Alternative
Elliptic Curve Representations

draft-struik-lwig-curve-representations-00

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Outline

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ECC Algorithm Zoo (1)

NIST curves:

Curve model:	Weierstrass curve
Curve equation:	$y^2 = x^3 + a \cdot x + b$ (**mod** p)
Base point:	$G=(G_x, G_y)$
Scalar multiplication:	addition formulae using, e.g., mixed Jacobian coordinates
Point representation:	both coordinates of point $P=(X, Y)$ (affine coordinates)
	0x04 || $\underline{X}$ || $\underline{Y}$ in most-significant-bit/octet first order
Examples:	NIST P-256 (ANSI X9.62, NIST SP 800-56a, SECG, etc.);
	Brainpool256r1 (RFC 5639)

ECDSA:

Signature:	$\underline{R}$ || $\underline{s}$ in most-significant-bit/octet first order
Signing equation:	$e = s \cdot k + d \cdot r$ (**mod** n), where e=Hash(m)
Example:	ECDSA, w/ P-256 and SHA-256 (FIPS 186-4, ANSI X9.62, etc.)
Note:	message m pre-hashed

ECC Algorithm Zoo (2)

<u>CFRG curves:</u>

Curve model:	Montgomery curve
Curve equation:	$B \cdot y^2 = x^3 + A \cdot x^2 + x \ (\mathbf{mod}\ p)$
Base point:	$G=(G_x, G_y)$
Scalar multiplication:	Montgomery ladder, using projective coordinates [X: :Z]
Point representation:	x-coordinate of point $P=(X, Y)$ (x-coordinate-only)
	<u>X</u> in least-significant-octet, most-significant-bit first order
Examples:	Curve25519, Curve448 (RFC 7748)

<u>DH Key agreement:</u>

Key shares:	x-coordinates of $X=xG$ (Party A) and $Y=yG$ (Party B)
Shared key:	x-coordinate of $K_A = (h \cdot x)Y = K_B = (h \cdot y)X$, where h co-factor
Notes:	Montgomery ladder can provide y-coordinate as well[1]
Examples:	X25519, X448 (RFC 7748)

[1]Not in RFC 7748 or most code (NaCl, etc.)

ECC Algorithm Zoo (3)

CFRG curves:

Curve model:	twisted Edwards curve
Curve equation:	$a \cdot x^2 + y^2 = 1 + d \cdot x^2 \cdot y^2$ (**mod** p)
Base point:	$G=(G_x, G_y)$
Scalar multiplication:	Dawson formulae, using extended coordinates (X: Y: T: Z)
Point representation:	compressed point $P=(Y, X')$, where $X'=\text{lsb}(X)$
	$\underline{Y} \| \| \underline{X}'$ in least-significant-octet/bit first order
Examples:	Edwards25519, Edwards448 (RFC 7748)

EdDSA:

Signature:	$\underline{R} \| \| \underline{s}$ in least significant bit/octet first order
Signing equation:	$s = k + e \cdot d$ (**mod** n), where e=Hash($\underline{Q} \| \| \underline{R} \| \| m$)
Example:	Ed25519-SHA-512, Ed448-SHAKE-256
Notes:	Deterministic signature, where k=Hash ($d' \| \| m$)
	Variant w/ pre-hashing uses Hash(m) instead of m

Implementation Detail

Aspect:	NIST P-256	Curve25519	Edwards25519
Curve model:	Weierstrass	Montgomery	twisted Edwards
Base point:	affine	x-coord only	affine
Internal coord:	Jacobian	x-projective	extended
Formulae:	Jacobian	Montgomery	Dawson
Wire format:	affine	x-coord only	compressed
Bit/Octet ordering:	MSB, msb	LSB, msb	LSB, lsb

Implementation drawback:
different arithmetic, different point format, different bit/octet encoding

Lots of code to implement...
a) key agreement co-factor ECDH using NIST P-256 + Curve25519 (TLS1.3);
b) key agreement + sign/verify Curve25519 + Ed25519 (JOSE [RFC 8037]);
c) key agreement + sign/verify P-256 + ECDSA & Curve25519 + Ed25519

How to Reuse Code (1)

