Pairing-Friendly Curves

draft-yonezawa-pairing-friendly-curves

Shoko YONEZAWA, Tsunekazu SAITO, Tetsutaro KOBAYASHI

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Brief Overview

- Problem statement
  - Pairing-based cryptography is getting widely used
  - The security evaluation of pairing-friendly curves, which realize pairing-based cryptography, has been changed due to the attack proposed in 2016
  - Introducing secure pairing-friendly curves are required
- Goal
  - Show the latest security evaluation of well-known pairing-friendly curves
  - Show the parameters of pairing-friendly curves with each security level
    - According to their security evaluations in several papers and implementation status in several libraries

Related RG Items

- BLS Signature Schemes (draft-boneh-bls-signature, to appear as draft-irtf-bls-signature)
  - Pairing-based schemes that enable signature aggregation
  - Pairing-friendly curves are necessary for construction
- Hashing to Elliptic Curves (draft-irtf-cfrg-hash-to-curve)
  - Most pairing-based schemes (including BLS signatures) require hashing to pairing-friendly curves

Pairing-Based Cryptography

- A kind of elliptic curve cryptography which utilizes “pairing”
- Thanks to the property of pairing, cryptographic algorithms and protocols with more functionalities are getting widely used

Standards

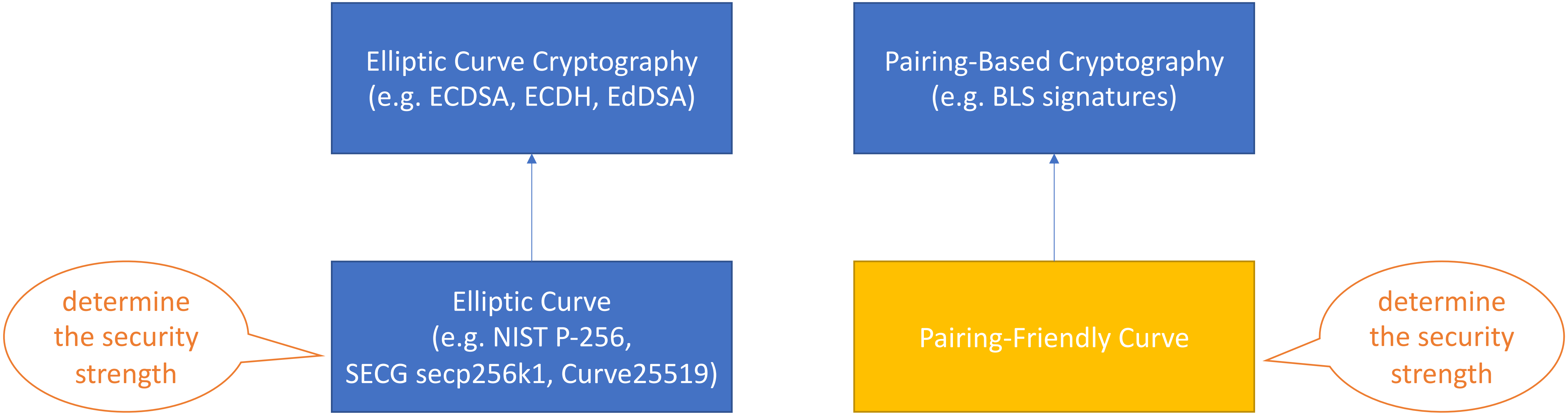
- Identity-based cryptography (IBCS) [RFC5091]
- Sakai-Kasahara Key Encryption (SAKKE) [RFC6508]
- Identity-based authenticated key exchange (IBAKE) [RFC6539]
- (Identity-based) key agreement (ISO/IEC)
- Elliptic Curve Direct Anonymous Attestation (ECDAA) (TCG, FIDO, W3C)
- MIKEY-SAKKE (3GPP) – key encryption

Implementations

- M-Pin (MIRACL) – multi-factor authentication protocol
- Intel SGX EPID (Intel) – remote anonymous attestation protocol
- Geo Key Manager (Cloudflare) – attribute-based encryption
- zk-SNARKs (Zcash) – zero-knowledge proof for blockchain
- Decentralized Random Beacon (DFINITY) – threshold signature
- BLS signature (Algorand) – aggregate signature

Pairing-Based Cryptography (cont.)

