

Computational analysis of EDHOC Stat-Stat

HYPOTHESIS

Notations

- $\Pi = (\mathcal{E}, \mathcal{D})$: One-Time Encryption scheme
- $\Pi' = (\mathcal{E}', \mathcal{D}')$: Authenticated Encryption scheme
- \mathcal{A} : Adversary

- \mathbb{G} : a cyclic group of order p

- sk_i : symmetric keys (denoted as k_i in the specification)
- x_s : Initiator long-term key (denoted as I in the specification)

Diffie-Hellman

- Computational Diffie-Hellman (CDH):
 - Given $(g^u, g^v) \in \mathbb{G}$, $(u, v) \leftarrow^{\$} \mathbb{Z}_p$, compute g^{uv}
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- Best attack : Baby-Step Giant-Step, $\mathcal{O}(\sqrt{p})$

Symmetric Encryption – One-Time Pad

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- One-Time indistinguishability:
 - $\mathcal{E}(k, m_0)$ and $\mathcal{E}(k, m_1)$ are indistinguishable
 - Notation: $\text{Adv}_{\Pi}^{\text{OT-ind}}(t)$

Symmetric Encryption – AEAD

- Indistinguishability:
 - Given access to an encryption and decryption oracles \mathcal{E} and \mathcal{D}
 - $\mathcal{E}'(k, m_0) \cong \mathcal{E}'(k, m_1)$ for a random $k \in \mathcal{K}$,
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 - Notation: $Adv_{\Pi'}^{ind}(t)$
- Unforgeability:
 - Given access to an encryption and decryption oracles \mathcal{E} and \mathcal{D}
 - Generate a valid ciphertext
 - Notation: $Adv_{\Pi'}^{uf-cma}(t)$

RESULTS

Notations

- q_{RO} : global number of queries to the random oracles
- n_{σ} : number of running sessions
- N : number of users
- ℓ_{hash} : hash digest length
- ℓ_{MAC} : MAC digest length

Key Privacy

- **Theorem 1:** under the Gap Diffie-Hellman problem in the Random Oracle model, and the injectivity of $(\mathcal{E}, \mathcal{D})$:

$$Adv_{EDHOC}^{kp-ake}(t; q_{RO}, n_{\sigma}, N) \leq Adv_{\mathbb{G}}^{GDH}(t, n_{\sigma} \cdot q_{RO}) + 2N \cdot Adv_{\mathbb{G}}^{GDH}(t, q_{RO}) + \frac{q_{RO}^2 + 4}{2^{\ell_{hash} + 1}}$$

- Best attack:
 - Baby-step Giant-step and the Birthday Paradox
 - $\mathcal{O}(\sqrt{p} + 2^{\ell_{hash}/2})$

Explicit Authentication – Responder

- **Theorem 2:** under the Gap Diffie-Hellman problem in the Random Oracle model, and the injectivity of $\Pi = (\mathcal{E}, \mathcal{D})$:

$$\begin{aligned} Adv_{EDHOC}^{auth-resp}(t; q_{RO}, n_\sigma, N) &\leq Adv_{\mathbb{G}}^{GDH}(t, n_\sigma \cdot q_{RO}) + 2N \cdot Adv_{\mathbb{G}}^{GDH}(t, q_{RO}) + \\ &+ \frac{q_{RO}^2 + 2}{2^{\ell_{hash} + 1}} + \frac{1}{2^{\ell_{MAC}}} \end{aligned}$$

- Best attack:
 - Guess the tag t_2
 - $\mathcal{O}(2^{\ell_{MAC}})$

Explicit Authentication – Initiator

- **Theorem 3:** under the Gap Diffie-Hellman problem in the Random Oracle model, and the injectivity of $\Pi = (\mathcal{E}, \mathcal{D})$.

$$\begin{aligned} Adv_{EDHOC}^{auth-init}(t; q_{RO}, n_{\sigma}, N) &\leq Adv_{\mathbb{G}}^{GDH}(t, n_{\sigma} \cdot q_{RO}) + 2N \cdot Adv_{\mathbb{G}}^{GDH}(t, q_{RO}) + \\ &+ \frac{q_{RO}^2 + 4}{2^{\ell_{hash} + 1}} + \frac{1}{2^{\ell_{MAC}}} \end{aligned}$$

