cfrg@IETF'102
Montreal, July 2018

draft-irft-cfrg-vrf-02

Verifiable Random Functions (VRF)

update on changes + some questions

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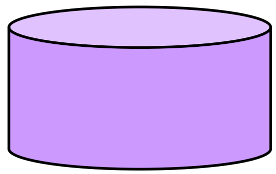
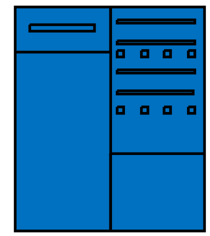
Jan Vcelak (ns1)

VRF: verifiable random function

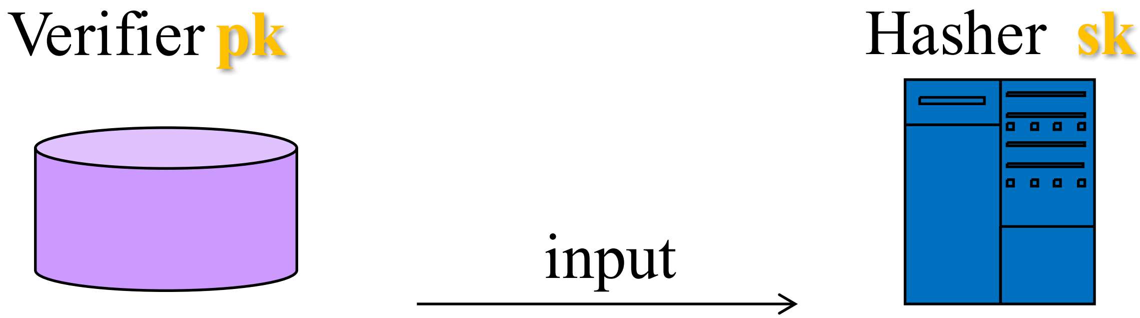
Verifier **pk**



Hasher **sk**



VRF: verifiable random function



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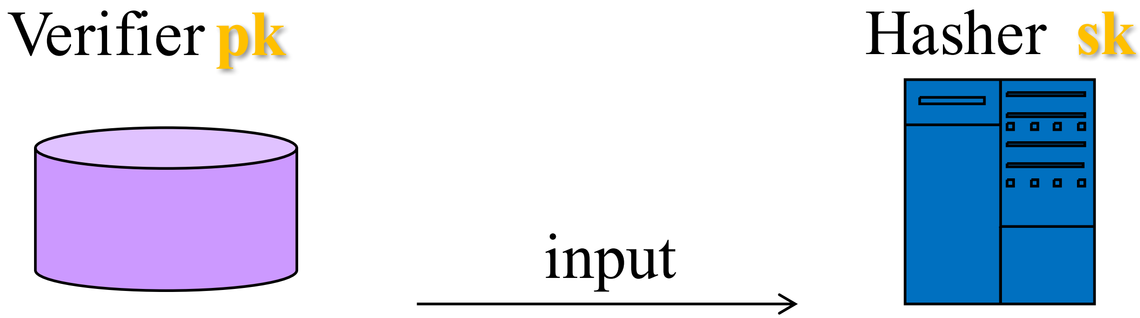
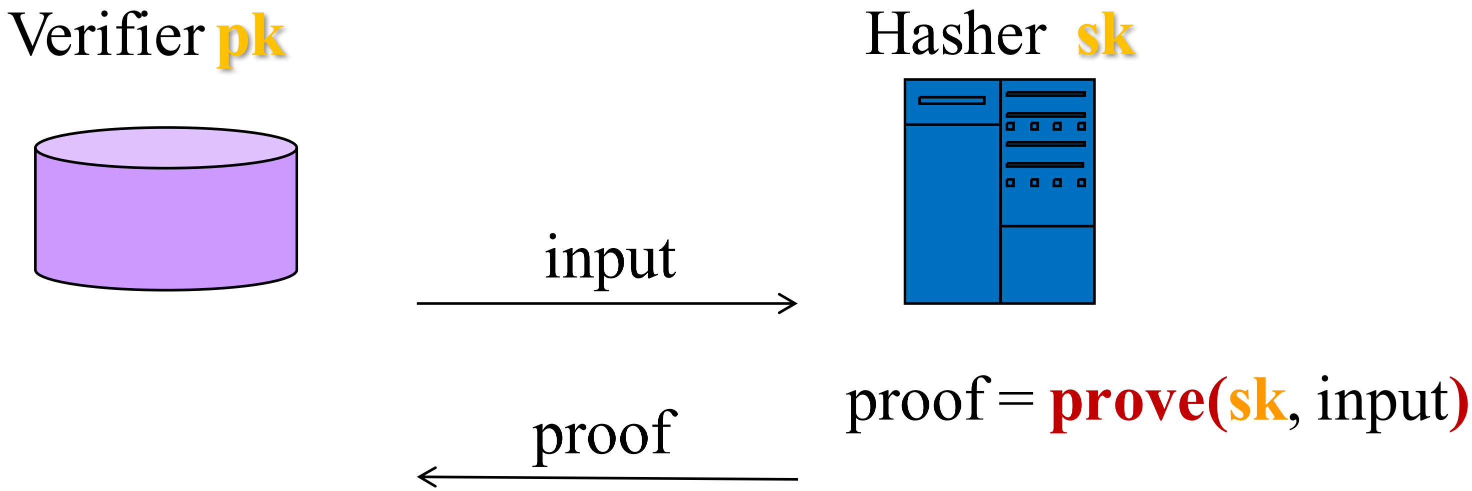


