (given a public key and an input, raises the input to the public RSA exponent modulo n)

MGF1 - Mask Generation Function based on the hash function Hash as defined in Section B.2.1 of [\[RFC8017\]](#) (given an input, produces a random-oracle-like output of desired length)

|| - octet string concatenation

4.1. RSA-FDH-VRF Proving

RSAPFDHVRP_prove(K, alpha_string[, MGF_salt])

Input:

K - RSA private key

alpha_string - VRF hash input, an octet string

Optional Input:

MGF_salt - a public octet string used as a hash function salt;
this input is not used when MGF_salt is specified as part of the
ciphersuite

Output:

pi_string - proof, an octet string of length k

Steps:

1. mgf_domain_separator = 0x01
2. EM = MGF1(suite_string || mgf_domain_separator || MGF_salt ||
alpha_string, k - 1)
3. m = OS2IP(EM)
4. s = RSASP1(K, m)
5. pi_string = I2OSP(s, k)
6. Output pi_string

4.2. RSA-FDH-VRF Proof to Hash

RSAFDHVRF_proof_to_hash(pi_string)

Input:

pi_string - proof, an octet string of length k

Output:

beta_string - VRF hash output, an octet string of length hLen

Important note:

RSAFDHVRF_proof_to_hash should be run only on pi_string that is
known to have been produced by RSAFDHVRF_prove, or from within
RSAFDHVRF_verify as specified in [Section 4.3](#).

Steps:

1. proof_to_hash_domain_separator = 0x02
2. beta_string = Hash(suite_string ||
proof_to_hash_domain_separator || pi_string)
3. Output beta_string

4.3. RSA-FDH-VRF Verifying

`RSAFDHVRF_verify((n, e), alpha_string, pi_string[, MGF_salt])`

Input:

`(n, e)` - RSA public key

`alpha_string` - VRF hash input, an octet string

`pi_string` - proof to be verified, an octet string of length `k`

Optional Input:

`MGF_salt` - a public octet string used as a hash function salt; this input is not used when `MGF_salt` is specified as part of the ciphersuite

Output:

Output:

`("VALID", beta_string)`, where `beta_string` is the VRF hash output, an octet string of length `hLen`; or

`"INVALID"`

Steps:

1. `s = OS2IP(pi_string)`
2. `m = RSAVP1((n, e), s)`; if `RSAVP1` returns "signature representative out of range", output "INVALID" and stop.
3. `mgf_domain_separator = 0x01`
4. `EM' = MGF1(suite_string || mgf_domain_separator || MGF_salt || alpha_string, k - 1)`
5. `m' = OS2IP(EM')`
6. If `m` and `m'` are equal, output `("VALID", RSAFDHVRF_proof_to_hash(pi_string))`; else output "INVALID".

4.4. RSA-FDH-VRF Ciphersuites

This document defines RSA-FDH-VRF-SHA256 as follows:

`*suite_string = 0x01`

*The hash function Hash is SHA-256 as specified in [[RFC6234](#)], with
hLen = 32

*MGF_salt = I2OSP(k, 4) || I2OSP(n, k)

This document defines RSA-FDH-VRF-SHA384 as follows:

*suite_string = 0x02

*The hash function Hash is SHA-384 as specified in [[RFC6234](#)], with
hLen = 48

*MGF_salt = I2OSP(k, 4) || I2OSP(n, k)

This document defines RSA-FDH-VRF-SHA512 as follows:

*suite_string = 0x03

*The hash function Hash is SHA-512 as specified in [[RFC6234](#)], with
hLen = 64

*MGF_salt = I2OSP(k, 4) || I2OSP(n, k)

5. Elliptic Curve VRF (ECVRF)

The Elliptic Curve Verifiable Random Function (ECVRF) is a VRF that, for suitable parameter choices, satisfies the "full uniqueness", "trusted collision resistance", and "full pseudorandomness properties" defined in [Section 3](#). If validate_key parameter given to the ECVRF_verify is TRUE, then the ECVRF additionally satisfies "full collision resistance" and "unpredictability under malicious key generation". See [Section 7](#) for further discussion. Formal security proofs are in [[PWHVNRG17](#)].

Notation used:

Elliptic curve operations are written in additive notation, with P+Q denoting point addition and x*P denoting scalar multiplication of a point P by a scalar x

x^y - x raised to the power y

x*y - x multiplied by y

s || t - concatenation of octet strings s and t

0xMN (where M and N are hexadecimal digits) - a single octet with value M*16+N; equivalently, int_to_string(M*16+N, 1), where int_to_string is as defined below.

Fixed options (specified in [Section 5.5](#)):

F - finite field

fLen - length, in octets, of an element in F encoded as an octet string

E - elliptic curve (EC) defined over F

ptLen - length, in octets, of a point on E encoded as an octet string

G - subgroup of E of large prime order

q - prime order of group G

qLen - length of q in octets, i.e., smallest integer such that $2^{(8qLen)} > q$

cLen - length, in octets, of a challenge value used by the VRF (note that in the typical case, cLen is qLen/2 or close to it)

cofactor - number of points on E divided by q

B - generator of group G

Hash - cryptographic hash function

hLen - output length in octets of Hash (hLen must be at least cLen; in the typical case, it is at least qLen)

ECVRF_encode_to_curve - a function that hashes strings to points on E.

ECVRF_nonce_generation - a function that derives a pseudorandom nonce from SK and the input as part of ECVRF proving.

suite_string - an octet string specifying the ECVRF ciphersuite, which determines the above options as well as type conversions and parameter generation

Type conversions (specified in [Section 5.5](#)):

int_to_string(a, len) - conversion of nonnegative integer a to octet string of length len

string_to_int(a_string) - conversion of an octet string a_string to a nonnegative integer

`point_to_string` - conversion of a point on E to an `ptLen`-octet string

`string_to_point` - conversion of an `ptLen`-octet string to a point on E. `string_to_point` returns `INVALID` if the octet string does not convert to a valid EC point on the curve E.

Note that with certain software libraries (for big integer and elliptic curve arithmetic), the `int_to_string` and `point_to_string` conversions are not needed, when the libraries encode integers and EC points in the same way as required by the ciphersuites. For example, in some implementations, EC point operations will take octet strings as inputs and produce octet strings as outputs, without introducing a separate elliptic curve point type.

Parameters used (the generation of these parameters is specified in [Section 5.5](#)):

`SK` - VRF private key

`x` - VRF secret scalar, an integer. Note: depending on the ciphersuite used, the VRF secret scalar may be equal to `SK`; else, it is derived from `SK`

$Y = x \cdot B$ - VRF public key, an point on E

`PK_string` = `point_to_string(Y)` - VRF public key represented as an octet string

`encode_to_curve_salt` - a public value used as a hash function salt

5.1. ECVRF Proving

`ECVRF_prove(SK, alpha_string[, encode_to_curve_salt])`

Input:

`SK` - VRF private key

`alpha_string` - input alpha, an octet string

Optional input:

`encode_to_curve_salt` - a public salt value, an octet string; this input is not used when `encode_to_curve_salt` is specified as part of the ciphersuite

Output:

pi_string - VRF proof, octet string of length ptLen+cLen+qLen

Steps:

1. Use SK to derive the VRF secret scalar x and the VRF public key

$$Y = x * B$$

(this derivation depends on the ciphersuite, as per [Section 5.5](#);

these values can be cached, for example, after key generation, and need not be rederived each time)

2. $H = \text{ECVRF_encode_to_curve}(\text{encode_to_curve_salt}, \text{alpha_string})$
(see [Section 5.4.1](#))

3. $\text{h_string} = \text{point_to_string}(H)$

4. $\text{Gamma} = x * H$

5. $k = \text{ECVRF_nonce_generation}(\text{SK}, \text{h_string})$ (see [Section 5.4.2](#))

6. $c = \text{ECVRF_challenge_generation}(Y, H, \text{Gamma}, k * B, k * H)$ (see [Section 5.4.3](#))

7. $s = (k + c * x) \bmod q$

8. $\text{pi_string} = \text{point_to_string}(\text{Gamma}) || \text{int_to_string}(c, \text{cLen}) || \text{int_to_string}(s, \text{qLen})$

9. Output pi_string

5.2. ECVRF Proof to Hash

ECVRF_proof_to_hash(pi_string)

Input:

pi_string - VRF proof, octet string of length ptLen+cLen+qLen

Output:

"INVALID", or

beta_string - VRF hash output, octet string of length hLen

Important note:

ECVRF_proof_to_hash should be run only on pi_string that is known to have been produced by ECVRF_prove, or from within ECVRF_verify as specified in [Section 5.3](#).

