Daala Transforms

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Transform Design Goals

- Exact integer implementation
  - Lots of iterated prediction with unstable (gain=1.0) filters, no drift acceptable
- Many variations
  - Low bit-depth, high bit-depth, rectangular, DCT, DST, etc.
- High accuracy
  - We don’t need to compromise quality for complexity
- Low software complexity
  - In particular implementation in SIMD
- Reasonable hardware complexity
  - Low latency for small sizes
  - Transform re-use/embedded designs

H.264 4-point DCT

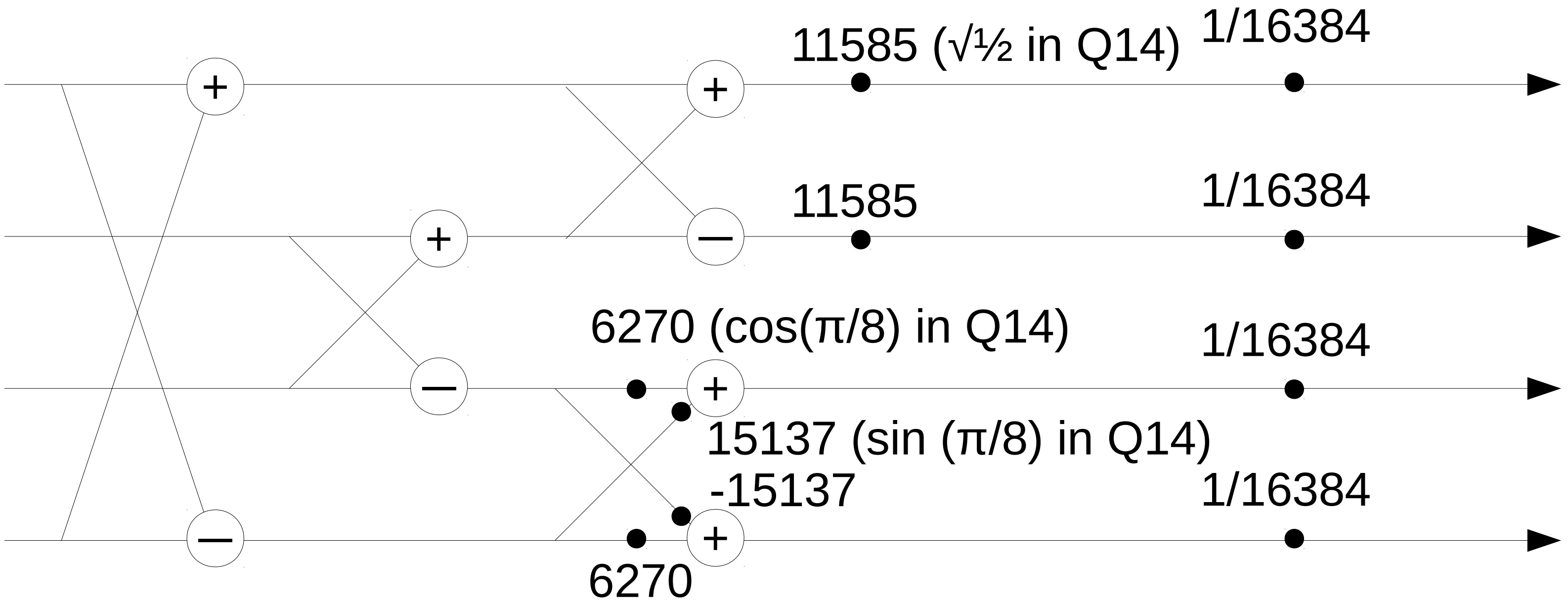
- Very low complexity (8 adds, 2 shifts):

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & -1 & -2 \\ 1 & -1 & -1 & 1 \\ 1 & -2 & 2 & -1 \end{bmatrix}$$

- Drawback: non-uniform scale
  - Saves one multiply/coeff. by combining it with quantization
  - But costs several multiplies/coeff. when rate-distortion optimizing coefficients in an encoder
    - Need uniform scale for distortion to make good trade-offs
    - Encoder costs multiplied by search space
  - Costs a large table of constants (very large for large transform sizes)
- New goal: uniform scaling (4 multiplies)
  - Achievable with much less than 1 multiply/coeff for large sizes