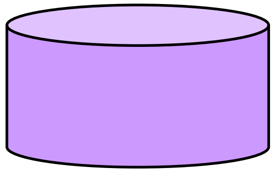
proof = **prove(sk**, input**)**

VRF: verifiable random function



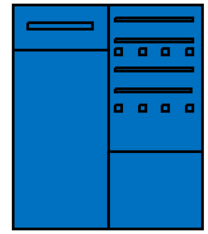
VRF: verifiable random function

Verifier pk



input

Hasher sk



proof = **prove(**sk, input**)**

proof

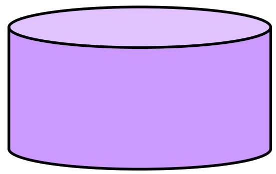
If **verify (**pk, input, proof**)**

hash = **proof2hash(**proof**)**

Else INVALID

VRF: verifiable random function

Verifier **pk**



input →

Hasher **sk**

proof = **prove(sk**, input**)**

← proof

If **verify (pk**, input, proof**)**

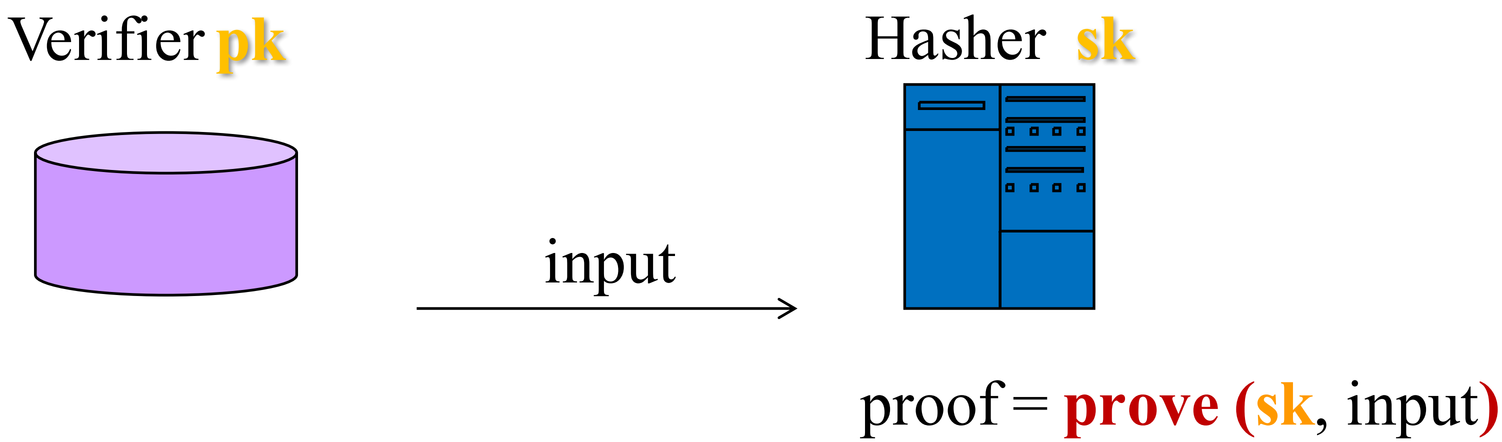
hash = **proof2hash(**proof**)**

Else INVALID

properties:

1. **uniqueness:** 1-to-1 relationship between input and hash.
2. **collision resistance:** hard to find two inputs with same hash