Steps:

1. D = ECVRF_decode_proof(pi_string) (see [Section 5.4.4](#))
2. If D is "INVALID", output "INVALID" and stop
3. (Gamma, c, s) = D
4. proof_to_hash_domain_separator_front = 0x03
5. proof_to_hash_domain_separator_back = 0x00
6. beta_string = Hash(suite_string ||
proof_to_hash_domain_separator_front ||
point_to_string(cofactor * Gamma) ||
proof_to_hash_domain_separator_back)
7. Output beta_string

5.3. ECVRF Verifying

ECVRF_verify(PK_string, alpha_string, pi_string[,
encode_to_curve_salt, validate_key])

Input:

PK_string - public key, an octet string

alpha_string - VRF input, octet string

pi_string - VRF proof, octet string of length ptLen+cLen+qLen

Optional input:

encode_to_curve_salt - a public salt value, an octet string; this input is not used when encode_to_curve_salt is specified as part of the ciphersuite

validate_key - a boolean. An implementation MAY support only the option of validate_key = TRUE, or only the option of validate_key = FALSE, in which case this input is not needed. If an implementation supports only one option, it MUST specify which option it supports.

Output:

("VALID", beta_string), where beta_string is the VRF hash output, octet string of length hLen; or

"INVALID"

Steps:

1. $Y = \text{string_to_point}(\text{PK_string})$
2. If Y is "INVALID", output "INVALID" and stop
3. If `validate_key`, run `ECVRF_validate_key(Y)` ([Section 5.4.5](#)); if it outputs "INVALID", output "INVALID" and stop
4. $D = \text{ECVRF_decode_proof}(\text{pi_string})$ (see [Section 5.4.4](#))
5. If D is "INVALID", output "INVALID" and stop
6. $(\text{Gamma}, c, s) = D$
7. $H = \text{ECVRF_encode_to_curve}(\text{encode_to_curve_salt}, \text{alpha_string})$ (see [Section 5.4.1](#))
8. $U = s*B - c*Y$
9. $V = s*H - c*\text{Gamma}$
10. $c' = \text{ECVRF_challenge_generation}(Y, H, \text{Gamma}, U, V)$ (see [Section 5.4.3](#))
11. If c and c' are equal, output ("VALID", `ECVRF_proof_to_hash(pi_string)`); else output "INVALID"

Note that the first three steps need to be performed only once for a given public key.

5.4. ECVRF Auxiliary Functions

5.4.1. ECVRF Encode to Curve

The `ECVRF_encode_to_curve` algorithm takes a public salt (see [Section 7.9](#)) and the VRF input α and converts it to H , an EC point in G . This algorithm is the only place the VRF input α is used for proving and verifying. See [Section 7.7](#) for further discussion.

This section specifies a number of such algorithms, which are not compatible with each other and are intended to use with various ciphersuites specified in [Section 5.5](#).

Input:

encode_to_curve_salt - public salt value, an octet string

alpha_string - value to be hashed, an octet string

Output:

H - hashed value, a point in G

5.4.1.1. ECVRF_encode_to_curve_try_and_increment

The following

ECVRF_encode_to_curve_try_and_increment(encode_to_curve_salt, alpha_string) algorithm implements ECVRF_encode_to_curve in a simple and generic way that works for any elliptic curve. To use this algorithm, hLen MUST be at least fLen.

The running time of this algorithm depends on alpha_string. For the ciphersuites specified in [Section 5.5](#), this algorithm is expected to find a valid curve point after approximately two attempts (i.e., when ctr=1) on average.

However, because the running time of algorithm depends on alpha_string, this algorithm SHOULD be avoided in applications where it is important that the VRF input alpha remain secret.

ECVRF_encode_to_curve_try_and_increment(encode_to_curve_salt, alpha_string)

Fixed option (specified in [Section 5.5](#)):

interpret_hash_value_as_a_point - a function that attempts to convert a cryptographic hash value to a point on E; may output INVALID.

Steps:

1. ctr = 0
2. encode_to_curve_domain_separator_front = 0x01
3. encode_to_curve_domain_separator_back = 0x00
4. H = "INVALID"
5. While H is "INVALID" or H is the identity element of the elliptic curve group:
 - a. ctr_string = int_to_string(ctr, 1)

- b. `hash_string = Hash(suite_string ||
 encode_to_curve_domain_separator_front ||
 encode_to_curve_salt || alpha_string || ctr_string ||
 encode_to_curve_domain_separator_back)`
- c. `H = interpret_hash_value_as_a_point(hash_string)`
- d. If H is not "INVALID" and cofactor > 1, set `H = cofactor * H`
- e. `ctr = ctr + 1`

6. Output H

Note even though the loop is infinite as written, and `int_to_string(ctr,1)` may fail when ctr reaches 256, `interpret_hash_value_as_a_point` functions specified in [Section 5.5](#) will succeed on roughly half hash_string values. Thus the loop is expected to stop after two iterations, and ctr is overwhelmingly unlikely (probability about 2^{-256}) to reach 256.

5.4.1.2. ECVRF_encode_to_curve_h2c_suite

The `ECVRF_encode_to_curve_h2c_suite(encode_to_curve_salt, alpha_string)` algorithm implements `ECVRF_encode_to_curve` using one of the several hash-to-curve options defined in [[I-D.irtf-cfrg-hash-to-curve](#)]. The specific choice of the hash-to-curve option (called Suite ID in [[I-D.irtf-cfrg-hash-to-curve](#)]) is given by the `h2c_suite_ID_string` parameter.

`ECVRF_encode_to_curve_h2c_suite(encode_to_curve_salt, alpha_string)`

Fixed option (specified in [Section 5.5](#)):

`h2c_suite_ID_string` - a hash-to-curve suite ID, encoded in ASCII
 (see discussion below)

Steps:

1. `string_to_be_hashed = encode_to_curve_salt || alpha_string`
2. `H = encode(string_to_be_hashed)`
 (the encode function is discussed below)

3. Output H

The encode function is provided by the hash-to-curve suite whose ID is `h2c_suite_ID_string`, as specified in [[I-D.irtf-cfrg-hash-to-](#)

[curve](#)], Section 8. The domain separation tag DST, a parameter to the hash-to-curve suite, SHALL be set to

"ECVRF_" || h2c_suite_ID_string || suite_string

where "ECVRF_" is represented as a 6-byte ASCII encoding (in hexadecimal, octets 45 43 56 52 46 5F).

5.4.2. ECVRF Nonce Generation

The following algorithms generate the nonce value k in a deterministic pseudorandom fashion. This section specifies a number of such algorithms, which are not compatible with each other. The choice of a particular algorithm from the options specified in this section depends on the ciphersuite, as specified in [Section 5.5](#).