Aspect:	**NIST P-256**	**Curve25519**	**Edwards25519**
Curve model:	Weierstrass	Montgomery	(t)Edwards
Base point:	affine	**affine**	affine
Internal coord:	Jacobian	x-projective	extended
Formulae:	Jacobian	Montgomery	Dawson
Wire format:	affine	**affine**	**affine**
Bit/Octet ordering:	MSB, msb	**MSB**, msb	**MSB**, **msb**

Implementation drawback:

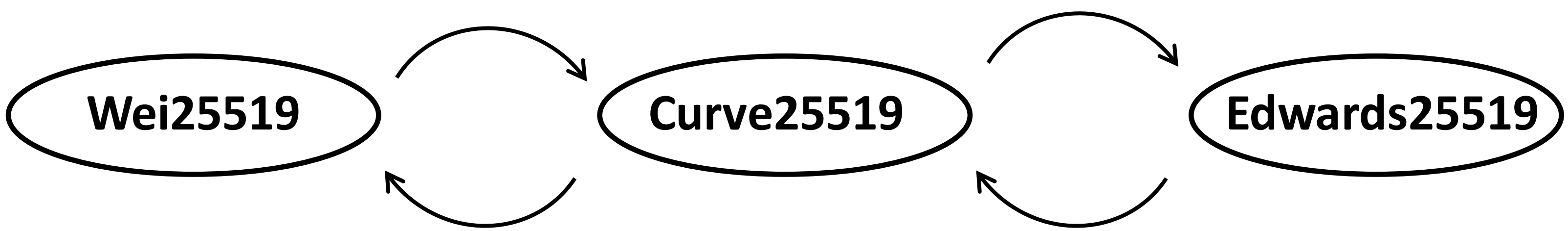
different arithmetic, ~~different point format, different bit/octet encoding~~

we will deal with the encoding mess later on ...

How to Reuse Code (2)

Aspect:	Wei25519	Curve25519	Edwards25519
Curve model:	Weierstrass	Montgomery	(t)Edwards
Base point:	affine	**affine**	affine
Internal coord:	Jacobian	–	–
Formulae:	Jacobian	–	–
Wire format:	affine	**affine**	**affine**
Bit/Octet ordering:	MSB, msb	**MSB**, msb	**MSB**, **msb**

Implementation can use mappings between different curve models:

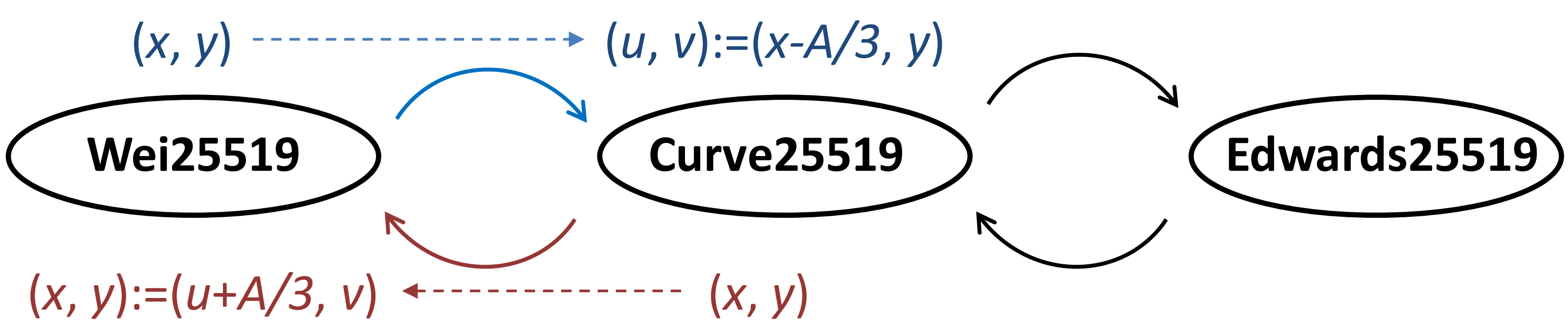


These mappings are so-called isomorphisms, so group structure remains the same

Mappings easy to implement at virtually zero incremental cost

How to Reuse Code (3)