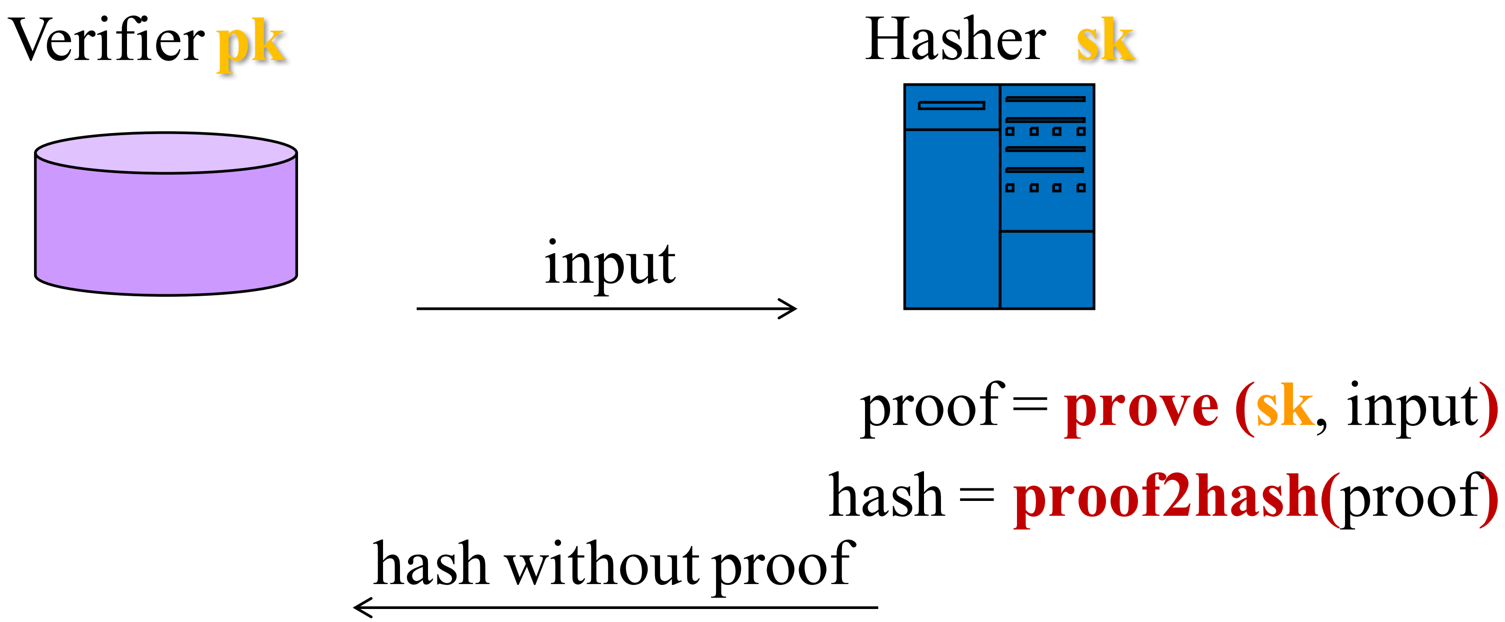
VRF: verifiable random function



properties:

1. **uniqueness:** 1-to-1 relationship between input and hash.
2. **collision resistance:** hard to find two inputs with same hash
3. **pseudorandomness:** only hasher can compute hash from input

VRF: verifiable random function



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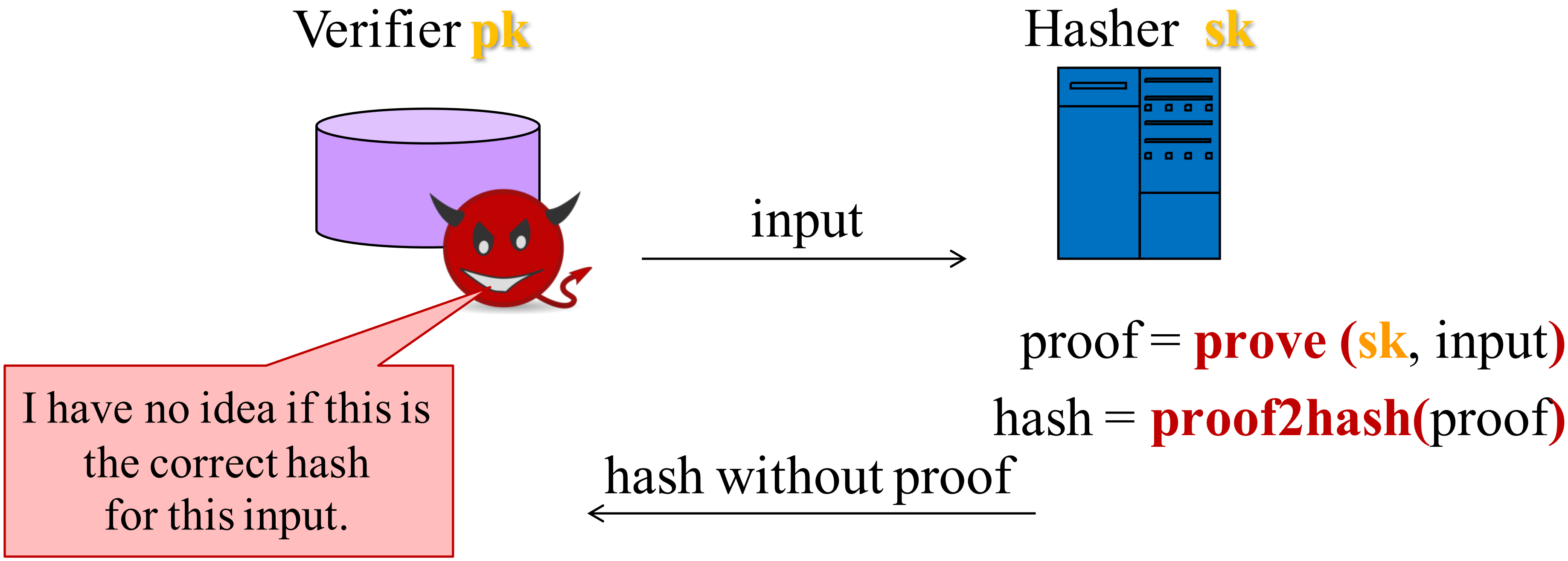
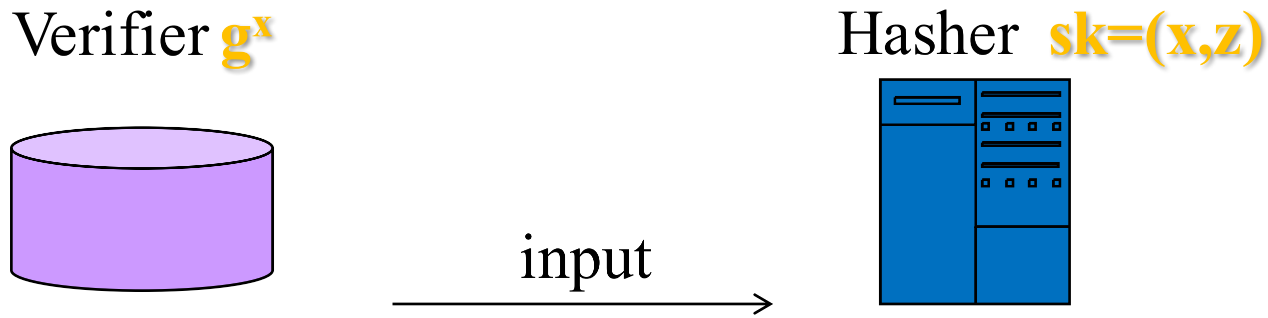


properties:

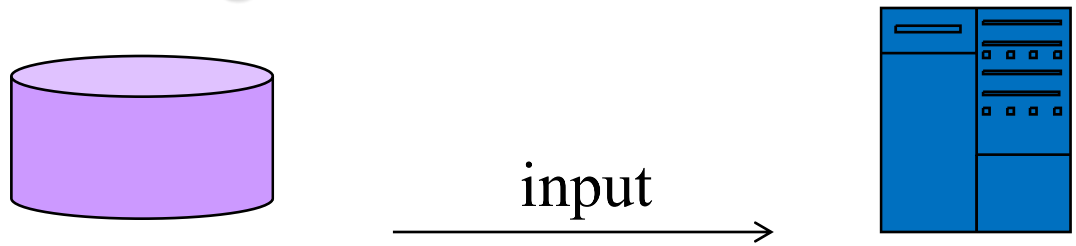
1. **uniqueness:** 1-to-1 relationship between input and hash.
2. **collision resistance:** hard to find two inputs with same hash
3. **pseudorandomness:** only hasher can compute hash from input

EC-VRF (elliptic curve VRF)



EC-VRF (elliptic curve VRF)

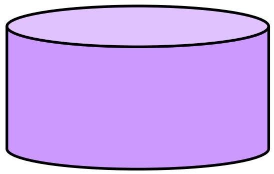
Verifier g^x Hasher $sk=(x,z)$



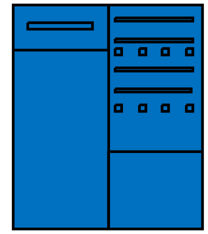
$\mathbf{h} = \text{hash\_to\_curve}(\text{suite}, \mathbf{g}^x, \text{input})$

EC-VRF (elliptic curve VRF)

Verifier g^x



Hasher **sk=(x,z)**



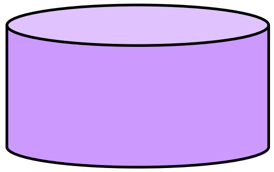
input →

$\mathbf{h} = \text{hash\_to\_curve}(\text{suite}, \mathbf{g}^x, \text{input})$

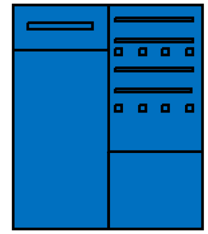
← proof: $(\mathbf{h}^x, \mathbf{c}, \mathbf{s})$

EC-VRF (elliptic curve VRF)

Verifier g^x



Hasher $sk=(x,z)$



input $\longrightarrow$

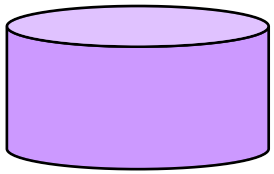
$\mathbf{h}$ = hash_to_curve (suite, $\mathbf{g}^x$, input)

proof: $(\mathbf{h}^x, \mathbf{c}, \mathbf{s})$ $\longleftarrow$

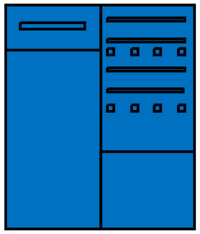
return hash($\mathbf{h}^x$)

EC-VRF **(elliptic curve VRF)**

Verifier g^x



Hasher $sk=(x,z)$



input →

$\mathbf{h}$ = hash_to_curve (suite, $\mathbf{g}^x$, input)

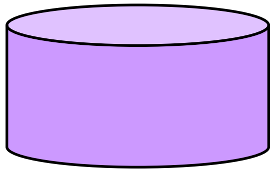
proof that
$\log_g \mathbf{g}^x = \log_h \mathbf{h}^x$

proof: $(\mathbf{h}^x, \mathbf{c}, \mathbf{s})$ ←

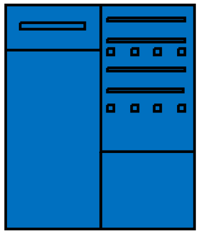
return hash($\mathbf{h}^x$)

EC-VRF (elliptic curve VRF)

Verifier g^x



Hasher sk=(x,z)



input →

$\mathbf{h}$ = hash_to_curve (suite, $\mathbf{g}^x$, input)

proof that $\log_g \mathbf{g}^x = \log_h \mathbf{h}^x$

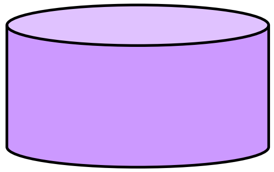
$$\left\{\begin{array}{l} \text{nonce } \mathbf{k} = \text{hash}(\ldots, \mathbf{z}, \mathbf{h}) \\ \mathbf{c} = \mathbf{H}(\mathbf{h}, \mathbf{h}^x, \mathbf{g}^k, \mathbf{h}^k) \\ \mathbf{s} = \mathbf{k} + \mathbf{cx} \bmod \mathbf{q} \end{array}\right.$$

← proof: $(\mathbf{h}^x, \mathbf{c}, \mathbf{s})$

return hash($\mathbf{h}^x$)

EC-VRF (elliptic curve VRF)