5.4.2.1. ECVRF Nonce Generation from RFC 6979

ECVRF_nonce_generation_RFC6979(SK, h_string)

Input:

SK - an ECVRF secret key

h_string - an octet string

Output:

k - an integer nonce between 1 and $q-1$

The ECVRF_nonce_generation function is as specified in [\[RFC6979\]](#) Section 3.2 where

Input m is set equal to h_string

The "suitable for DSA or ECDSA" check in step h.3 is omitted

The hash function H is Hash and its output length $hlen$ (in bits) is set as $hlen*8$

The secret key x is set equal to the VRF secret scalar x

The prime q is the same as in this specification

$qlen$ is the binary length of q , i.e., the smallest integer such that $2^{qlen} > q$ (this $qlen$ is not to be confused with $qlen$ in this document, which is the length of q in octets)

All the other values and primitives as defined in [\[RFC6979\]](#)

5.4.2.2. ECVRF Nonce Generation from RFC 8032

The following is from Steps 2-3 of Section 5.1.6 in [[RFC8032](#)]. To use this algorithm, hLen MUST be at least 64.

ECVRF_nonce_generation_RFC8032(SK, h_string)

Input:

SK - an ECVRF secret key

h_string - an octet string

Output:

k - an integer nonce between 0 and q-1

Steps:

1. hashed_sk_string = Hash(SK)
2. truncated_hashed_sk_string =
 hashed_sk_string[32]...hashed_sk_string[63]
3. k_string = Hash(truncated_hashed_sk_string || h_string)
4. k = string_to_int(k_string) mod q

5.4.3. ECVRF Challenge Generation

ECVRF_challenge_generation(P1, P2, P3, P4, P5)

Input:

P1, P2, P3, P4, P5 - EC points

Output:

c - challenge value, integer between 0 and $2^{(8*cLen)}-1$

Steps:

1. challenge_generation_domain_separator_front = 0x02
2. Initialize str = suite_string ||
 challenge_generation_domain_separator_front
3. for PJ in [P1, P2, P3, P4, P5]:

 str = str || point_to_string(PJ)

4. challenge_generation_domain_separator_back = 0x00
5. str = str || challenge_generation_domain_separator_back
6. c_string = Hash(str)
7. truncated_c_string = c_string[0]...c_string[cLen-1]
8. c = string_to_int(truncated_c_string)
9. Output c

5.4.4. ECVRF Decode Proof

ECVRF_decode_proof(pi_string)

Input:

pi_string - VRF proof, octet string (ptLen+cLen+qLen octets)

Output:

"INVALID", or

Gamma - a point on E

c - integer between 0 and $2^{(8*cLen)}-1$

s - integer between 0 and q-1

Steps:

1. gamma_string = pi_string[0]...pi_string[ptLen-1]
2. c_string = pi_string[ptLen]...pi_string[ptLen+cLen-1]
3. s_string = pi_string[ptLen+cLen]...pi_string[ptLen+cLen+qLen-1]
4. Gamma = string_to_point(gamma_string)
5. if Gamma = "INVALID" output "INVALID" and stop
6. c = string_to_int(c_string)
7. s = string_to_int(s_string)
8. if s >= q output "INVALID" and stop
9. Output Gamma, c, and s

5.4.5. ECVRF Validate Key

ECVRF_validate_key(Y)

Input:

Y - public key, a point on E

Output:

"VALID" or "INVALID"

Important note: the public key Y given to this procedure MUST be a valid point on E.

Steps:

1. Let $Y' = \text{cofactor} * Y$
2. If Y' is the identity element of the elliptic curve group, output "INVALID" and stop
3. Output "VALID"

Note that if the cofactor = 1, then Step 1 simply sets $Y' = Y$. In particular, for the P-256 curve, ECVRF_validate_key simply ensures that Y is not the point at infinity.

Also note that if the cofactor is small, the total number of Y values that could cause Step 2 to output "INVALID" may be small, and it may be more efficient to simply check Y against a fixed list of such points. For example, the following algorithm can be used for the edwards25519 curve:

1. PK_string = point_to_string(Y)
2. oneTwentySeven_string = 0x7F
3. y_string[31] = y_string[31] & oneTwentySeven_string
(this step clears the high-order bit of octet 31)
4. bad_pk[0] = int_to_string(0, 32)
5. bad_pk[1] = int_to_string(1, 32)
6. bad_y2 =
2707385501144840649318225287225658788936804267575313519463743609750303402022
7. bad_pk[2] = int_to_string(bad_y2, 32)

8. `bad_pk[3] = int_to_string(p-bad_y2, 32)`
9. `bad_pk[4] = int_to_string(p-1, 32)`
10. `bad_pk[5] = int_to_string(p, 32)`
11. `bad_pk[6] = int_to_string(p+1, 32)`
12. If `y_string` is in the list `[bad_pk[0], ..., bad_pk[6]]`, output "INVALID" and stop
13. Output `Y`

(`bad_pk[0]`, `bad_pk[2]`, `bad_pk[3]` each match two bad public keys, depending on the sign of the x-coordinate, which was cleared in step 5, in order to make sure that it does not affect the comparison. `bad_pk[1]` and `bad_pk[4]` each match one bad public key, because x-coordinate is 0 for these two public keys. `bad_pk[5]` and `bad_pk[6]` are simply `bad_pk[0]` and `bad_pk[1]` shifted by `p`, in case the y-coordinate had not been modular reduced by `p`. There is no need to shift the other `bad_pk` values by `p`, because they will exceed 2^{255} . These bad keys, which represent all points of order 1, 2, 4, and 8, have been obtained by converting the points specified in [\[X25519\]](#) to Edwards coordinates.)

5.5. ECVRF Ciphersuites

This document defines ECVRF-P256-SHA256-TAI as follows:

`*suite_string = 0x01.`

*The EC group `G` is the NIST P-256 elliptic curve, with curve parameters as specified in [\[FIPS-186-4\]](#) (Section D.1.2.3) and [\[RFC5114\]](#) (Section 2.6). For this group, `fLen = qLen = 32` and `cofactor = 1`.

`*cLen = 16.`

*The key pair generation primitive is specified in Section 3.2.1 of [\[SECG1\]](#) (`q`, `B`, `SK`, and `Y` in this document correspond to `n`, `G`, `d`, and `Q` in Section 3.2.1 of [\[SECG1\]](#)). In this ciphersuite, the secret scalar `x` is equal to the private key `SK`.

`*encode_to_curve_salt = PK_string`

*The `ECVRF_nonce_generation` function is as specified in [Section 5.4.2.1](#).

*The `int_to_string` function is the I2OSP function specified in Section 4.1 of [\[RFC8017\]](#). (This is big-endian representation.)

*The `string_to_int` function is the OS2IP function specified in Section 4.2 of [[RFC8017](#)]. (This is big-endian representation.)

*The `point_to_string` function converts a point on E to an octet string according to the encoding specified in Section 2.3.3 of [[SECG1](#)] with point compression on. This implies `ptlen = flen + 1 = 33`. (Note that certain software implementations do not introduce a separate elliptic curve point type and instead directly treat the EC point as an octet string per above encoding. When using such an implementation, the `point_to_string` function can be treated as the identity function.)

*The `string_to_point` function converts an octet string to an a point on E according to the encoding specified in Section 2.3.4 of [[SECG1](#)]. This function MUST output INVALID if the octet string does not decode to a point on the curve E.

*The hash function Hash is SHA-256 as specified in [[RFC6234](#)], with `hlen = 32`.

*The `ECVRF_encode_to_curve` function is as specified in [Section 5.4.1.1](#), with `interpret_hash_value_as_a_point(s) = string_to_point(0x02 || s)`.