VP9 4-point DCT

- 6 multiplies (full 32-bit products needed), 8 adds (2 at 32 bits), 4 shifts



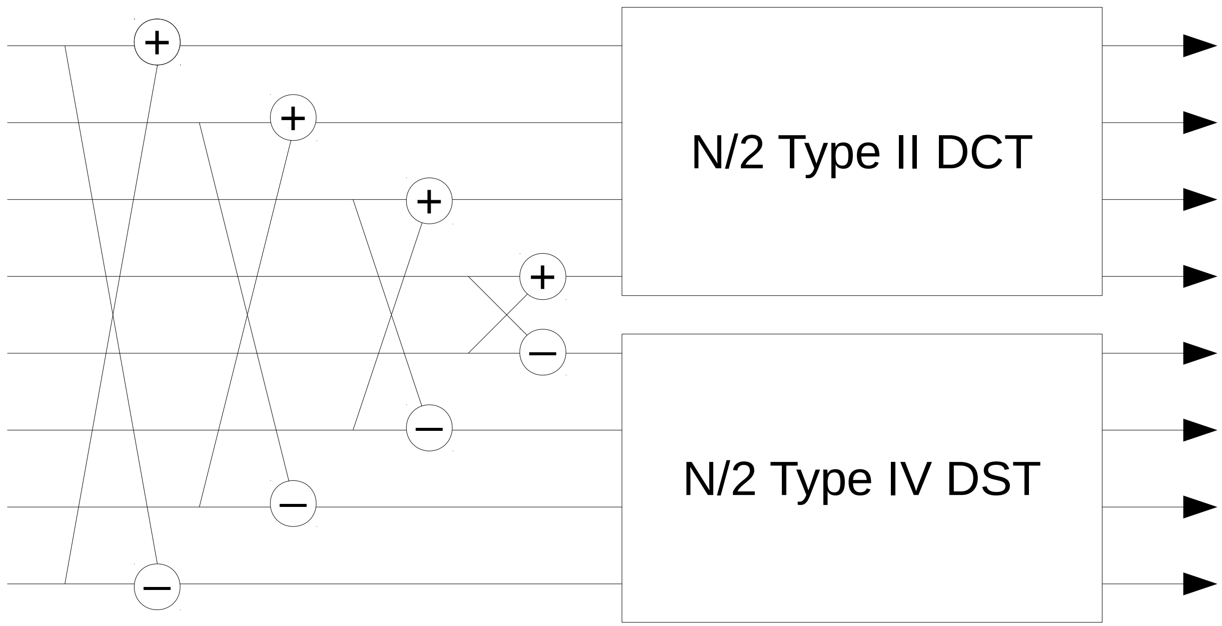
Avenues for Improvement

- Simplify the multiplies
  - Just scaling the output of the H.264 transform only costs 4 multiplies (but less accurate)
- Scaling
  - Adds a factor of √2 relative to a unitary transform
  - VP9 adds an additional √2 each time the size doubles
  - When $\log_2(\text{width}) + \log_2(\text{height})$ is even, correct with a shift
  - But it's odd for rectangular transforms (e.g., 8x4)
  - Costs 1 multiply/coeff. to correct for

