ristretto255 & decaf448

draft-irtf-cfrg-ristretto255-decaf448

what problem are we solving?

2.4 Notation

Let $\mathbb{G}$ denote a cyclic group of prime order p,

what kind of elliptic curve?

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prime order	✅	❌
fastest formulas	❌	✅
complete formulas	😭	✅
easy in constant time	❌	✅

okay, it’s only a small conceptual mismatch…

…so what’s wrong with
a small cofactor?

well…

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ad-hoc protocol tweaks are specific to each protocol

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e.g., rfc8032 does not require ed25519 implementations to agree on whether a signature is valid

- different behaviour between batch, single verification
- very bad for applications involving consensus
- also incompatible with hierarchical key derivation

…or catastrophic failures

how do we fix this mismatch?

decaf & ristretto

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“batteries included”: a single hash-to-group method how does this work?

three families of curves

montgomery curves

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require few constraints in circuits

edwards curves

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$$\mathcal{E}_{a,d} : \quad ax^2 + y^2 = 1 + dx^2y^2$$

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formulas allow parallelism inside a curve operation

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easy to write down 4 points of order 2

we can efficiently encode $(s,t) \bmod \mathcal{J}[2]$

linking curves with isogenies

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$$\begin{tikzcd}
\mathcal{E}_{a,d} \arrow[dr, dash] & & \mathcal{E}_{-a,d-a} \arrow[dl, dash] \\
& \mathcal{J}_{a^2,a-2d} \arrow[d, dash] & \\
& \mathcal{M}_{a,2-4d/a} \arrow[r, dashed, dash] & \mathcal{E}_{a',d'}
\end{tikzcd}$$

encoding with isogenies

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specify an encoding on the jacobi quartic

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isogenies “transport” the encoding to other curve shapes

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extra step to handle cofactor 8 instead of cofactor 4

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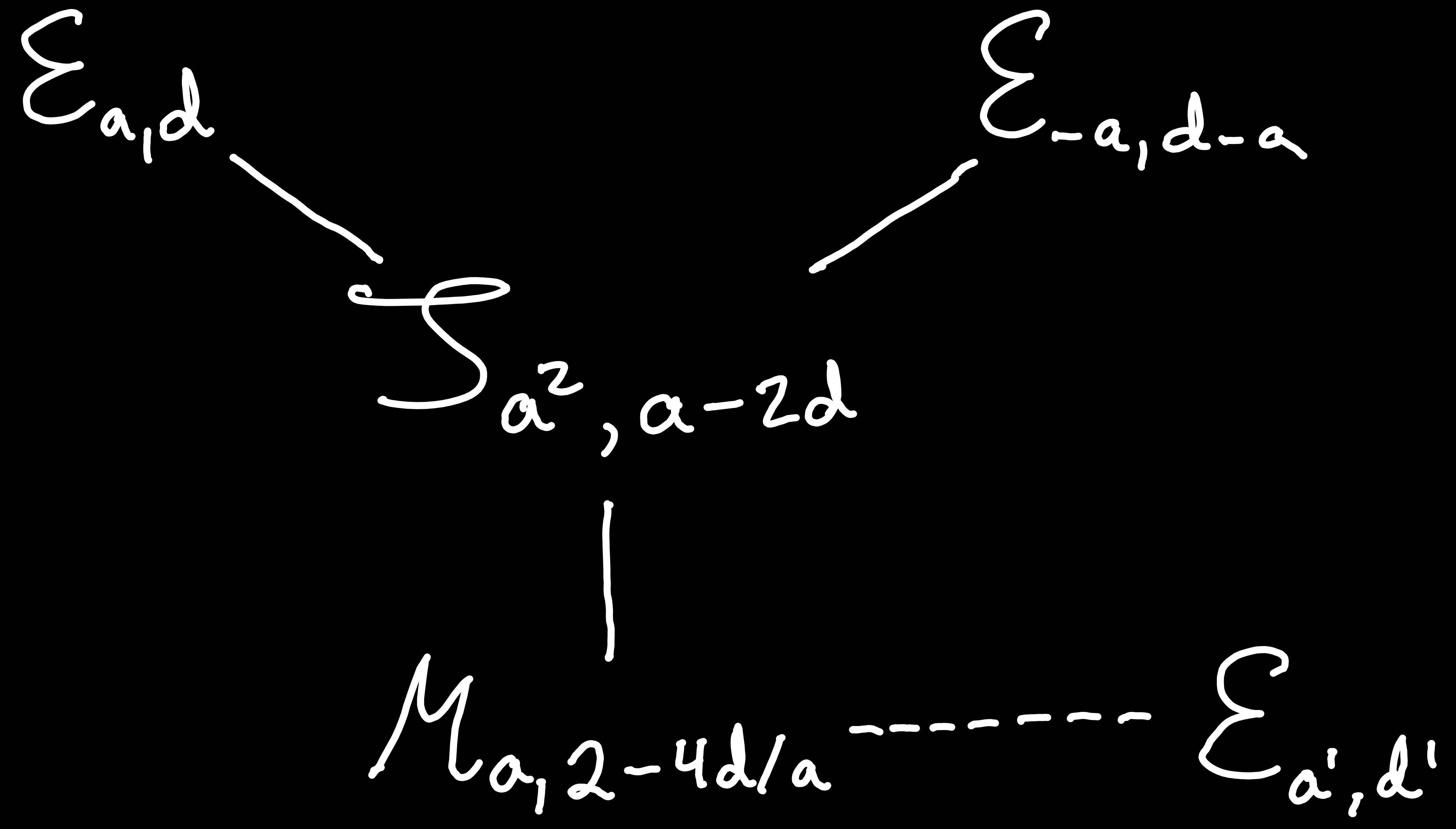
ristretto transports the encoding to edwards via montgomery + cofactor 8



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either can use any curve internally

$$
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$$

concrete parameterizations

ristretto255

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can be implemented using curve25519

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~128-bit security level

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~128-bit security level

increasing adoption: zk proofs, psi, pake, etc

decaf448

decaf448

can be implemented using edwards448

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~224-bit security level

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suitable where ed448 would be used

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no outstanding issues, ready to go