This document defines ECVRF-P256-SHA256-SSWU as identical to ECVRF-P256-SHA256-TAI, except that:

*`suite_string = 0x02`.

*the `ECVRF_encode_to_curve` function is as specified in [Section 5.4.1.2](#) with `h2c_suite_ID_string = P256_XMD:SHA-256_SSWU_NU_` (the suite is defined in [[I-D.irtf-cfrg-hash-to-curve](#)] Section 8.2)

This document defines ECVRF-EDWARDS25519-SHA512-TAI as follows:

*`suite_string = 0x03`.

*The EC group G is the edwards25519 elliptic curve with parameters defined in Table 1 of [[RFC8032](#)]. For this group, `flen = qlen = 32` and `cofactor = 8`.

*`clen = 16`.

*The private key and generation of the secret scalar and the public key are specified in Section 5.1.5 of [[RFC8032](#)].

*`encode_to_curve_salt = PK_string`

*The `ECVRF_nonce_generation` function is as specified in [Section 5.4.2.2](#).

*The `int_to_string` function as specified in the first paragraph of Section 5.1.2 of [\[RFC8032\]](#). (This is little-endian representation.)

*The `string_to_int` function interprets the string as an integer in little-endian representation.

*The `point_to_string` function converts a point on E to an octet string according to the encoding specified in Section 5.1.2 of [\[RFC8032\]](#). This implies `ptLen = fLen = 32`. (Note that certain software implementations do not introduce a separate elliptic curve point type and instead directly treat the EC point as an octet string per above encoding. When using such an implementation, the `point_to_string` function can be treated as the identity function.)

*The `string_to_point` function converts an octet string to a point on E according to the encoding specified in Section 5.1.3 of [\[RFC8032\]](#). This function MUST output `INVALID` if the octet string does not decode to a point on the curve E.

*The hash function Hash is SHA-512 as specified in [\[RFC6234\]](#), with `hLen = 64`.

*The `ECVRF_encode_to_curve` function is as specified in [Section 5.4.1.1](#), with `interpret_hash_value_as_a_point(s) = string_to_point(s[0]...s[31])`.

This document defines ECVRF-EDWARDS25519-SHA512-ELL2 as identical to ECVRF-EDWARDS25519-SHA512-TAI, except:

*`suite_string = 0x04`.

*the `ECVRF_encode_to_curve` function is as specified in [Section 5.4.1.2](#) with `h2c_suite_ID_string = edwards25519_XMD:SHA-512_ELL2_NU_` (the suite is defined in [\[I-D.irtf-cfrg-hash-to-curve\]](#) Section 8.5).

6. Implementation Status

Note to RFC editor: Remove before publication

A reference C++ implementation of ECVRF-P256-SHA256-TAI, ECVRF-P256-SHA256-SSWU, ECVRF-EDWARDS25519-SHA512-TAI, and ECVRF-EDWARDS25519-SHA512-ELL2 is available at <https://github.com/reyzin/ecvrf>. This implementation is neither secure nor especially efficient, but can be used to generate test vectors.

A Python implementation of an older version of ECVRF-EDWARDS25519-SHA512-ELL2 from the -05 version of this draft is available at <https://github.com/integritychain/draft-irtf-cfrg-vrf-05>.

A C implementation of an older version of ECVRF-EDWARDS25519-SHA512-ELL2 from the -03 version of this draft is available at https://github.com/algorand/libsodium/tree/draft-irtf-cfrg-vrf-03/src/libsodium/crypto_vrf/ietf-draft03.

A Rust implementation of an older version of ECVRF-P256-SHA256-TAI from the -05 version of this draft, as well as variants for the sect163k1 and secp256k1 curves, is available at <https://crates.io/crates/vrf>.

A C implementation of a variant of ECVRF-P256-SHA256-TAI from the -05 version of this draft adapted for the secp256k1 curve is available at <https://github.com/aergoio/secp256k1-vrf>.

An implementation of an earlier version of RSA-FDH-VRF (SHA-256) and ECVRF-P256-SHA256-TAI was first developed as a part of the NSEC5 project [I-D.vcelak-nsec5] and is available at <http://github.com/fcelda/nsec5-crypto>.

The Key Transparency project at Google uses a VRF implementation that is similar to the ECVRF-P256-SHA256-TAI, with a few changes including the use of SHA-512 instead of SHA-256. Its implementation is available at <https://github.com/google/keytransparency/blob/master/core/crypto/vrf/>

An implementation by Ryuji Ishiguro following an older version of ECVRF-EDWARDS25519-SHA512-TAI from the -00 version of this draft is available at <https://github.com/r2ishiguro/vrf>.

An implementation similar to ECVRF-EDWARDS25519-SHA512-ELL2 (with some changes, including the use of SHA-3) is available as part of the CONIKS implementation in Golang at <https://github.com/coniks-sys/coniks-go/tree/master/crypto/vrf>.

Open Whisper Systems also uses a VRF similar to ECVRF-EDWARDS25519-SHA512-ELL2, called VXEdDSA, and specified here <https://whispersystems.org/docs/specifications/xeddsa/> and here <https://moderncrypto.org/mail-archive/curves/2017/000925.html>.

Implementations in C and Java are available at <https://github.com/signalapp/curve25519-java> and <https://github.com/wavesplatform/curve25519-java>.

7. Security Considerations

7.1. Key Generation

Applications that use the VRFs defined in this document MUST ensure that the VRF key is generated correctly, using good randomness.

7.1.1. Uniqueness and collision resistance with untrusted keys

The RSA-FDH-VRF satisfies the "trusted uniqueness" (see [Section 3.1](#)) and "trusted collision resistance" (see [Section 3.2](#)) properties as long as the VRF keys are generated correctly. Uniqueness and collision resistance may not hold if the keys are generated adversarially (specifically, if the RSA function specified in the public key is not bijective because the modulus n or the exponent e are chosen not in compliance with the standard); thus, RSA-FDH-VRF defined in this document does not have "full uniqueness" and "full collision resistance". Therefore, if adversarial key generation is a concern, the RSA-FDH-VRF has to be enhanced by additional cryptographic checks that its public key has the right form. These enhancements are left for future specification.

For the ECVRF, the Verifier MUST obtain E and B from a trusted source, such as a ciphersuite specification, rather than from the prover. If the verifier does so, then the ECVRF satisfies the "full uniqueness" (see [Section 3.1](#)) and "trusted collision resistance" (see [Section 3.2](#)) properties. It additionally satisfies "full collision resistance" if `validate_key` parameter given to the `ECVRF_verify` is `TRUE`.

7.1.2. Pseudorandomness with untrusted keys

Without good randomness, the "pseudorandomness" properties of the VRF may not hold. Note that it is not possible to guarantee pseudorandomness in the face of adversarially generated VRF keys. This is because an adversary can always use bad randomness to generate the VRF keys, and thus, the VRF output may not be pseudorandom.

7.2. Security Levels

As shown in [[PWHVNRG17](#)], RSA-FDH-VRF satisfies the trusted uniqueness property unconditionally. The security level of the RSA-FDH-VRF, measured in bits, for the other two properties is as follows (in the random oracle model for the functions MGF1 and Hash):

*For trusted collision resistance: approximately $8 \cdot \min(k/2, \text{hLen}/2)$ (as shown in [[PWHVNRG17](#)]).

*For selective pseudorandomness: approximately as strong as the security, in bits, of the RSA problem for the key (n, e) (as shown in [\[GNPRVZ15\]](#)).

As shown in [\[PWHVNRG17\]](#), the security level of the ECVRF, measured in bits, is as follows (in the random oracle model for the functions Hash and ECVRF_encode_to_curve):

*For trusted uniqueness: approximately $8 \cdot \min(qLen, cLen)$.