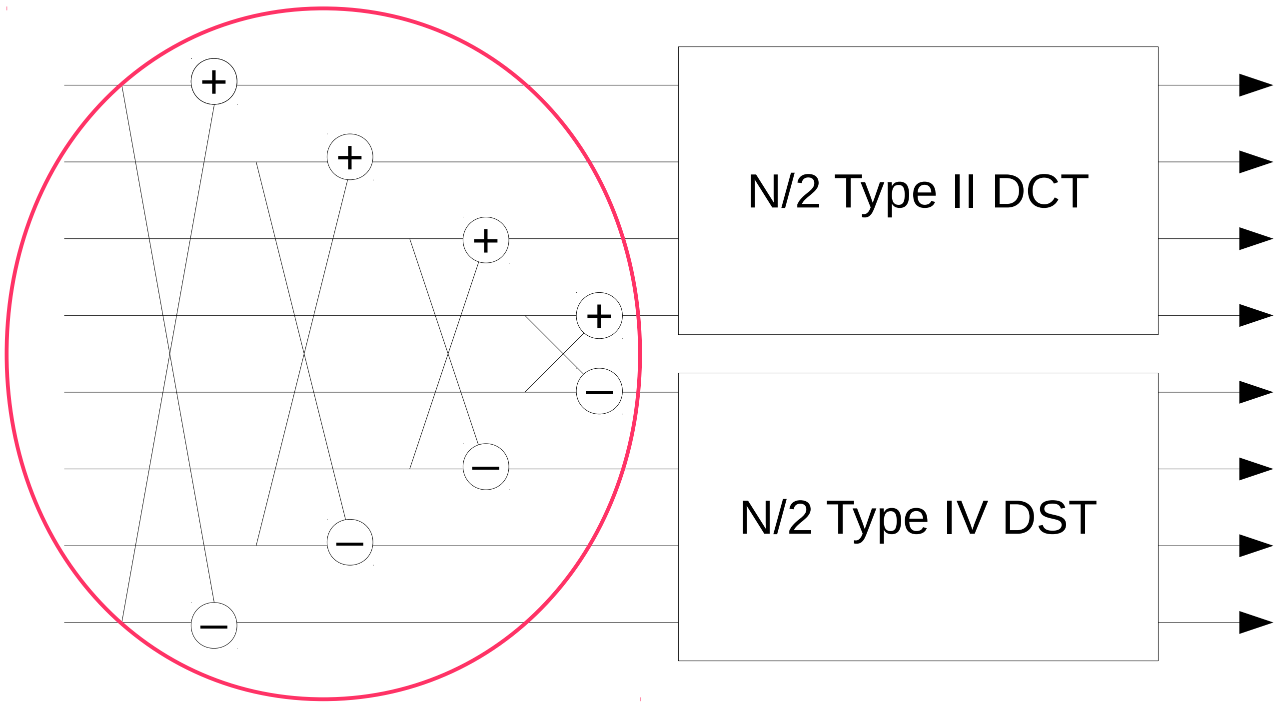
Extra Scaling

- Where does this scaling come from structurally?