- Like standard elliptic curve cryptography, pairing-based cryptography requires underlying elliptic curves
- Such elliptic curves are called pairing-friendly curves
- The security of pairing-based cryptography relies on the security of underlying pairing-friendly curves



Pairing

- Pairing (a.k.a. bilinear map) is a map from G_1 and G_2 onto G_T

$$e: G_1 \times G_2 \ \rightarrow \ G_T$$

  satisfying

$$e([a]S, [b]T) = \ e(S, T)^{ab}.$$

- In general, G_1, G_2 and G_T are chosen as follows.
  - G_1 : a subgroup of the group defined over an elliptic curve E
  - G_2 : a subgroup of the group defined over a twisted curve of E
  - G_T : a multiplicative group of finite field
- Various pairings
  - Weil pairing
  - Tate pairing
  - Optimal Ate pairing ← most efficient and popular

Pairing-Friendly Curves

- A special kind of elliptic curves where pairing is efficiently computable
- Examples curves
  - Barreto-Naehrig (BN) Curve
  - Barreto-Lynn-Scott (BLS) Curve
    - BLS12 (embedded degree = 12)
    - BLS24 (embedded degree = 24)
    - BLS48 (embedded degree = 48), etc.
  - Kachisa-Schaefer-Scott (KSS) Curve
  - Miyaji-Nakabayashi-Takano (MNT) Curve
  - etc.
- Pairing-friendly curves vary in parameters (key length), which determine the security strength
  - ex. BN254, BN256, BLS12-381, ...

Security of Pairing-Friendly Curves

$$e : G_1 \times G_2 \rightarrow G_T$$

Elliptic Curve Discrete Logarithm Problem (ECDLP)

Elliptic Curve Discrete Logarithm Problem (ECDLP)

Finite Field Discrete Logarithm Problem (FFDLP)

- Since the security of most pairing-based cryptography is reduced to the difficulty of these problems, we can only consider these DLPs.
- We should evaluate FFDLP in G_T as well as ECDLP in G_1 and G_2

Impact of Attack to Pairing-friendly Curves

- In 2016, Kim and Barbulescu presented a new number field sieve algorithm, exTNFS, at CRYPTO 2016 [KB16]
- Attacking by exTNFS affected the difficulty of FFDLP
- Due to the attack, the security level of ALL pairing-friendly curves has fallen
  - ex. BN256: 128-bit secure → 100-bit secure

[KB16] T. Kim and R. Barbulescu, Extended Tower Number Field Sieve: A New Complexity for the Medium Prime Case," CRYPTO 2016.

Security Evaluation of Pairing-Friendly Curves

- After exTNFS, BN256 (regarded as 128-bit secure so far) achieves at most 100 bits of security now
- Introducing new parameters for each security level is required
  - 128 bits of security
  - 192 bits of security
  - 256 bits of security
- We select the parameters in terms of
  - Security
  - Efficiency
  - Wide use

128 / 256 Bits of Security

- 128 bits
  - BN462
    - Evaluated as approx. 133.49 bits of security [BD18] – conservative
    - Implementation available
  - BLS12-381
    - Evaluated as approx. 117 - 120 bits of security [NCCG] – optimistic
    - Implementation available and widely used
- 256 bits
  - BLS48-581
    - Evaluated as approx. 256 bits of security [KIK+17]
    - Implementation available

[BD18] R. Barbulescu and S. Duquesne, “Updating Key Size Estimations for Pairings,” Journal of Cryptology, 2018.
[NCCG] NCC Group, “Zcash Overwinter Consensus and Sapling Cryptography Review,”
https://www.nccgroup.trust/us/our-research/zcash-overwinter-consensus-and-sapling-cryptography-review/
[KIK+17] Y. Kiyomura et al. “Secure and Efficient Pairing at 256-Bit Security Level,” ACNS 2017.

Open Issue : 192 Bits of Security

- Candidate curve : BLS24
- Several papers for 192bit-secure pairing-friendly curves
- NO implementation published
  - RELIC – preparing BLS24-477 but no executable code
  - AMCL – implementing BLS24 curve but not published

- QUESTION: How can we treat 192-bit parameters ?