*For collision resistance (trusted or full, depending on whether validation is performed as explained in [Section 7.1.1](#)): approximately $8 \cdot \min(qLen/2, hLen/2)$.

*For the selective pseudorandomness property: approximately as strong as the security, in bits, of the decisional Diffie-Hellman problem in the group G (which is at most $8 \cdot qLen/2$).

See [Section 3](#) for the definitions of these security properties. See [Section 7.3](#) for the discussion of full pseudorandomness.

7.3. Selective vs. Full Pseudorandomness

[\[PWHVNRG17\]](#) presents cryptographic reductions to an underlying hard problem (namely, the RSA problem for RSA-FDH-VRF and the Decisional Diffie-Hellman problem for the ECVRF) to prove that the VRFs specified in this document possess not only selective pseudorandomness, but also full pseudorandomness (see [Section 3.3](#) for an explanation of these notions). However, the cryptographic reductions are tighter for selective pseudorandomness than for full pseudorandomness. Specifically, the approximate provable security level, measured in bits, for full pseudorandomness may be obtained from the provable security level for selective pseudorandomness (given in [Section 7.2](#)) by subtracting the binary logarithm of the number of proofs produced for a given secret key. This holds for both the RSA-FDH-VRF and the ECVRF.

While no known attacks against full pseudorandomness are stronger than similar attacks against selective pseudorandomness, some applications may be concerned about tightness of cryptographic reductions. Such applications may consider the following two options:

*They may choose to ensure that selective pseudorandomness is sufficient for the application. That is, that pseudorandomness of outputs matters only for inputs that are chosen independently of the VRF key.

*They may increase security parameters to make up for the loose security reduction. For RSA-FDH-VRF, this means increasing the

RSA key length. For ECVRF, this means increasing the cryptographic strength of the EC group G by specifying a new ciphersuite.

7.4. Proper pseudorandom nonce for ECVRF

The security of the ECVRF defined in this document relies on the fact that the nonce k used in the ECVRF_prove algorithm is chosen uniformly and pseudorandomly modulo q , and is unknown to the adversary. Otherwise, an adversary may be able to recover the private VRF key x (and thus break pseudorandomness of the VRF) after observing several valid VRF proofs π_i . The nonce generation methods specified in the ECVRF ciphersuites of [Section 5.5](#) are designed with this requirement in mind.

7.5. Side-channel attacks

Side channel attacks on cryptographic primitives are an important issue. Implementers should take care to avoid side-channel attacks that leak information about the VRF private key SK (and the nonce k used in the ECVRF), which is used in VRF_prove. In most applications, VRF_proof_to_hash and VRF_verify algorithms take only inputs that are public, and thus side channel attacks are typically not a concern for these algorithms.

The VRF input α may be also a sensitive input to VRF_prove and may need to be protected against side channel attacks. Below we discuss one particular class of such attacks: timing attacks that can be used to leak information about the VRF input α .

The ECVRF_encode_to_curve_try_and_increment algorithm defined in [Section 5.4.1.1](#) SHOULD NOT be used in applications where the VRF input α is secret and is hashed by the VRF on-the-fly. This is because the algorithm's running time depends on the VRF input α , and thus creates a timing channel that can be used to learn information about α . That said, for most inputs the amount of information obtained from such a timing attack is likely to be small (1 bit, on average), since the algorithm is expected to find a valid curve point after only two attempts. However, there might be inputs which cause the algorithm to make many attempts before it finds a valid curve point; for such inputs, the information leaked in a timing attack will be more than 1 bit.

ECVRF-P256-SHA256-SSWU and ECVRF-EDWARDS25519-SHA512-ELL2 can be made to run in time independent of α , following recommendations in [\[I-D.irtf-cfrg-hash-to-curve\]](#).

7.6. Proofs provide no secrecy for the VRF input

The VRF proof π is not designed to provide secrecy and, in general, may reveal the VRF input α . Anyone who knows PK and π is able to perform an offline dictionary attack to search for α , by verifying guesses for α using `VRF_verify`. This is in contrast to the VRF hash output β which, without the proof, is pseudorandom and thus is designed to reveal no information about α .

7.7. Prehashing

The VRFs specified in this document allow for read-once access to the input α for both signing and verifying. Thus, additional prehashing of α (as specified, for example, in [\[RFC8032\]](#) for EdDSA signatures) is not needed, even for applications that need to handle long α or to support the Initialize-Update-Finalize (IUF) interface (in such an interface, α is not supplied all at once, but rather in pieces by a sequence of calls to `Update`). The ECVRF, in particular, uses α only in `ECVRF_encode_to_curve`. The curve point H becomes the representative of α thereafter.

7.8. Hash function domain separation

Hashing is used for different purposes in the two VRFs (namely, in the RSA-FDH-VRF, in MGF1 and in `proof_to_hash`; in the ECVRF, in `encode_to_curve`, `nonce_generation`, `challenge_generation`, and `proof_to_hash`). The theoretical analysis treats each of these functions as a separate hash function, modeled as a random oracle. This analysis still holds even if the same hash function is used, as long as the four queries made to the hash function for a given SK and α are overwhelmingly unlikely to equal each other or to any queries made to the hash function for the same SK and different α . This is indeed the case for the RSA-FDH-VRF defined in this document, because the second octets of the input to the hash function used in MGF1 and in `proof_to_hash` are different.

This is also the case for the ECVRF ciphersuites defined in this document, because:

- *inputs to the hash function used during `nonce_generation` are unlikely to equal inputs used in `encode_to_curve`, `proof_to_hash`, and `challenge_generation`. This follows since `nonce_generation` inputs a secret to the hash function that is not used by honest parties as input to any other hash function, and is not available to the adversary.

- *the second octets of the inputs to the hash function used in `proof_to_hash`, `challenge_generation`, and `encode_to_curve_try_and_increment` are all different.

*the last octet of the input to the hash function used in proof_to_hash, challenge_generation, and encode_to_curve_try_and_increment is always zero, and therefore different from the last octet of the input to the hash function used in ECVRF_encode_to_curve_h2c_suite, which is set equal to the nonzero length of the domain separation tag by [[I-D.irtf-cfrg-hash-to-curve](#)].

7.9. Hash function salting

In case a hash collision is found, in order to make it more difficult for the adversary to exploit such a collision, the MGF1 function for the RSA-FDH-VRF and ECVRF_encode_to_curve function for the ECVRF use a public value in addition to alpha (as a so-called salt). This value is determined by the ciphersuite. For the ciphersuites defined in this document, it is set equal to the string representation of the RSA modulus and EC public key, respectively. Implementations that do not use one of the ciphersuites (see [Section 7.10](#)) MAY use a different salt. For example, if a group of public keys to share the same salt, then the hash of the VRF input alpha will be the same for the entire group of public keys, which may aid in some protocol that uses the VRF.

7.10. Futureproofing

if future designs need to specify variants (e.g., additional ciphersuites) of the RSA-FDH-VRF or the ECVRF in this document, then, to avoid the possibility that an adversary can obtain a VRF output under one variant, and then claim it was obtained under another variant, they should specify a different suite_string constant. The suite_string constants in this document are all single octets; if a future suite_string constant is longer than one octet, then it should start with a different octet than the suite_string constants in this document. Then, for the RSA-FDH-VRF, the inputs to the hash function used in MGF1 and proof_to_hash will be different from other ciphersuites. For the ECVRF, the inputs ECVRF_encode_to_curve hash function used in producing H are then guaranteed to be different from other ciphersuites; since all the other hashing done by the prover depends on H, inputs to all the hash functions used by the prover will also be different from other ciphersuites as long as ECVRF_encode_to_curve is collision resistant.