N-point Type II DCT



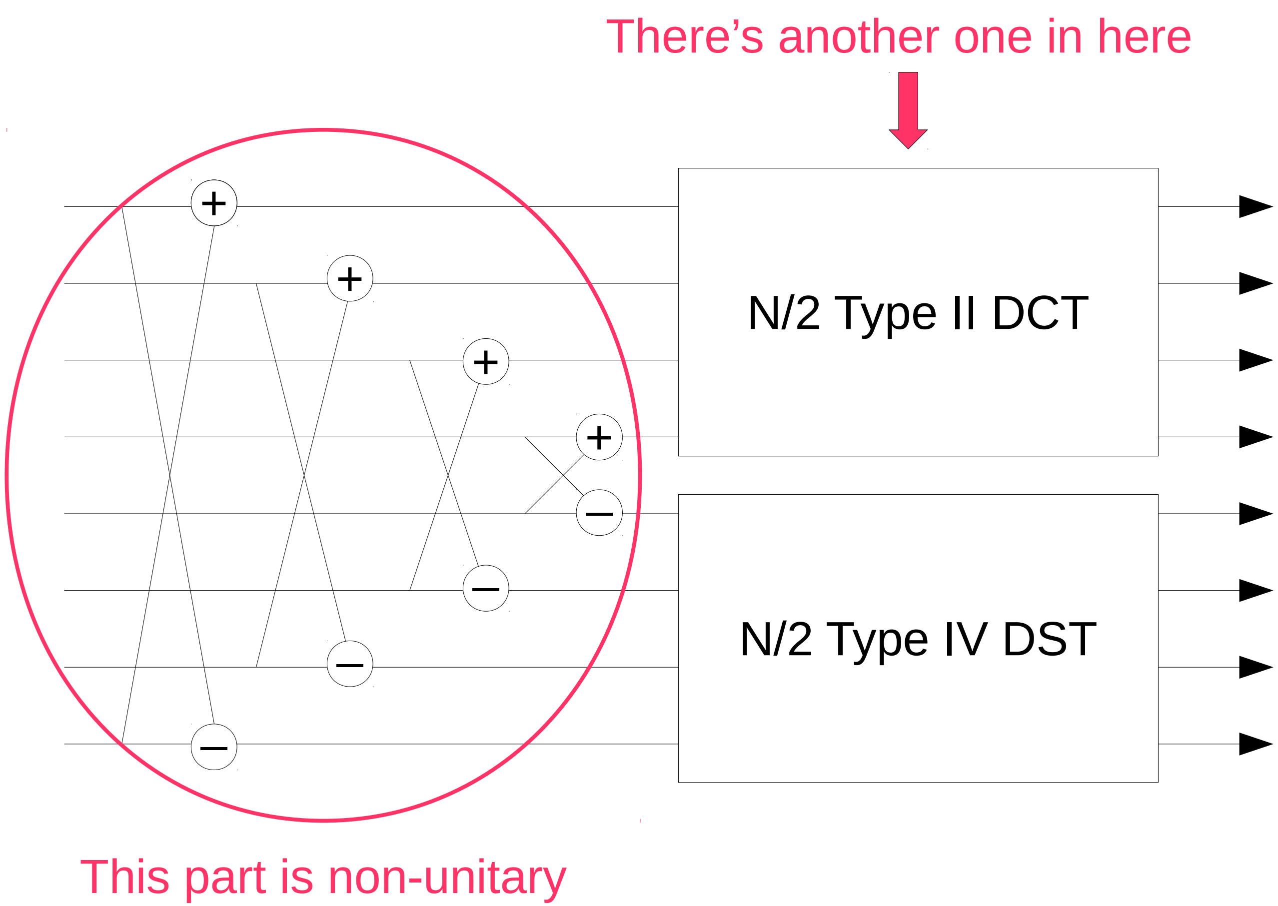
N-point Type II DCT



This part is non-unitary

sqrt(1^2 + 1^2) = √2

N-point Type II DCT

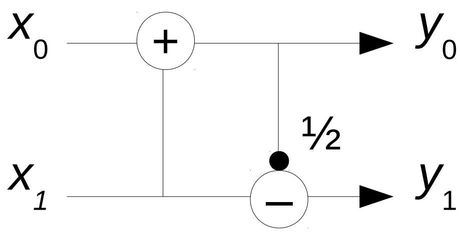


sqrt($1^2 + 1^2$) = √2

Getting Rid of the Extra Scaling

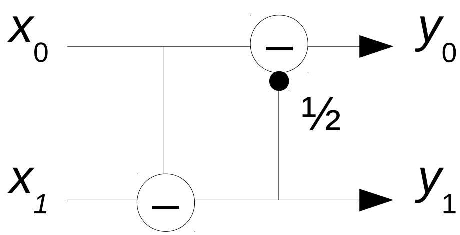
- Can use multiplies
  - Source of 2 of the multiplies in VP9’s 4-point DCT
  - Kind of expensive
- Another approach:
  - Restrict ourselves to shifts and adds
  - Use *asymmetric scaling*

Asymmetric Scaling (1)



$$y_0 = x_0 + x_1 \qquad = x_0 + x_1$$
$$y_1 = (y_0 >> 1) - x_1 \qquad = (x_0 - x_1)/2$$

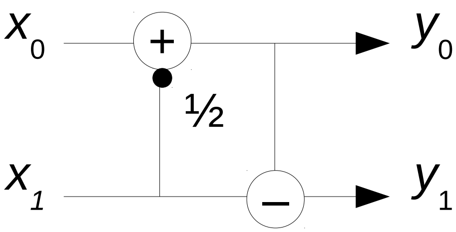
OR



$$y_0 = x_0 - (y_1 >> 1) \qquad = (x_0 + x_1)/2$$
$$y_1 = x_0 - x_1 \qquad = x_0 - x_1$$

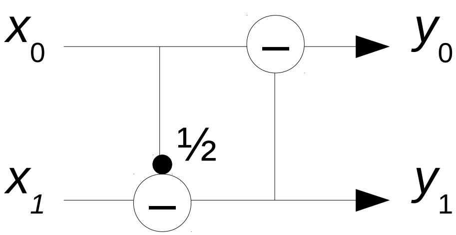
- Asymmetric *output* scales
- Overall scaling remains unity
- Cancel out the asymmetry in subsequent steps

Asymmetric Scaling (2)



$$y_0 = x_0 + (x_1 >> 1) \quad = x_0 + x_1/2$$
$$y_1 = y_0 - x_1 \quad = x_0 - x_1/2$$

OR



$$y_0 = x_0 - y_1 \quad = x_0/2 + x_1$$
$$y_1 = (x_0 >> 1) - x_1 \quad = x_0/2 - x_1$$

- Asymmetric *input* scales
- Cancels out the asymmetry from previous steps

Simplifying the Multiplies

- Multiplies arise from *plane rotations* between two variables

$$\begin{bmatrix} y_0 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \end{bmatrix}$$

- Can trade one multiply for one addition

$$p_0 = x_0 - \frac{\cos(\theta) - 1}{\sin(\theta)} x_1$$

$$y_1 = x_1 - \sin(\theta) p_0$$

$$y_0 = p_0 - \frac{\cos(\theta) - 1}{\sin(\theta)} y_1$$

Asymmetric Scaling Multiplies

- Can also arbitrarily scale inputs and outputs

$$\begin{bmatrix} y_0 \\ y_1 \end{bmatrix} = \begin{bmatrix} u & 0 \\ 0 & \frac{1}{stu} \end{bmatrix} \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} s & 0 \\ 0 & t \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \end{bmatrix}$$