Verifier g^x



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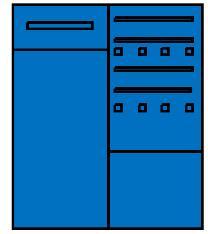


input $\longrightarrow$

$\mathbf{h} = \text{hash\_to\_curve}(\text{suite}, \mathbf{g}^x, \text{input})$

proof that $\log_g \mathbf{g}^x = \log_h \mathbf{h}^x$

nonce $\mathbf{k} = \text{hash}(\ldots, \mathbf{z}, \mathbf{h})$

$\mathbf{c} = \mathbf{H}(\mathbf{h}, \mathbf{h}^x, \mathbf{g}^k, \mathbf{h}^k)$

$\mathbf{s} = \mathbf{k} + \mathbf{cx} \bmod \mathbf{q}$

$\longleftarrow$ proof: $(\mathbf{h}^x, \mathbf{c}, \mathbf{s})$

$\mathbf{u} = \mathbf{g}^s / (\mathbf{g}^x)^c$

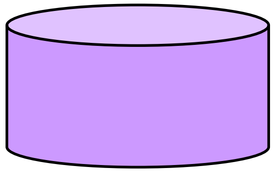
$\mathbf{h} = \text{hash\_to\_curve}(\text{input})$

$\mathbf{v} = \mathbf{h}^s / (\mathbf{h}^x)^c$

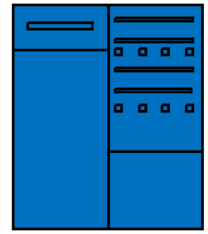
return hash($\mathbf{h}^x$)

EC-VRF (elliptic curve VRF)

Verifier g^x



Hasher $sk=(x,z)$



input

$\mathbf{h} = \text{hash\_to\_curve}(\text{suite}, \mathbf{g}^x, \text{input})$

proof that
$\log_g \mathbf{g}^x = \log_h \mathbf{h}^x$

nonce $\mathbf{k} = \text{hash}(\ldots, \mathbf{z}, \mathbf{h})$

$\mathbf{c} = \mathbf{H}(\mathbf{h}, \mathbf{h}^x, \mathbf{g}^k, \mathbf{h}^k)$

$\mathbf{s} = \mathbf{k} + \mathbf{cx} \bmod \mathbf{q}$

proof: $(\mathbf{h}^x, \mathbf{c}, \mathbf{s})$

$\mathbf{u} = \mathbf{g}^s / (\mathbf{g}^x)^c$

$\mathbf{h} = \text{hash\_to\_curve}(\text{input})$

$\mathbf{v} = \mathbf{h}^s / (\mathbf{h}^x)^c$

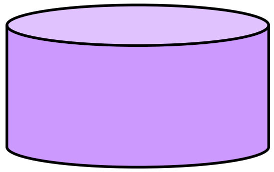
If $\mathbf{c} = \mathbf{H}(\mathbf{h}, \mathbf{h}^x, \mathbf{u}, \mathbf{v})$

return hash($\mathbf{h}^x$)

Else return INVALID

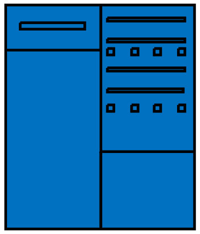
EC-VRF features

Verifier g^x



input

Hasher sk=(x,z)



$\mathbf{h} = \text{hash\_to\_curve}(\text{suite}, \mathbf{g}^{\mathbf{x}}, \text{input})$

nonce $\mathbf{k} = \text{hash}(\ldots, \mathbf{z}, \mathbf{h})$

$\mathbf{c} = \mathbf{H}(\mathbf{h}, \mathbf{h}^{\mathbf{x}}, \mathbf{g}^{\mathbf{k}}, \mathbf{h}^{\mathbf{k}})$

$\mathbf{s} = \mathbf{k} + \mathbf{cx} \bmod \mathbf{q}$

proof: $(\mathbf{h}^{\mathbf{x}}, \mathbf{c}, \mathbf{s})$

$\mathbf{u} = \mathbf{g}^{\mathbf{s}} / (\mathbf{g}^{\mathbf{x}})^{\mathbf{c}}$

$\mathbf{h} = \text{hash\_to\_curve}(\text{input})$

$\mathbf{v} = \mathbf{h}^{\mathbf{s}} / (\mathbf{h}^{\mathbf{x}})^{\mathbf{c}}$

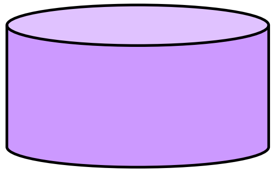
If $\mathbf{c} = \mathbf{H}(\mathbf{h}, \mathbf{h}^{\mathbf{x}}, \mathbf{u}, \mathbf{v})$

return hash($\mathbf{h}^{\mathbf{x}}$)

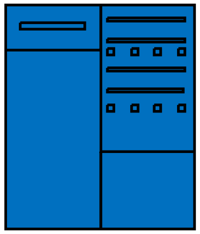
Else return INVALID

EC-VRF features

Verifier g^x



Hasher sk=(x,z)



input

$\mathbf{h}$ = hash_to_curve (suite, $\mathbf{g}^x$, input)

nonce $\mathbf{k}$ = hash(…, $\mathbf{z}$, $\mathbf{h}$)

$\mathbf{c} = \mathbf{H}(\mathbf{h}, \mathbf{h}^x, \mathbf{g}^k, \mathbf{h}^k)$

$\mathbf{s} = \mathbf{k} + \mathbf{cx} \bmod \mathbf{q}$

Short! Just 128 bits!

proof: $(\mathbf{h}^x, \mathbf{c}, \mathbf{s})$

$\mathbf{u} = \mathbf{g}^s / (\mathbf{g}^x)^c$

$\mathbf{h}$ = hash_to_curve (input)