Fact: 192 Bits of Security

- US CNSA Suite
  - In order to protect up to TOP SECRET, the security parameters for asymmetric cryptography are set to satisfy 192 bits of security.

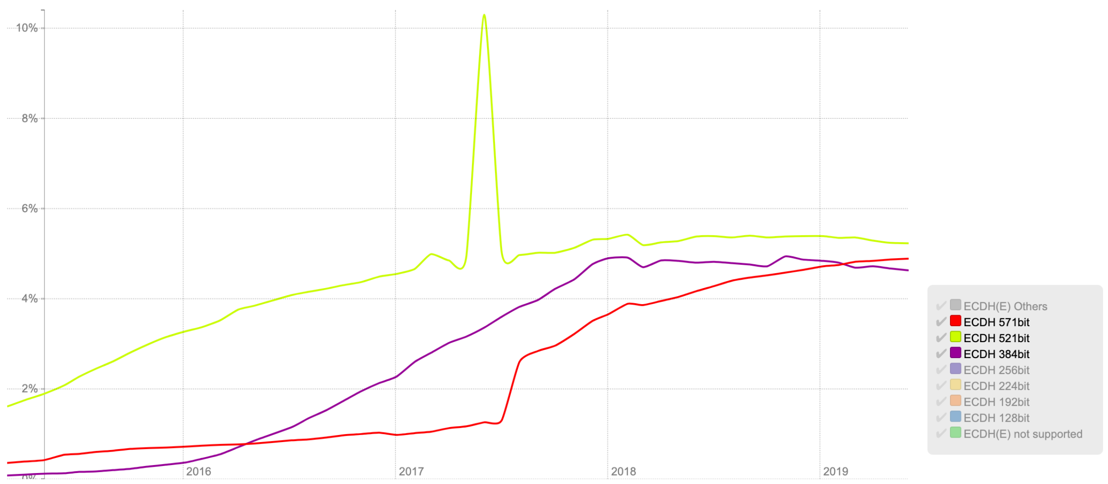
Transition Algorithms

Algorithm	Function	Specification	Parameters
Advanced Encryption Standard (AES)	Symmetric block cipher used for information protection	FIPS Pub 197	Use 256 bit keys to protect up to TOP SECRET
Elliptic Curve Diffie-Hellman (ECDH) Key Exchange	Asymmetric algorithm used for key establishment	NIST SP 800-56A	Use Curve P-384 to protect up to TOP SECRET.
Elliptic Curve Digital Signature Algorithm (ECDSA)	Asymmetric algorithm used for digital signatures	FIPS Pub 186-4	Use Curve P-384 to protect up to TOP SECRET.
Secure Hash Algorithm (SHA)	Algorithm used for computing a condensed representation of information	FIPS Pub 180-4	Use SHA-384 to protect up to TOP SECRET.
Diffie-Hellman (DH) Key Exchange	Asymmetric algorithm used for key establishment	IETF RFC 3526	Minimum 3072-bit modulus to protect up to TOP SECRET
RSA	Asymmetric algorithm used for key establishment	NIST SP 800-56B rev 1	Minimum 3072-bit modulus to protect up to TOP SECRET
RSA	Asymmetric algorithm used for digital signatures	FIPS PUB 186-4	Minimum 3072 bit-modulus to protect up to TOP SECRET.

https://apps.nsa.gov/iaarchive/programs/iad-initiatives/cnsa-suite.cfm

- SSL Pulse Trends (June 2019)
  - As for the key length of ECDH(E) in TLS servers, 5.23% of the servers supports 521bit, 4.89% supports 571bit while 4.63% supports 381bit.

Key Size for ECDH(E) Key Exchange



https://kjur.github.io/www/sslpulsetrend/index.html#kxecdh

History and Next Steps

- 00 version
  - Initial submission
- 01 version
  - Added pseudo-codes for pairing computation (from Kenny)
  - Added example parameters and test vectors of each curve (from Kenny)
- 02 version
  - Added 192 bits of security (no parameter provided yet) (from John)
  - Resolved comments from ML (from Mike, David, Marek and John)
  - Updated the status on applications and libraries

- Next Steps
  - Adoption call if interested