- Best attack:
 - Guess the tag t_3
 - $\mathcal{O}(2^{\ell_{MAC}})$
 - **No security provided by sk_3 as any imposter can compute it.**

Identity Protection - Responder

- **Theorem 4:** under the Gap Diffie-Hellman problem in the Random Oracle model, the injectivity and the semantic security of $\Pi = (\mathcal{E}, \mathcal{D})$.

$$\begin{aligned} Adv_{EDHOC}^{IdP-resp}(t; q_{RO}, n_\sigma, N) &\leq Adv_{\mathbb{G}}^{GDH}(t, n_\sigma \cdot q_{RO}) + 2N \cdot Adv_{\mathbb{G}}^{GDH}(t, q_{RO}) + \\ &+ \frac{q_{RO}^2 + 2}{2^{\ell_{hash} + 1}} + Adv_{\Pi}^{ind}(t) \end{aligned}$$

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Identity Protection - Initiator

- **Theorem 5:** under the Gap Diffie-Hellman problem in the Random Oracle model, the injectivity and the semantic security of $\Pi' = (\mathcal{E}, \mathcal{D})$.

$$\begin{aligned} Adv_{EDHOC}^{IdP-init}(t; q_{RO}, n_\sigma, N) &\leq Adv_{\mathbb{G}}^{GDH}(t, n_\sigma \cdot q_{RO}) + 2N \cdot Adv_{\mathbb{G}}^{GDH}(t, q_{RO}) + \\ &+ \frac{q_{RO}^2 + 2}{2^{\ell_{hash} + 1}} + Adv_{\Pi'}^{ind}(t) \end{aligned}$$

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Summary

- Considering Cipher Suites 0 and 2:

AEAD	Hash	MAC length
AES-CCM-16-64-128	SHA-256 (256 bits digest)	64

- **Key Privacy** : \approx 128 bits security
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- **Key Privacy** : \approx 128 bits security
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 - **Identity Protection** : \approx 128 bits security each
- Unuse of the unforgeability of Π'
 - sk_3 is independant of x_s
 - Can be computed by an impostor

IMPROVEMENTS

Improved message_3 - $\ell_{MAC} = 128$

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- **Impacts:**

- Same security (Key Privacy, Mutual Authentication, Identity Protection)

- Shorter message : $\text{len}(\text{CIPHERTEXT}_3) = \text{len}(\text{PLAINTEXT}_3) \leq 128 \times \left(\left\lfloor \frac{\text{len}(\text{PLAINTEXT}_3 - 1)}{128} \right\rfloor + 1 \right)$

Number of blocks
to encrypt with Π'



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- **Impacts:**

- Increased Initiator Authentication Security (+ around 64 bits thanks to the unforgeability of Π' with sk_4)
- If $\text{len}(ID_I || EAD_3) \bmod 128 \leq 64$ bits, longer message_3
- Otherwise, shorter message_3