$\mathbf{v} = \mathbf{h}^s / (\mathbf{h}^x)^c$

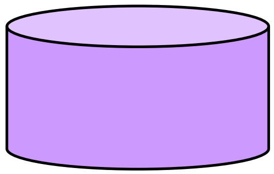
If $\mathbf{c} = \mathbf{H}(\mathbf{h}, \mathbf{h}^x, \mathbf{u}, \mathbf{v})$

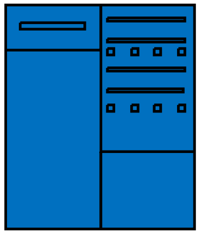
  return hash($\mathbf{h}^x$)

Else return INVALID

EC-VRF features

Verifier g^x | Hasher $sk=(x,z)$





input $\longrightarrow$

$\mathbf{h} = \text{hash\_to\_curve}(\text{suite}, \mathbf{g^x}, \text{input})$

nonce $\mathbf{k} = \text{hash}(\ldots, \mathbf{z}, \mathbf{h})$

$\mathbf{c} = \mathbf{H}(\mathbf{h}, \mathbf{h^x}, \mathbf{g^k}, \mathbf{h^k})$

$\mathbf{s} = \mathbf{k + cx \bmod q}$

NEW Future proofing!

Short! Just 128 bits!

proof: $(\mathbf{h^x}, \mathbf{c}, \mathbf{s})$ $\longleftarrow$

$\mathbf{u} = \mathbf{g^s} / (\mathbf{g^x})^{\mathbf{c}}$

$\mathbf{h} = \text{hash\_to\_curve}(\text{input})$

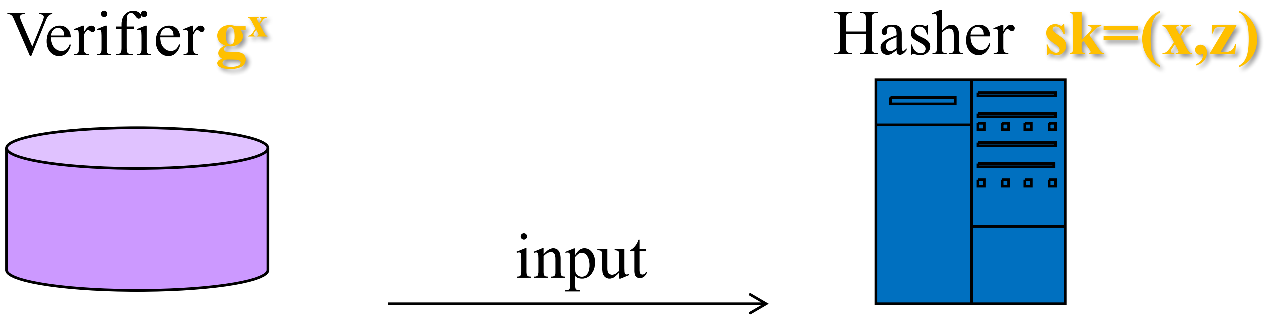
$\mathbf{v} = \mathbf{h^s} / (\mathbf{h^x})^{\mathbf{c}}$

If $\mathbf{c} = \mathbf{H}(\mathbf{h}, \mathbf{h^x}, \mathbf{u}, \mathbf{v})$

return hash($\mathbf{h^x}$)

Else return INVALID

Ciphersuite EC-VRF-P256-SHA256



$\mathbf{h} = \text{hash\_to\_curve}(\text{suite}, \mathbf{g}^{\mathbf{x}}, \text{input})$

$\text{nonce } \mathbf{k} = \text{hash}(\ldots, \mathbf{z}, \mathbf{h})$

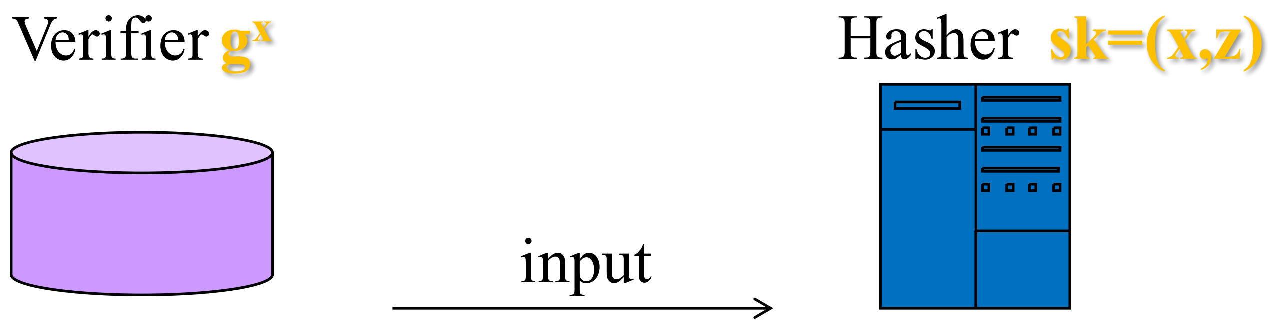
$\mathbf{c} = \mathbf{H}(\mathbf{h}, \mathbf{h}^{\mathbf{x}}, \mathbf{g}^{\mathbf{k}}, \mathbf{h}^{\mathbf{k}})$

$\mathbf{s} = \mathbf{k} + \mathbf{cx} \bmod \mathbf{q}$

Ciphersuite choices:

- **Curve:** NIST P-256
- **Hash:** SHA256
- NEW **Nonce:** Deterministic, identical to RFC 6979 [Determist'c ECDSA]
- **Hash-to-curve:** Try-and-increment with SHA256
- **Key generation:** Same as SECG1

Ciphersuite EC-VRF-ED25519-SHA512



$\mathbf{h}$ = hash_to_curve (suite, $\mathbf{g}^x$, input)

nonce $\mathbf{k}$ = hash(…, $\mathbf{z}$, $\mathbf{h}$)

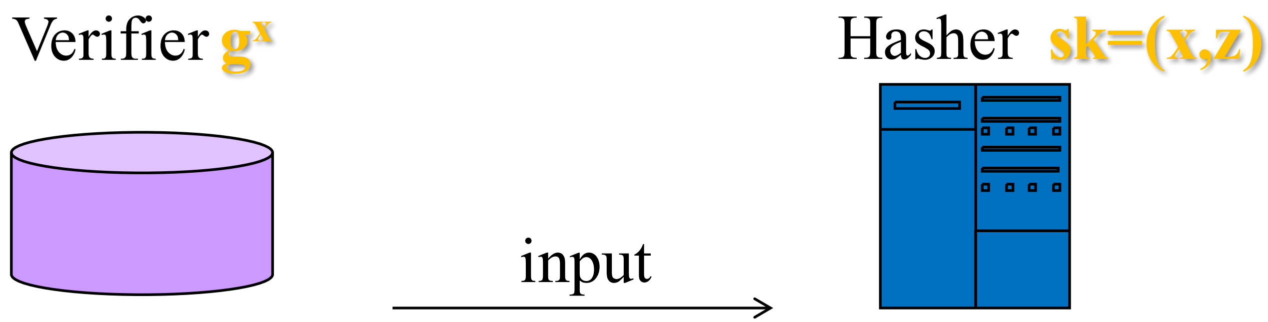
$\mathbf{c} = \mathbf{H}(\mathbf{h}, \mathbf{h}^x, \mathbf{g}^k, \mathbf{h}^k)$

$\mathbf{s} = \mathbf{k} + \mathbf{cx} \bmod \mathbf{q}$

Ciphersuite choices:

- **Curve:** Ed25519
- NEW **Hash:** SHA512
- NEW **Nonce:** Deterministic, based on RFC 8032 [EdDSA]
- **Hash-to-curve:** Try-and-increment with SHA512
- **Key generation:** Same as RFC 8032 [EdDSA]

Ciphersuite EC-VRF-ED25519-SHA512-Elligator2



$\mathbf{h}$ = hash_to_curve (suite, $\mathbf{g}^{\mathbf{x}}$, input)

nonce $\mathbf{k} =$ hash(…, $\mathbf{z}$, $\mathbf{h}$)

$\mathbf{c} = \mathbf{H}(\mathbf{h}, \mathbf{h}^{\mathbf{x}}, \mathbf{g}^{\mathbf{k}}, \mathbf{h}^{\mathbf{k}})$

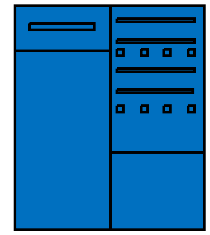
$\mathbf{s} = \mathbf{k} + \mathbf{cx} \bmod \mathbf{q}$

Ciphersuite choices:

- **Curve:** Ed25519
- **Hash:** SHA512
- **Nonce:** Deterministic, based on RFC 8032 [EdDSA]
- **Hash-to-curve:** Elligator2
- **Key generation:** Same as RFC 8032 [EdDSA]

decision: domain separation strategy

Hasher **sk=(x,z)**



Domain separation goal:
hash inputs should be distinct,
even under adversarial inputs

$\mathbf{h} = \text{hash\_to\_curve}\,(\mathbf{suite},\ \ \mathbf{g^x}, \text{input})$

nonce $\mathbf{k} = \text{hash}(\ldots, \mathbf{z}, \mathbf{h})$

$\mathbf{c} = \mathbf{H}(\mathbf{suite},\ \ \mathbf{h}, \mathbf{h^x}, \mathbf{g^k}, \mathbf{h^k})$

$\mathbf{s} = \mathbf{k} + \mathbf{cx} \bmod \mathbf{q}$

proof: $(\mathbf{h^x}, \mathbf{c}, \mathbf{s})$

$\mathbf{u} = \mathbf{g^s} / (\mathbf{g^x})^{\mathbf{c}}$

$\mathbf{h} = \text{hash\_to\_curve}\,(\mathbf{suite},\ \ \text{input})$

$\mathbf{v} = \mathbf{h^s} / (\mathbf{h^x})^{\mathbf{c}}$

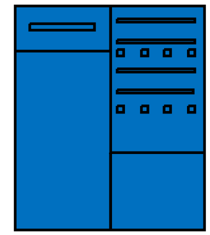
If $\mathbf{c} = \mathbf{H}(\mathbf{suite},\ \ \mathbf{h}, \mathbf{h^x}, \mathbf{u}, \mathbf{v})$

return hash($\mathbf{suite},\ \ \mathbf{h^x}$)

Else return INVALID

NEW decision: domain separation strategy

Hasher sk=(x,z)



Domain separation goal:
hash inputs should be distinct, even under adversarial inputs

Solution:
- **nonce generation uses secret z;**
- **for the other three hashes, use one-octet suite id + one-octet prefix 0x01, 0x02, or 0x03**

$\mathbf{h}$ = hash_to_curve (**suite, 1,** $\mathbf{g^x}$, input)

nonce $\mathbf{k}$ = hash(…, **z,** $\mathbf{h}$)