Aspect:	Wei25519	Curve25519	Edwards25519
Curve model:	Weierstrass	Montgomery	(t)Edwards
Common parms:			
prime p		$p=2^{255}-19$	
co-factor h		$h=8$	
group size n:		$n=2^{252}+$ 0x14def9de a2f79cd6 5812631a 5cf5d3ed	
Specific parms:			
Base point G:	$(9+A/3, \ldots)$	$(9, \ldots)$	$(c\cdot 4/5, \ldots)$
Constants:	$a= \ldots$	A=486662	$a=-1$
	$b= \ldots$	B=1	$d=-121665/121666$



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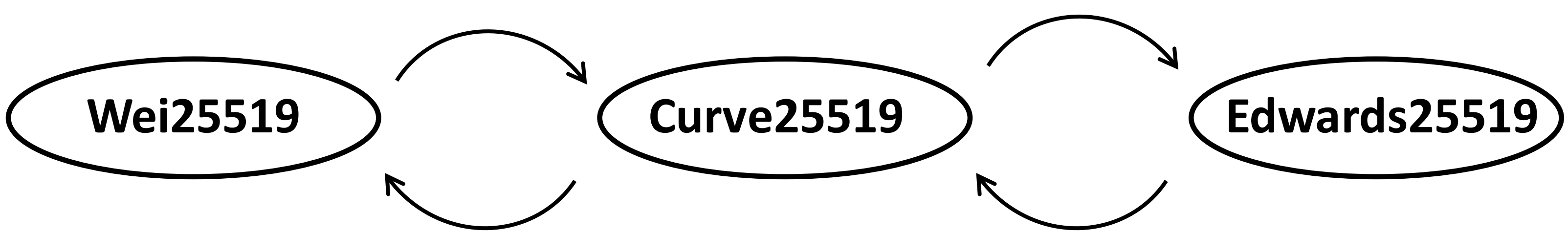
How to Reuse Code (4)

Aspect:	NIST P-256	Wei25519
Curve model:	Weierstrass	Weierstrass
Base point:	affine	affine
Internal coord:	Jacobian	Jacobian
Formulae:	Jacobian	Jacobian
Wire format:	affine	affine
Bit/Octet ordering:	MSB, msb	MSB, msb

Implementations can **re-use all code for Weierstrass curves** under the hood, using the mappings between different curve models.



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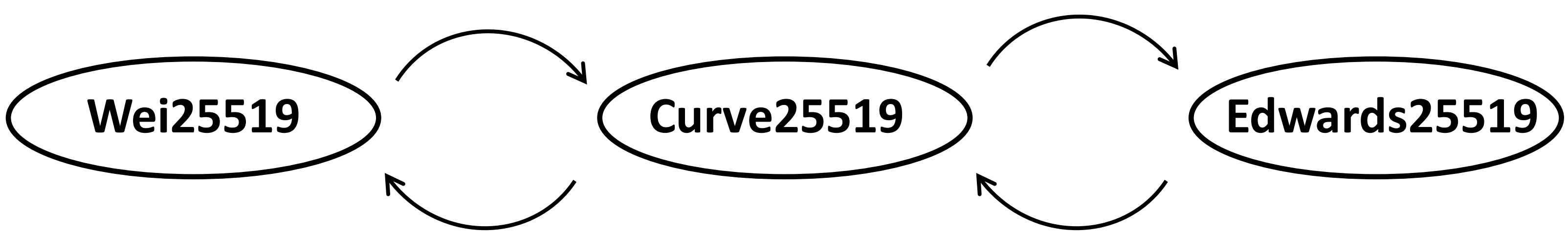
How to Reuse Code (5)

Aspect:	Wei25519	Curve25519	Edwards25519
Curve model:	Weierstrass	Montgomery	twisted Edwards
Base point:	affine	**x-coord only**	affine
Internal coord:	Jacobian	x-projective	extended
Formulae:	Jacobian	Montgomery	Dawson
Wire format:	affine	**x-coord only**	**compressed**
Bit/Octet ordering:	MSB, msb	**LSB**, msb	**LSB**, **lsb**

Implementations can **re-use all code for Weierstrass curves** under the hood, using the mapping between different curve models. *One can deal with encoding conversion as pre- or postprocessing operation* (or, fix encoding if one can).