- Becomes

$$p_0 = x_0 - \frac{t}{s} \frac{\cos(\theta) - su}{\sin(\theta)} x_1$$

$$y_1 = x_1 - \frac{\sin(\theta)}{tu} p_0$$

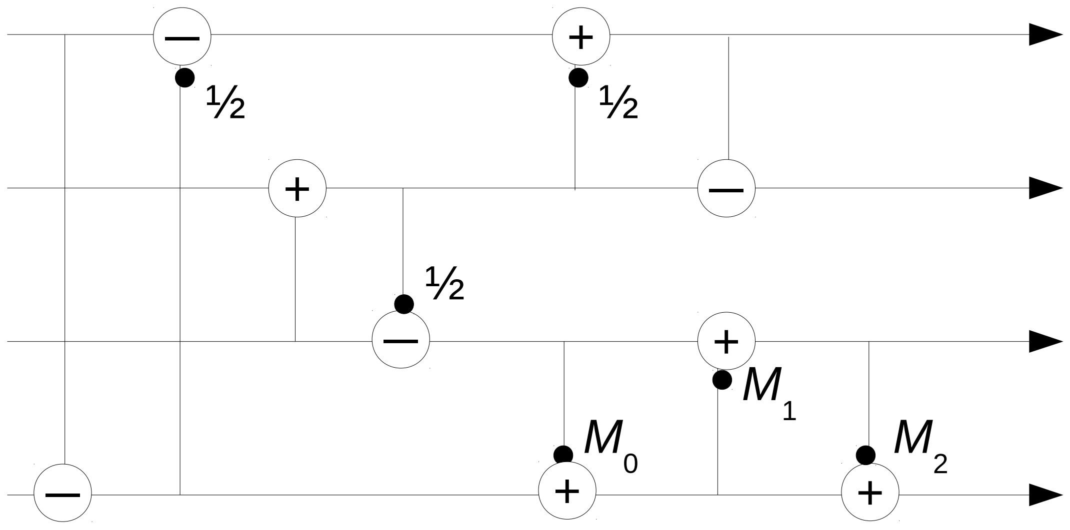
$$y_0 = p_0 - tu \frac{su\cos(\theta) - 1}{\sin(\theta)} y_1$$

Advantages

- 25% fewer multiplies
  - Much more expensive than adds
- All have x += a*y structure
- Becomes x += (a*y + 16384) >> 15 in fixed point
  - Only need top part of multiplier output
  - 16-bit SIMD stays in 16 bits
    - Going to 32 bits halves SIMD throughput
  - SSSE3 and NEON both have an instruction for this
    - PMULHRSW (parallel multiply high, round, and shift word)
    - VQRDMULH.S16 (vector saturated rounding doubled multiply high)
  - Single instruction to multiply, add rounding offset, and shift down

Putting It All Together

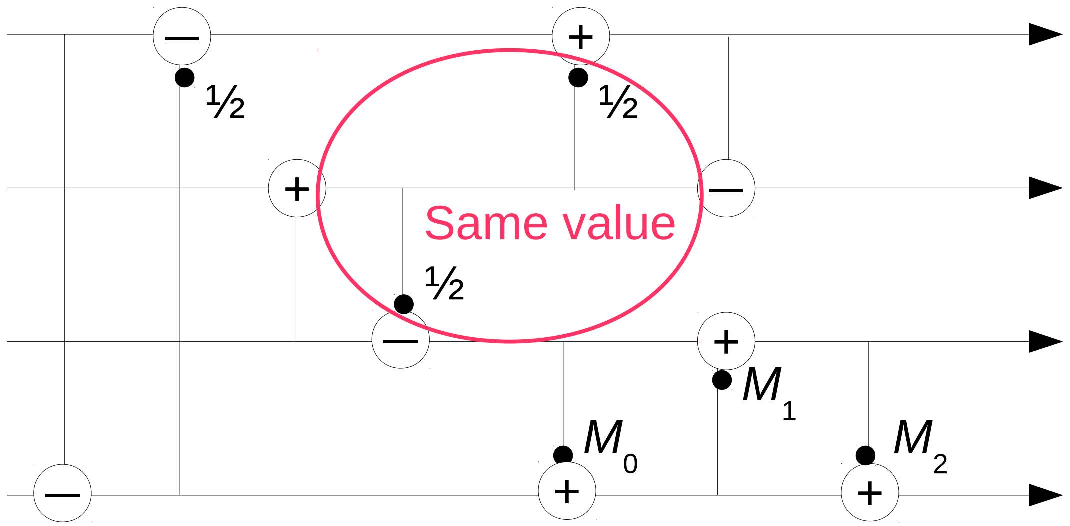
- 9 adds, 3 multiplies, 2 shifts



$$M_0 = \frac{2\cos\left(\frac{3\pi}{8}\right) - \sqrt{2}}{\sin\left(\frac{3\pi}{8}\right)} \quad M_1 = \sqrt{\frac{1}{2}}\sin\left(\frac{3\pi}{8}\right) \quad M_2 = \frac{\cos\left(\frac{3\pi}{8}\right) - \sqrt{2}}{\sin\left(\frac{3\pi}{8}\right)}$$