8. Change Log

Note to RFC Editor: if this document does not obsolete an existing RFC, please remove this appendix before publication as an RFC.

00 - Forked this document from draft-goldbe-vrf-01.

01 - Minor updates, mostly highlighting TODO items.

02 - Added specification of Elligator2 for Curve25519, along with ciphersuites for ECVRF-ED25519-SHA512-Elligator. Changed ECVRF-ED25519-SHA256 suite_string to ECVRF-ED25519-SHA512. (This change made because Ed25519 in [\[RFC8032\]](#) signatures use SHA512 and not SHA256.) Made ECVRF nonce generation a separate component, so that nonces are deterministic. In ECVRF proving, changed + to - (and made corresponding verification changes) in order to be consistent with EdDSA and ECDSA. Highlighted that ECVRF_hash_to_curve acts like a prehash. Added "suites" variable to ECVRF for futureproofing. Ensured domain separation for hash functions by modifying hash_points and added discussion about domain separation. Updated todos in the "additional pseudorandomness property" section. Added a discussion of secrecy into security considerations. Removed B and PK=Y from ECVRF_hash_points because they are already present via H, which is computed via hash_to_curve using the suite_string (which identifies B) and Y.

03 - Changed Ed25519 conversions to little-endian, to match RFC 8032; added simple key validation for Ed25519; added Simple SWU cipher suite; clarified Elligator and removed the extra x0 bit, to make Montgomery and Edwards Elligator the same; added domain separation for RSA VRF; improved notation throughout; added nonce generation as a section; changed counter in try-and-increment from four bytes to one, to avoid endian issues; renamed try-and-increment ciphersuites to -TAI; added qLen as a separate parameter; changed output length to hLen for ECVRF, to match RSAVRF; made Verify return beta so unverified proofs don't end up in proof_to_hash; added test vectors.

04 - Clarified handling of optional arguments x and PK in ECVRF_prove. Edited implementation status to bring it up to date.

05 - Renamed ed25519 into the more commonly used edwards25519. Corrected ECVRF_nonce_generation_RFC6979 (thanks to Gorka Irazoqui Apecechea and Mario Cao Cueto for finding the problem) and corresponding test vectors for the P256 suites. Added a reference to the Rust implementation.

06 - Made some variable names more descriptive. Added a few implementation references.

07 - Incorporated hash-to-curve draft by reference to replace our own Elligator2 and Simple SWU. Clarified discussion of EC parameters and functions. Added a 0 octet to all hashing to enforce domain separation from hashing done inside hash-to-curve.

08 - Incorporated suggestions from crypto panel review by Chloe Martindale. Changed Reyzin's affiliation. Updated references.

09 - Added a note to remove the implementation page before publication.

10 - Added a check in ECVRF_decode_proof to ensure that s is reduced mod q . Connected security properties (Section 3) and security considerations (Section 7) with more cross-references.

11 - Processed last call comments. Clarified various notation, including lengths of various parameters for ECVRF; added error handling to RSA-FDH-VRF; added security levels section; clarified full vs trusted uniqueness and full vs selective pseudorandomness; added RSA ciphersuites; made key validation clearer; renamed hash_to_curve to encode_to_curve to be consistent with the hash_to_curve draft; allowed a more general salt in hashing, added the public key as input to ECVRF_challenge_generation, and added an explanation about the salt.

12 - Added k_string to edwards25519 test vectors

9. Contributors

This document also would not be possible without the work of Moni Naor, Sachin Vasant, and Asaf Ziv. Chloe Martindale provided a thorough cryptographer's review. Liliya Akhmetzyanova, Tony Arcieri, Gary Belvin, Mario Cao Cueto, Brian Chen, Sergey Gorbunov, Shumon Huque, Gorka Irazoqui Apecechea, Marek Jankowski, Burt Kaliski, David C. Lawrence, Derek Ting-Haye Leung, Antonio Marcedone, Piotr Nojszewski, Chris Peikert, Trevor Perrin, Sam Scott, Stanislav Smyshlyaev, Adam Suhl, Nick Sullivan, Christopher Wood, Jiayu Xu, and Annie Yousar provided valuable input to this draft. Riad Wahby helped this document align with draft-irtf-cfrg-hash-to-curve.

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Appendix A. Test Vectors for the ECVRFs

The test vectors in this section were generated using the reference implementation at <https://github.com/reyzin/ecvrf>.

A.1. ECVRF-P256-SHA256-TAI

The example secret keys and messages in Examples 1 and 2 are taken from Appendix A.2.5 of [RFC6979].

Example 1:

```
SK = x =
c9afa9d845ba75166b5c215767b1d6934e50c3db36e89b127b8a622b120f6721
PK =
0360fed4ba255a9d31c961eb74c6356d68c049b8923b61fa6ce669622e60f29fb6

alpha = 73616d706c65 (ASCII "sample")
try_and_increment succeeded on ctr = 1
```

H =
0272a877532e9ac193aff4401234266f59900a4a9e3fc3cfc6a4b7e467a15d06d4

k =
0d90591273453d2dc67312d39914e3a93e194ab47a58cd598886897076986f77

U = k*B =
02bb6a034f67643c6183c10f8b41dc4babf88bff154b674e377d90bde009c21672

V = k*H =
02893ebee7af9a0faa6da810da8a91f9d50e1dc071240c9706726820ff919e8394

pi =
035b5c726e8c0e2c488a107c600578ee75cb702343c153cb1eb8dec77f4b5071b4a53f0a46f018bc2c56e5

beta =
a3ad7b0ef73d8fc6655053ea22f9bede8c743f08bbbed3d38821f0e16474b505e

Example 2:

SK = x =
c9afa9d845ba75166b5c215767b1d6934e50c3db36e89b127b8a622b120f6721
PK =
0360fed4ba255a9d31c961eb74c6356d68c049b8923b61fa6ce669622e60f29fb6

alpha = 74657374 (ASCII "test")
try_and_increment succeeded on ctr = 3
H =
02173119b4fff5e6f8afed4868a29fe8920f1b54c2cf89cc7b301d0d473de6b974

k =
5852353a868bdce26938cde1826723e58bf8cb06dd2fed475213ea6f3b12e961
U = k*B =
022779a2cafc6b65414c4a04a4b4d2adf4c50395f57995e89e6de823250d91bc48e

V = k*H =
033b4a14731672e82339f03b45ff6b5b13dee7ada38c9bf1d6f8f61e2ce5921119

pi =
034dac60aba508ba0c01aa9be80377ebd7562c4a52d74722e0abae7dc3080ddb56c19e067b15a8a8174905

beta =
a284f94ceec2ff4b3794629da7cbafa49121972671b466cab4ce170aa365f26d

The example secret key in Example 3 is taken from Appendix L.4.2 of
[[ANSI.X9-62-2005](#)].

Example 3:

```
SK = x =
2ca1411a41b17b24cc8c3b089cfd033f1920202a6c0de8abb97df1498d50d2c8
PK =
03596375e6ce57e0f20294fc46bdfcfd19a39f8161b58695b3ec5b3d16427c274d

alpha =
4578616d706c65207573696e67204543445341206b65792066726f6d20417070656e646978204c2e342e32
(ASCII "Example using ECDSA key from Appendix L.4.2 of
ANSI.X9-62-2005")
try_and_increment succeeded on ctr = 1
H =
0258055c26c4b01d01c00fb57567955f7d39cd6f6e85fd37c58f696cc6b7aa761d

k =
5689e2e08e1110b4dda293ac21667eac6db5de4a46a519c73d533f69be2f4da3
U = k*B =
020f465cd0ec74d2e23af0abde4c07e866ae4e5138bde5dd1196b8843f380db84

V = k*H =
036cb6f811428fc4904370b86c488f60c280fa5b496d2f34ff8772f60ed24b2d1d

pi =
03d03398bf53aa23831d7d1b2937e005fb0062cbefa06796579f2a1fc7e7b8c667d091c00b0f5c3619d10e

beta =
90871e06da5caa39a3c61578ebb844de8635e27ac0b13e829997d0d95dd98c19
```

A.2. ECVRF-P256-SHA256-SSWU

The example secret keys and messages in Examples 4 and 5 are taken from Appendix A.2.5 of [[RFC6979](#)].