$\mathbf{c} = \mathbf{H}$(**suite, 2,** $\mathbf{h}$, $\mathbf{h^x}$, $\mathbf{g^k}$, $\mathbf{h^k}$)

$\mathbf{s} = \mathbf{k + cx \bmod q}$

proof: ($\mathbf{h^x}$, $\mathbf{c}$, $\mathbf{s}$)

$\mathbf{u} = \mathbf{g^s / (g^x)^c}$

$\mathbf{h}$ = hash_to_curve (**suite, 1,** input)

$\mathbf{v} = \mathbf{h^s / (h^x)^c}$

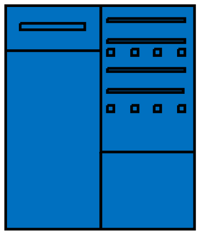
If $\mathbf{c} = \mathbf{H}$(**suite, 2,** $\mathbf{h}$, $\mathbf{h^x}$, $\mathbf{u}$, $\mathbf{v}$)

  return hash(**suite, 3,** $\mathbf{h^x}$)

Else return INVALID

decision: no prehash ciphersuites

Current design



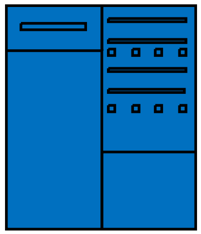
h = hash_to_curve (… , input)

nonce **k** = hash(…,z, **h**)

$\mathbf{c} = \mathbf{H}(\mathbf{h}, \mathbf{h}^{\mathbf{x}}, \mathbf{g}^{\mathbf{k}}, \mathbf{h}^{\mathbf{k}})$

$\mathbf{s} = \mathbf{k} + \mathbf{cx} \bmod \mathbf{q}$

Possible prehash suite



prehash = prehash (input)

h = hash_to_curve (…, **prehash**)

nonce **k =** hash(…. , z, **prehash**)

$\mathbf{c} = \mathbf{H}(\mathbf{prehash}, \mathbf{h}^{\mathbf{x}}, \mathbf{g}^{\mathbf{k}}, \mathbf{h}^{\mathbf{k}})$

$\mathbf{s} = \mathbf{k} + \mathbf{cx} \bmod \mathbf{q}$

our claim: hash_to_curve already acts like a prehash!

seeking feedback: ciphersuites

Specified ciphersuites:

1. EC-VRF-P256-SHA256
2. EC-VRF-ED25519-SHA512
3. EC-VRF-ED25519-SHA512-Elligator 

non-constant time

Q: Do we need all three ciphersuites?

We could easily kill #2.

seeking feedback: ED25519-SHA512-x nonce gen

Q: Do we "copy" ED25519 nonce generation from RFC8032?

Nonce generation in RFC 8032 [EdDSA]:

3 options (to support prehash and/or context)

nonce =
- hash(hash(z), **input**) mod 2^{255}-19

seeking feedback: ED25519-SHA512-x nonce gen

Q: Do we "copy" ED25519 nonce generation from RFC8032?

Nonce generation in RFC 8032 [EdDSA]:

3 options (to support prehash and/or context)

nonce =

- hash(hash(z), **input**) mod 2^{255}-19
- hash("sigEd25519 no Ed25519 collisions", **0**, ctxlen, ctx, hash(z), **input**) mod 2^{255}-19
- hash("sigEd25519 no Ed25519 collisions", **1**, ctxlen, ctx, hash(z), **prehash(input)**) mod …

seeking feedback: ED25519-SHA512-x nonce gen

Q: Do we "copy" ED25519 nonce generation from RFC8032?

Nonce generation in RFC 8032 [EdDSA]:

3 options (to support prehash and/or context)

nonce =
- hash(hash(z), **input**) mod 2^{255}-19
- hash("sigEd25519 no Ed25519 collisions", **0**, ctxlen, ctx, hash(z), **input**) mod 2^{255}-19
- hash("sigEd25519 no Ed25519 collisions", **1**, ctxlen, ctx, hash(z), **prehash(input)**) mod …

Nonce generation in our draft:

nonce = hash(hash(z), **h**) mod 2^{255}-19 (where **h** = hash_to_curve (suite, 0x01, $\mathbf{g}^x$, input))

seeking feedback: P256-SHA256 nonce gen

Q: Do we copy P256 nonce generation from RFC6979 (determistic ECDSA)?

Nonce generation in RFC 6979 uses HMAC_DRBG

pros: already implemented for deterministic ECDSA

cons: needs at least 10 applications of a hash. (slower!)

very small probability of a timing side channel

$K_1 = HMAC_0(1 ,0, z, h)$

$V_1 = HMAC_{K1}(1)$

$K_2 = HMAC_{K1}(V_1 ,1, z, h)$

timing sidechannel

$V_2 = HMAC_{K2}(V_1)$

If $V_3 = HMAC_{K2}(V_2) <$ prime, output V_3; else repeat this step.

ALTERNATIVE:

Use SHA512 in this suite with ED25519-style nonce gen.

seeking feedback: nits

Q: We use exponential notation. Switch to multiplicative?

h^x **vs** xH

Q: We do not support "contexts". Should we?

h = hash_to_curve (suite, g^x, input)

vs

h = hash_to_curve (suite, g^x, **contextlen, context,** input,)

Q: Take the "first n octets" or the "last n octets" of a hash?
Also: terminology: "first" vs "leftmost"?

Q: Do we add domain separation, context to the RSA VRF?
Easy for us to copy from EC VRF.