Putting It All Together

- 9 adds, 3 multiplies, 2 shifts



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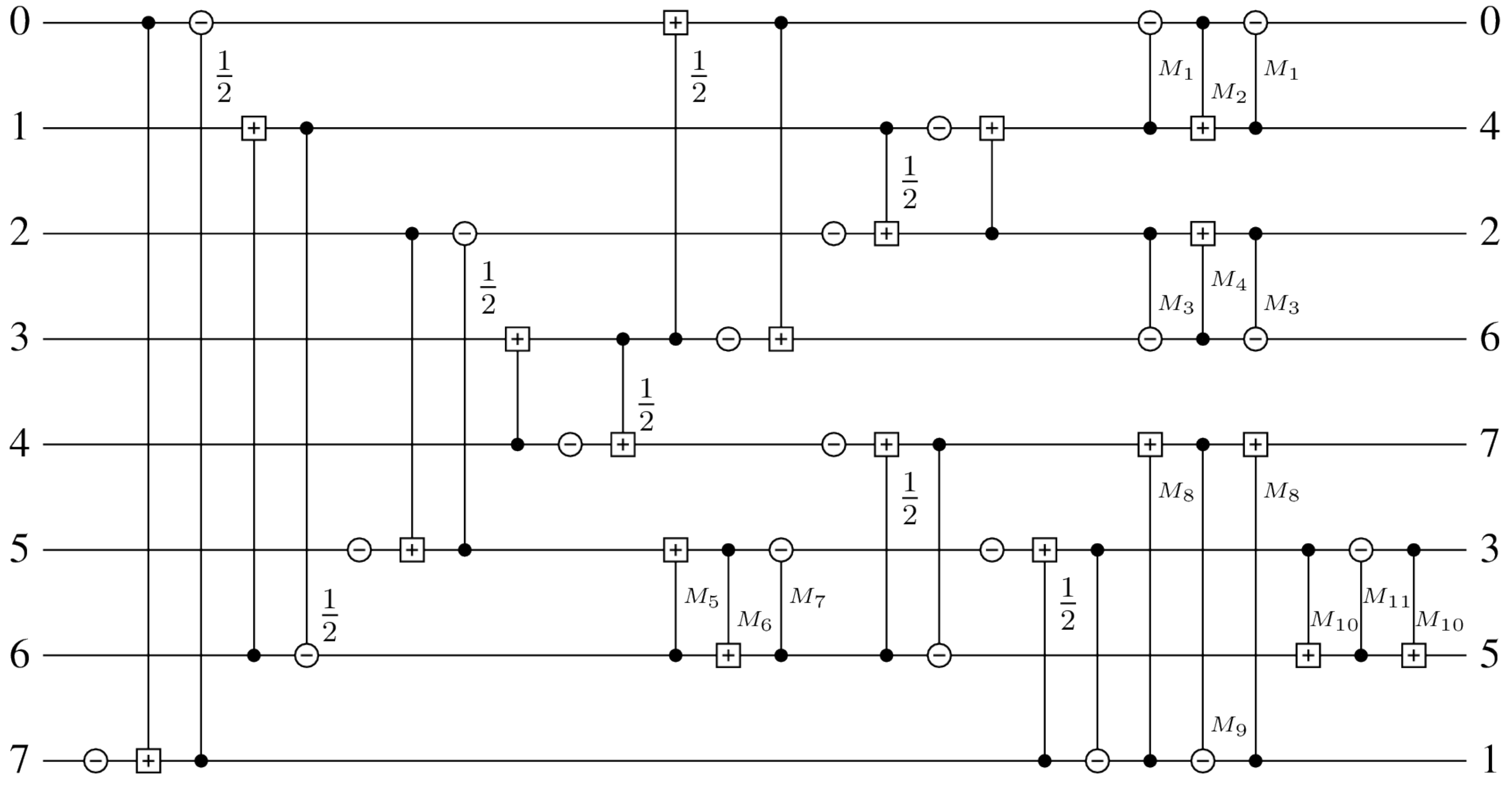
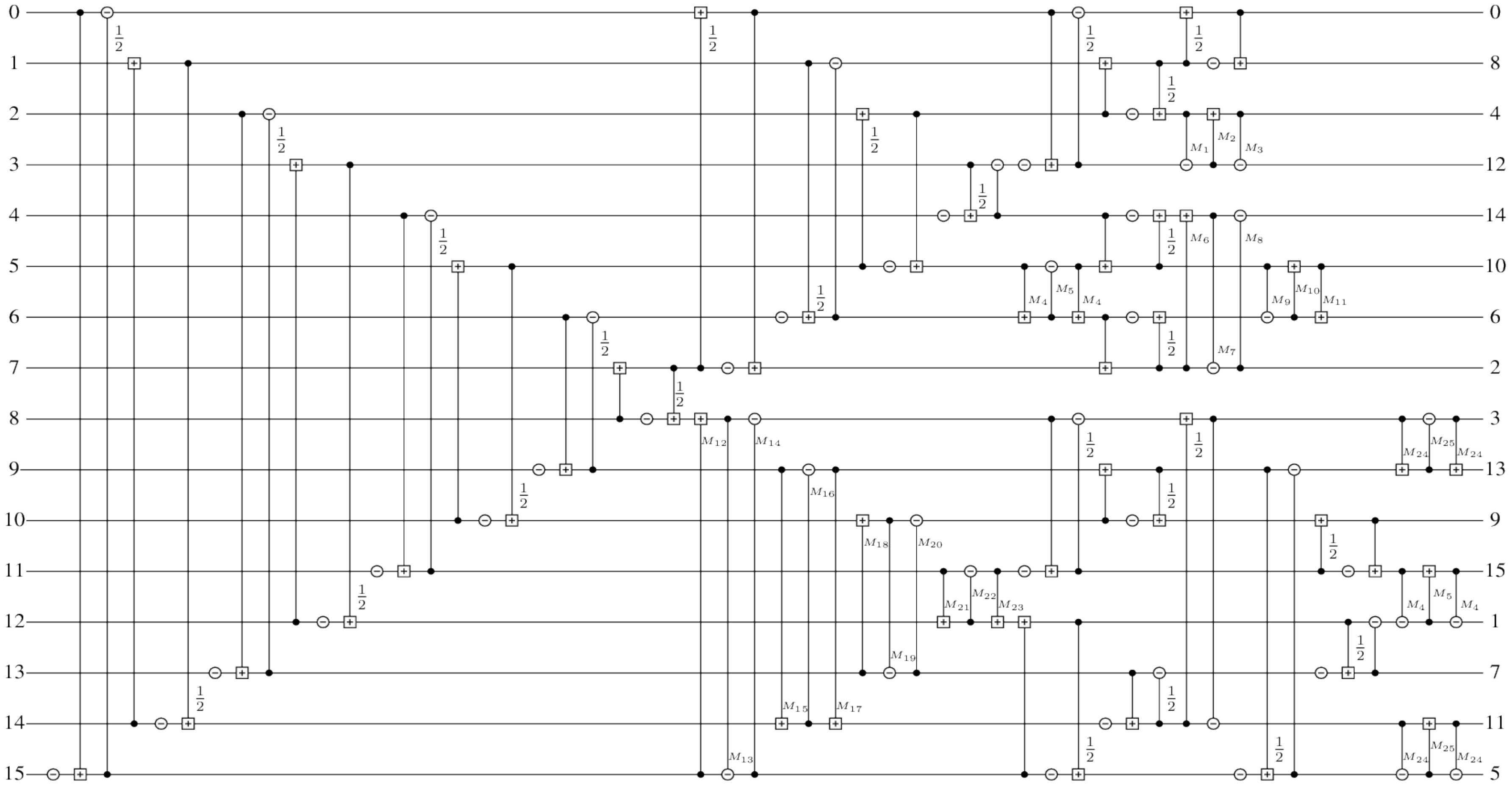
8-Point DCT