Example 4:

```
SK = x =
c9afa9d845ba75166b5c215767b1d6934e50c3db36e89b127b8a622b120f6721
PK =
0360fed4ba255a9d31c961eb74c6356d68c049b8923b61fa6ce669622e60f29fb6

alpha = 73616d706c65 (ASCII "sample")
In SSWU: uniform_bytes =
5024e98d6067dec313af09ff0cbe78218324a645c2a4b0aae2453f6fe91aa3bd9471f7b4a5fbf128e4b53f

In SSWU: u =
df565615a2372e8b31b8771f7503bafc144e48b05688b97958cc27ce29a8d810
In SSWU: x1 =
e7e39eb8a4c982426fcff629e55a3e13516cfeb62c02c369b1e750316f5e94eb
```

In SSWU: gx_1 is a nonsquare

$H =$

02b31973e872d4a097e2cfae9f37af9f9d73428fde74ac537dda93b5f18dbc5842

$k =$

e92820035a0a8afe132826c6312662b6ea733fc1a0d33737945016de54d02dd8

$U = k*B =$

031490f49d0355ffcdf66e40df788bee93861917ee713acff79be40d20cc91a30a

$V = k*H =$

03701df0228138fa3d16612c0d720389326b3265151bc7ac696ea4d0591cd053e3

$\pi =$

0331d984ca8fece9cbb9a144c0d53df3c4c7a33080c1e02ddb1a96a365394c7888782ffde7b842c38c20c

$\beta =$

21e66dc9747430f17ed9efeda054cf4a264b097b9e8956a1787526ed00dc664b

Example 5:

$SK = x =$

c9afa9d845ba75166b5c215767b1d6934e50c3db36e89b127b8a622b120f6721

$PK =$

0360fed4ba255a9d31c961eb74c6356d68c049b8923b61fa6ce669622e60f29fb6

$\alpha = 74657374$ (ASCII "test")

In SSWU: $\text{uniform_bytes} =$

910cc66d84a57985a1d15843dad83fd9138a109afb243b7fa5d64d766ec9ca3894fdcf46eb21a3972eb4

In SSWU: $u =$

d8b0107f7e7aa36390240d834852f8703a6dc407019d6196bda5861b8fc00181

In SSWU: $x_1 =$

ccc747fa7318b9486ce4044adbbecaa084c27be6eda88eb7b7f3d688fd0968c7

In SSWU: gx_1 is a square

$H =$

03ccc747fa7318b9486ce4044adbbecaa084c27be6eda88eb7b7f3d688fd0968c7

$k =$

febc3451ea7639fde2cf41ffd03f463124ecb3b5a79913db1ed069147c8a7dea

$U = k*B =$

031200f9900e96f811d1247d353573f47e0d9da601fc992566234fc1a5b37749ae

$V = k*H =$

02d3715dcfee136c7ae50e95ffca76f4ca6c29ddfb92a39c31a0d48e75c6605cd1

$\pi =$

03f814c0455d32dbc75ad3aea08c7e2db31748e12802db23640203aebf1fa8db2743aad348a3006dc1caad

beta =
8e7185d2b420e4f4681f44ce313a26d05613323837da09a69f00491a83ad25dd

The example secret key in Example 6 is taken from Appendix L.4.2 of [\[ANSI.X9-62-2005\]](#).

Example 6:

SK = x =
2ca1411a41b17b24cc8c3b089cfd033f1920202a6c0de8abb97df1498d50d2c8
PK =
03596375e6ce57e0f20294fc46bdfcfd19a39f8161b58695b3ec5b3d16427c274d

alpha =
4578616d706c65207573696e67204543445341206b65792066726f6d20417070656e646978204c2e342e32
(ASCII "Example using ECDSA key from Appendix L.4.2 of
ANSI.X9-62-2005")
In SSWU: uniform_bytes =
9b81d55a242d3e8438d3bcfb1bee985a87fd144802c9268cf9adeeee160e6e9ff765569797a0f701cb43160

In SSWU: u =
e43c98c2ae06d13839fedb0303e5ee815896beda39be83fb11325b97976efdce
In SSWU: x1 =
be9e195a50f175d3563aed8dc2d9f513a5536c1e9aee1757d86c08d32d582a86
In SSWU: gx1 is a nonsquare
H =
022dd5150e5a2a24c66feab2f68532be1486e28e07f1b9a055cf38ccc16f6595ff

k =
8e29221f33564f3f66f858ba2b0c14766e1057adbd422c3e7d0d99d5e142b613
U = k*B =
03a8823ff9fd16bf879261c740b9c7792b77fee0830f21314117e441784667958d

V = k*H =
02d48fbb45921c755b73b25be2f23379e3ce69294f6cee9279815f57f4b422659d

pi =
039f8d9cdc162c89be2871cbcb1435144739431db7fab437ab7bc4e2651a9e99d5488405a11a6c7fc8defd

beta =
4fbadf33b42a5f42f23a6f89952d2e634a6e3810f15878b46ef1bb85a04fe95a

A.3. ECVRF-EDWARDS25519-SHA512-TAI

The example secret keys and messages in Examples 7, 8, and 9 are taken from Section 7.1 of [\[RFC8032\]](#).

Example 7:

```
SK =
9d61b19def fd5a60ba844af492ec2cc44449c5697b326919703bac031cae7f60
PK =
d75a980182b10ab7d54bfed3c964073a0ee172f3daa62325af021a68f707511a
alpha = (the empty string)
x =
307c83864f2833cb427a2ef1c00a013cfdff2768d980c0a3a520f006904de94f
try_and_increment succeeded on ctr = 0
H =
91bbbed02a99461df1ad4c6564a5f5d829d0b90cfc7903e7a5797bd658abf3318
k_string =
7100f3d9eadb6dc4743b029736ff283f5be494128df128df2817106f345b8594b6d6da2d6fb0b4c0257eb30

k =
8a49edbd1492a8ee09766befe50a7d563051bf3406cbfffc20a88def030730f0f
U = k*B =
aef27c725be964c6a9bf4c45ca8e35df258c1878b838f37d9975523f09034071
V = k*H =
5016572f71466c646c119443455d6cb9b952f07d060ec8286d678615d55f954f
pi =
8657106690b5526245a92b003bb079ccd1a92130477671f6fc01ad16f26f723f26f8a57ccaed74ee1b190b0

beta =
90cf1df3b703cce59e2a35b925d411164068269d7b2d29f3301c03dd757876ff66b71dda49d2de59d03450a
```