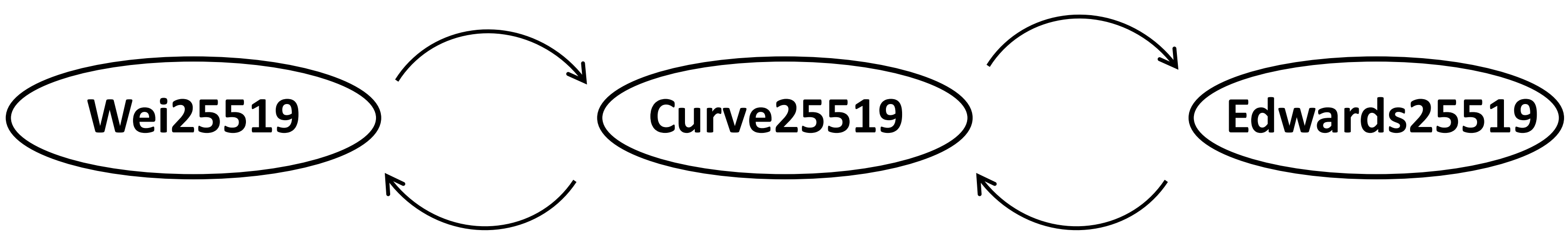
How to Reuse Code (6)

Efficiency:

– Conversion between different curve models comes at negligible cost (only +0.1%)

– Point representation can impact efficiency (if no affine coordinates available):
  Example: TLS1.3 purposely uses x-coordinate only representation Curve25519, thus handicapping conversion to Weierstrass format (+12% cost due to point decompression [affine format would have done away with this])

– Encoding mess re endianness octet/bit ordering [RFC 7748, RFC 8032] does not help, but comes at negligible computational cost



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How to Reuse Existing Standards (1)

Lots of code to implement... and ways to alleviate this

a) key agreement co-factor ECDH using NIST P-256 + Curve25519 (TLS1.3)

– Use co-factor ECDH with Wei25519 curve and NIST encodings (@NIST SP 800-56a)

b) key agreement + sign/verify Curve25519 + Ed25519 (JOSE [RFC 8037])

– Use ECDSA/Wei25519/SHA-256 (@FIPS 186-4)

– Use Schnorr/Wei25519/SHA-256 and NIST encodings (@BSI ECC2.0 spec)

c) key agreement + sign/verify P-256+ ECDSA & Curve25519 + Ed25519

– As above, but now Wei25519 + P-256 with NIST encodings allow single implementation (simply invoke with different domain parms)

Alternatively, one can reuse Weierstrass code under the hood if implementing Curve25519 or Ed25519, *provided one deals with encoding conversion as pre- and post-processing operation*.

NOTE: if one uses Ed25519 with SHA-256 one can reuse curve arithmetic and hash function ECDSA/P-256/SHA-256, but **not** secure computation of (e, s) values.

How to Reuse Existing Standards (2)

Defining new schemes using existing standards

a) MQV key agreement using NIST P-256 + Curve25519 (DTLS1.3)

– Use MQV with Wei25519 curve and NIST encodings (@NIST SP 800-56a)

b) X509 Certificate Infrastructure with CA key on Curve25519 [RFC 5280]

– Use curve Wei25519 with X509 and NIST encodings (@RFC 5280)

c) Implicit certificates using NIST P-256 + Curve25519

– Use curve Wei25519 with NIST encodings (@SEC4, P1609.2, Autonomous Vehicles, US Dept of Transport)

Conclusions

1. Different curve models can be implemented using the same code if one uses the short-Weierstrass model.
2. One can thereby reuse not just code, but also existing standards, thus significantly reducing standards development cycles.
3. Encoding format issues may negatively impact code reuse and reuse of existing standards, since can be used as artifical ‟moat around a solution”, making code reuse or algorithm agility economically less viable than these could/should be.
4. Representation conventions require more careful considerations by IETF in the future than has happened so far (in TLS1.3, CFRG).