16-Point DCT



And more...

- Up to 64-point DCT implemented
  - The margin of this slide is too small to contain...
- Embedded structure
  - Both N-point DCT and N-point DST are embedded in the 4N-point DCT
    - Embedding skips a level because of the asymmetries

Accuracy (1)

- Right shifts and multiplies introduce rounding errors
- Want to keep these as small as possible
- Solution?
  - Shift up input
  - Forward transform, quantize, code, inverse transform
  - Shift down output
- Diminishing returns at 4 bits (for 8-bit input)
  - Enough to make all DCTs match a double-precision floating point implementation after rounding to nearest integer
    - Error ≤ 0.5

Accuracy (2)

- How does this compare with VP9?
  - Also shifts up inputs (by a smaller amount)
  - And shifts down outputs (by a larger amount)
    - Sometimes between row and column transforms, too
- Scale of VP9 coefficients grows as transform progresses
  - Rounding errors early in process get magnified
- Daala: all stages have the same scale
  - All errors injected at the same level
  - Accumulate, but aren’t magnified

High Bit Depth

- Accuracy less important for higher bit depths (10 or 12 bits)
  - Importance is accuracy *relative* to quantizer, and higher bit depths use larger quantizers
- We shift up less for higher bit depths
  - 10 bits = 2 bit shift
  - 12 bits = no shift
- Result: Can use same transforms for all bit depths

Dynamic Range (1)

- Everything has orthonormal (unitary) scaling
- *Dynamic range* of the outputs still increases
  - Dynamic range = minimum/maximum output values
  - Unitary transforms are N-dimensional rotations
  - If the input is a box, the length of the diagonal is longer than the length of an edge
    - By a factor of $\sqrt{2}$ every time N doubles
- So how big can the outputs be?

Dynamic Range (2)

- All transforms with 64 pixels or less fit in 16 bits
  - 9-bit residual + 4-bit up shift + 3 bits of dynamic range expansion
  - Includes 4x4, 4x8, 8x4, 8x8, 4x16, 16x4
- All column transforms fit in 16 bits
  - Maximum size needed for hardware transpose buffer
- VP9 has larger intermediaries in the transforms, but shifts final coefficients down to fit in 16 bits
  - Think this is a mis-optimization
  - Just as easy to pack during quantization
  - Avoids double-rounding, simplifies RDO (no special cases)

Reversibility (1)

- Steps of the form

$$x_i = x_i + f\left(x_{0,} \ldots, x_{i-1}, x_{i+1}, \ldots, x_N\right)$$

are called *lifting steps*

- Exactly reversible:

$$x_i = x_i - f\left(x_{0,} \ldots, x_{i-1}, x_{i+1}, \ldots, x_N\right)$$

- Inverse transform: just reverse all of the steps
- Why is this good?