Example 8:

```
SK =
4ccd089b28ff96da9db6c346ec114e0f5b8a319f35aba624da8cf6ed4fb8a6fb
PK =
3d4017c3e843895a92b70aa74d1b7ebc9c982ccf2ec4968cc0cd55f12af4660c
alpha = 72 (1 byte)
x =
68bd9ed75882d52815a97585caf4790a7f6c6b3b7f821c5e259a24b02e502e51
try_and_increment succeeded on ctr = 1
H =
5b659fc3d4e9263fd9a4ed1d022d75eaacc20df5e09f9ea937502396598dc551
k_string =
42589bbf0c485c3c91c1621bb4bfe04aed7be76ee48f9b00793b2342acb9c167cab856f9f9d4febc3113300

k =
d8c3a66921444cb3427d5d989f9b315aa8ca3375e9ec4d52207711a1fdb44107
U = k*B =
1dcb0a4821a2c48bf53548228b7f170962988f6d12f5439f31987ef41f034ab3
V = k*H =
fd03c0bf498c752161bae4719105a074630a2aa5f200ff7b3995f7bfb1513423
```



```
pi =
f3141cd382dc42909d19ec5110469e4feae18300e94f304590abdc48aed5933bf0864a62558b3ed7f2fe

beta =
eb4440665d3891d668e7e0fc587f1b4bd7fbfe99d0eb2211ccec90496310eb5e33821bc613efb94db5e5
```

Example 9:

```
SK =
c5aa8df43f9f837bedb7442f31dcb7b166d38535076f094b85ce3a2e0b4458f7
PK =
fc51cd8e6218a1a38da47ed00230f0580816ed13ba3303ac5deb911548908025
alpha = af82 (2 bytes)
x =
909a8b755ed902849023a55b15c23d11ba4d7f4ec5c2f51b1325a181991ea95c
try_and_increment succeeded on ctr = 0
H =
bf4339376f5542811de615e3313d2b36f6f53c0acfebb482159711201192576a
k_string =
38b868c335ccda94a088428cbf3ec8bc7955bfafe1f3bd2aa2c59fc31a0febc59d0e1af3715773ce11b3b

k =
5ffdbc72135d936014e8ab708585fda379405542b07e3bd2c0bd48437fbac60a
U = k*B =
2bae73e15a64042fceb062abe7e432b2eca6744f3e8265bc38e009cd577ecd5
V = k*H =
88cba1cb0d4f9b649d9a86026b69de076724a93a65c349c988954f0961c5d506
pi =
9bc0f79119cc5604bf02d23b4caede71393cedfbb191434dd016d30177ccbf8096bb474e53895c362d8628

beta =
645427e5d00c62a23fb703732fa5d892940935942101e456ecca7bb217c61c452118fec1219202a0edcf03
```

A.4. ECVRF-EDWARDS25519-SHA512-ELL2

The example secret keys and messages in Examples 10, 11, and 12 are taken from Section 7.1 of [[RFC8032](#)].

Example 10:

```
SK =
9d61b19deffd5a60ba844af492ec2cc44449c5697b326919703bac031cae7f60
PK =
d75a980182b10ab7d54bfed3c964073a0ee172f3daa62325af021a68f707511a
alpha = (the empty string)
x =
307c83864f2833cb427a2ef1c00a013cfdff2768d980c0a3a520f006904de94f
```

```

In Elligator2: uniform_bytes =
d620782a206d9de584b74e23ae5ee1db5ca5298b3fc527c4867f049dee6dd419b3674967bd614890f621c12

In Elligator2: u =
30f037b9745a57a9a2b8a68da81f397c39d46dee9d047f86c427c53f8b29a55c
In Elligator2: gx1 =
8cb66318fb2cea01672d6c27a5ab662ae33220961607f69276080a56477b4a08
In Elligator2: gx1 is a square
H =
b8066ebbb706c72b64390324e4a3276f129569eab100c26b9f05011200c1bad9
k_string =
b5682049fee54fe2d519c9afff73bbfad724e69a82d5051496a42458f817bed7a386f96b1a78e5736756192

k =
55cbb247af9b8372259a97b2cfec656d78868deb33b203d51b9961c364522400
U = k*B =
762f5c178b68f0cddcc1157918edf45ec334ac8e8286601a3256c3bbf858edd9
V = k*H =
4652eba1c4612e6fce762977a59420b451e12964adbe4fbecd58a7aef5860af
pi =
7d9c633ffeee27349264cf5c667579fc583b4bda63ab71d001f89c10003ab46f14adf9a3cd8b8412d903853

beta =
9d574bf9b8302ec0fc1e21c3ec5368269527b87b462ce36dab2d14ccf80c53cccf6758f058c5b1c856b1163

```

Example 11:

```

SK =
4ccd089b28ff96da9db6c346ec114e0f5b8a319f35aba624da8cf6ed4fb8a6fb
PK =
3d4017c3e843895a92b70aa74d1b7ebc9c982ccf2ec4968cc0cd55f12af4660c
alpha = 72 (1 byte)
x =
68bd9ed75882d52815a97585caf4790a7f6c6b3b7f821c5e259a24b02e502e51
In Elligator2: uniform_bytes =
04ae20a9ad2a2330fb33318e376a2448bd77bb99e81d126f47952b156590444a9225b84128b66a2f15b4129

In Elligator2: u =
3092f033b16d4d5f74a3f7dc7091fe434b449065152b95476f121de899bb773d
In Elligator2: gx1 =
25d7fe7f82456e7078e99fdb24ef2582b4608357cdba9c39a8d535a3fd98464d
In Elligator2: gx1 is a nonsquare
H =
76ac3ccb86158a9104dff819b1ca293426d305fd76b39b13c9356d9b58c08e57
k_string =
88bf479281fd29a6cbdf67e2c5ec0024d92f14eaed58f43f22f37c4c37f1d41e65c036fbf01f9fba11d53

```

```
k =
9565956daeedf376cad61b829b2a4d21ba1b52e9b3e2457477a64630a9711003
U = k*B =
8ec26e77b8cb3114dd2265fe1564a4efb40d109aa3312536d93dfe3d8d80a061
V = k*H =
fe799eb5770b4e3a5a27d22518bb631db183c8316bb552155f442c62a47d1c8b
pi =
47b327393ff2dd81336f8a2ef10339112401253b3c714eeda879f12c509072ef055b48372bb82efbdce8e10

beta =
38561d6b77b71d30eb97a062168ae12b667ce5c28cacddf76bc88e093e4635987cd96814ce55b4689b3dd20
```

Example 12:

```
SK =
c5aa8df43f9f837bedb7442f31dcb7b166d38535076f094b85ce3a2e0b4458f7
PK =
fc51cd8e6218a1a38da47ed00230f0580816ed13ba3303ac5deb911548908025
alpha = af82 (2 bytes)
x =
909a8b755ed902849023a55b15c23d11ba4d7f4ec5c2f51b1325a181991ea95c
In Elligator2: uniform_bytes =
be0aed556e36cdfddf8f1eeddbb7356a24fad64cf95a922a098038f215588b216beabbfe6acf20256188e80

In Elligator2: u =
f6675dc6d17fc790d4b3f1c6acf689a13d8b5815f23880092a925af94cd6fa24
In Elligator2: gx1 =
a63d48e3247c903e22fdfb88fd9295e396712a5fe576af335dbe16f99f0af26c
In Elligator2: gx1 is a square
H =
13d2a8b5ca32db7e98094a61f656a08c6c964344e058879a386a947a4e189ed1
k_string =
a7ddd74a3a7d165d511b02fa268710ddbb3b939282d276fa2efcfa5aaf79cf576087299ca9234aacd7cd670

k =
1fda4077f737098b3f361c33a36cccafd7e9e9b720e1f84011254e25f37eed02
U = k*B =
a012f35433df219a88ab0f9481f4e0065d00422c3285f3d34a8b0202f20bac60
V = k*H =
fb613986d171b3e98319c7ca4dc44c5dd8314a6e5616c1a4f16ce72bd7a0c25a
pi =
926e895d308f5e328e7aa159c06eddbbe56d06846abf5d98c2512235eaa57fdce35b46edfc655bc828d44ad0

beta =
121b7f9b9aaaa29099fc04a94ba52784d44eac976dd1a3cca458733be5cd090a7b5fbd148444f17f8daf1f0
```

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