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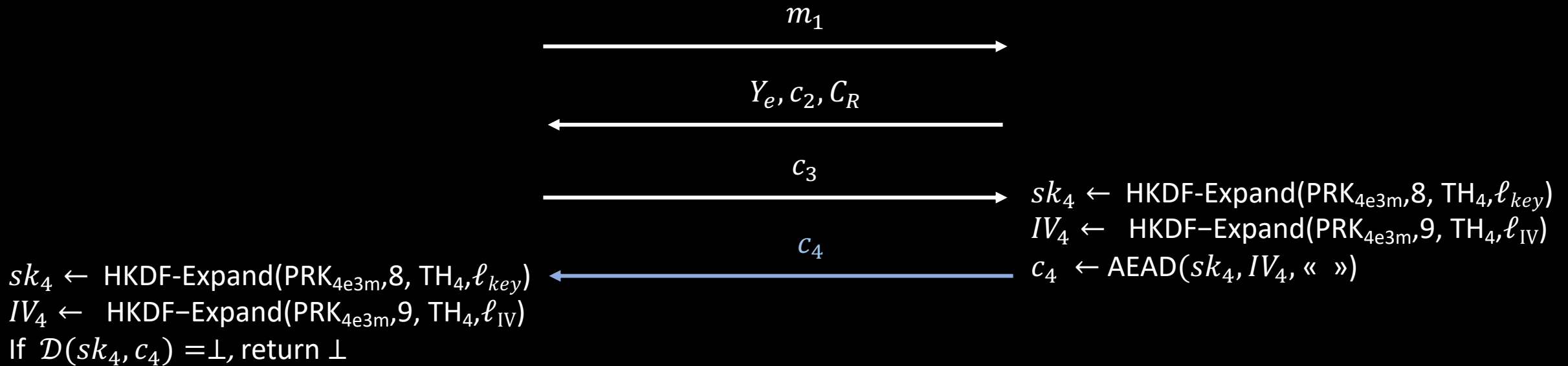
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- Similar idea can be used for message_2

Better explicit authentication for the responder

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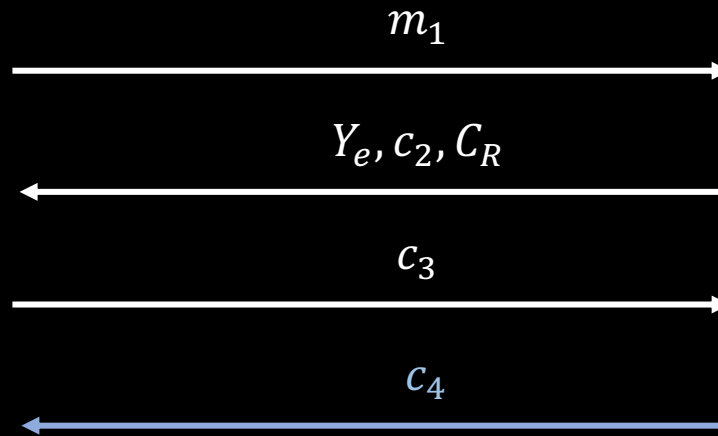
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Depends of y_s

$sk_4 \leftarrow \text{HKDF-Expand}(\text{PRK}_{4e3m}, 8, \text{TH}_4, \ell_{key})$
 $IV_4 \leftarrow \text{HKDF-Expand}(\text{PRK}_{4e3m}, 9, \text{TH}_4, \ell_{IV})$
 $c_4 \leftarrow \text{AEAD}(sk_4, IV_4, \ll \gg)$

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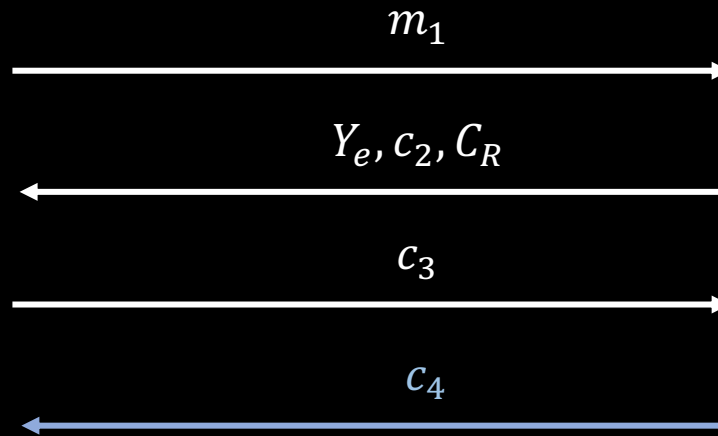
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- Advantage multiplied by $Adv_{\Pi}^{uf-cma}(t) \approx 2^{-64}$
- Responder authentication security ≈ 128 bits

Improved Key Schedule

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- **Idea:** Use TH_2 instead
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- **Impacts:**
 - No extra cost
 - Advantage becomes $\text{Adv}_{\mathbb{G}}^{\text{GDH}}(t, q_{\text{RO}})$
 - Security independant of n_σ .

Thanks!