Reversibility in Daala

- Daala used lapping instead of a deblocking filter
- Deblocking filters are *low pass*
  - Tend to blur out details over consecutive frames
- Forward and inverse lapping are *matched*
  - No low passing
- If that match is not *exact*, errors will build up over multiple frames
  - Costs bits to correct

Reversibility (2)

- Do we need perfect reversibility?
  - It seems to help (small coding gain improvements)
  - Probably not *required*, but it's basically free
  - Don't actually have it in Daala anymore
    - 4 bit down shift after inverse breaks it
    - Using 12-bit references (even for 8-bit data) restores it [1]
    - But using CLPF/deringing also solves the problem
      - Adds the low pass filter we were missing from deblocking

[1] https://people.xiph.org/~xiphmont/demo/daala/random.shtml

Reversibility and Dynamic Range

- Transform coefficients values are larger than pixel values
  - Forward transform expands dynamic range
- Inverse transform is also an N-dim. rotation
  - How do we know it doesn't expand dynamic range?
- E.g., if x_0 and x_1 just barely fit in 16 bits, how do we know $x_0 + x_1$ won't overflow?
- Answer: Reversibility
  - Values computed in inverse same as forward transform
    - ± quantization error
  - Only guaranteed if coefficients result of transforming pixels

Type IV vs. Type VII DST

- For intra prediction residuals, prediction error is asymmetric
  - Less error closer to edges we're predicting from
- Want an asymmetric transform to code them
- Optimal transform is a Type VII DST
  - Compute correlation matrix, solve eigensystem problem in the limit as the correlation approaches 1
- Type VII DST factorizations are much nastier than Type IVs

Type VII vs. Type IV DST

- Type IV

$$y_k = \sum_{n=0}^{N-1} x_n \sin\left(\frac{\pi}{N}\left(n+\frac{1}{2}\right)\left(k+\frac{1}{2}\right)\right)$$

- Type VII

$$y_k = \sqrt{\frac{2}{N+\frac{1}{2}}} \sum_{n=0}^{N-1} x_n \sin\left(\frac{\pi}{N+\frac{1}{2}}(n+1)\left(k+\frac{1}{2}\right)\right)$$

Type VII vs. Type IV

- Type IV transforms *almost* as good, and already embedded inside our DCTs
- Current approach
  - Use Type VII for small DSTs (4-point and 8-point)
  - Use embedded Type IV for larger DSTs

Overall Complexity

		Daala TX			TXMG	
	muls/coeff	adds/coeff	shifts/coeff	muls/coeff	adds/coeff	shifts/coeff
DCT 4	1	2	0.5	2	2	1
DST 4	1.25	2.75	0.5	3	2.5	1.5
DCT 8 [1]	1.875	3.875	0.625	2.5	3.25	1.25
DST 8 [2]	2.625	9.375	2.25	4	4	2
DCT 16	2.0625	5.1875	1	3.25	4.625	1.625
DST 16	3.1875	6.1875	1.25	5	5.5	2.5
DCT 32	2.7188	6.2188	1.1875	4.125	6.0625	2.0625
DST 32	3.6562	7.6562	1.125	6	7	3

- [1] SIMD benchmarked at 26.2% faster
- [2] Daala TX uses a Type VII DST, while TXMG uses a Type IV

Hardware Considerations (1)

- Intra prediction requires reconstructed pixels from neighboring blocks
- This serializes reconstruction of these blocks
  - Including the inverse transform
  - Particularly a problem for encoders
- Our 3-multiply rotations chain them all consecutively
- This is a bottleneck for small transform sizes

Low-Latency Small Transforms

- 4-point DCT: replace 3-multiply block with 4-multiply version
  - All multiplies can proceed in parallel
  - Still only use top part of multiply
    - Full SIMD throughput
- 4-point Type VII DST:
  - Use custom factorization with 5 parallel multiplies
- These are not exactly reversible

Hardware Considerations (2)

- Most hardware already “multi-standard”
  - Including VP9
- Dedicates a lot of gates to parallel multipliers
- Can replace serial multiplies in rotations with parallel multiplies

$$u_0 = x_0 + a x_1$$
$$y_1 = u_0 + b u_0$$
$$y_0 = y_1 + a x_1$$

becomes

$$u_0 = x_0 + x_1$$
$$u_1 = (1 + ab) u_0$$
$$u_2 = (b(a-1) + 1) x_0$$
$$u_3 = (a + (a-1)(1+ab)) x_1$$
$$y_0 = u_1 + u_3$$
$$y_1 = u_1 - u_2$$

- Still experimenting to see impact on accuracy, potential for overflows

Questions?