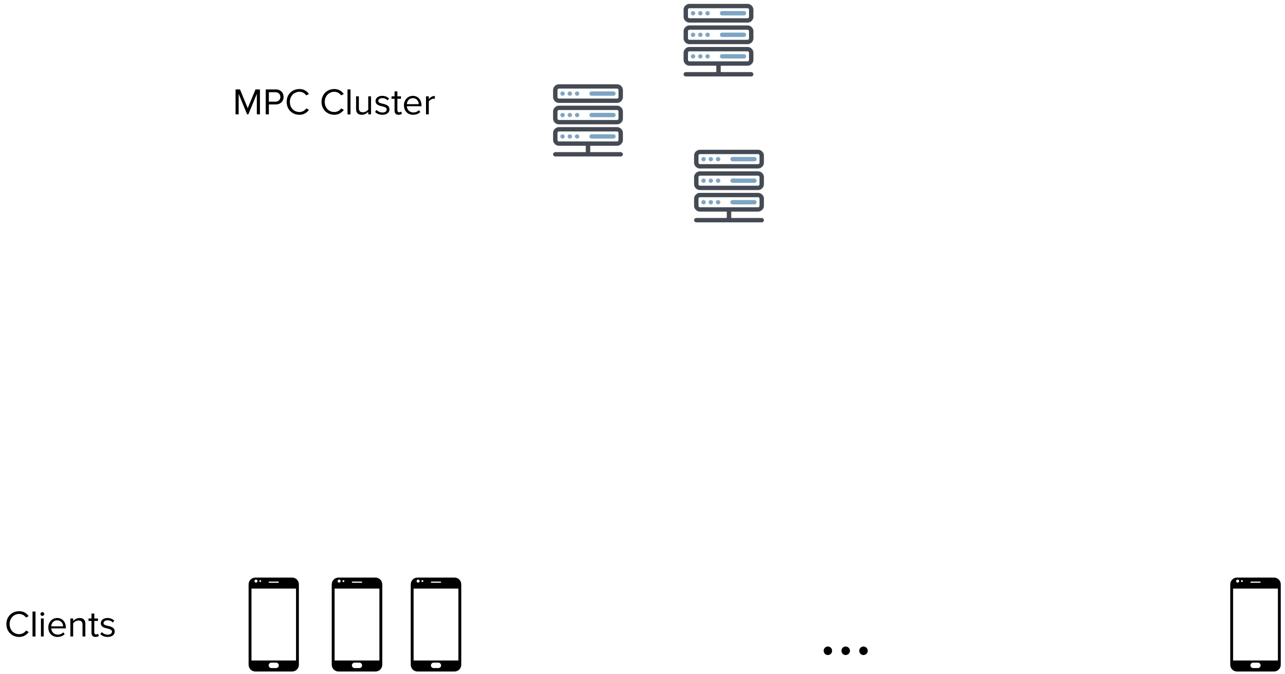
Secure Partitioning Protocols

Phillipp Schoppmann – IETF 116

Background: Aggregate Statistics Measurements



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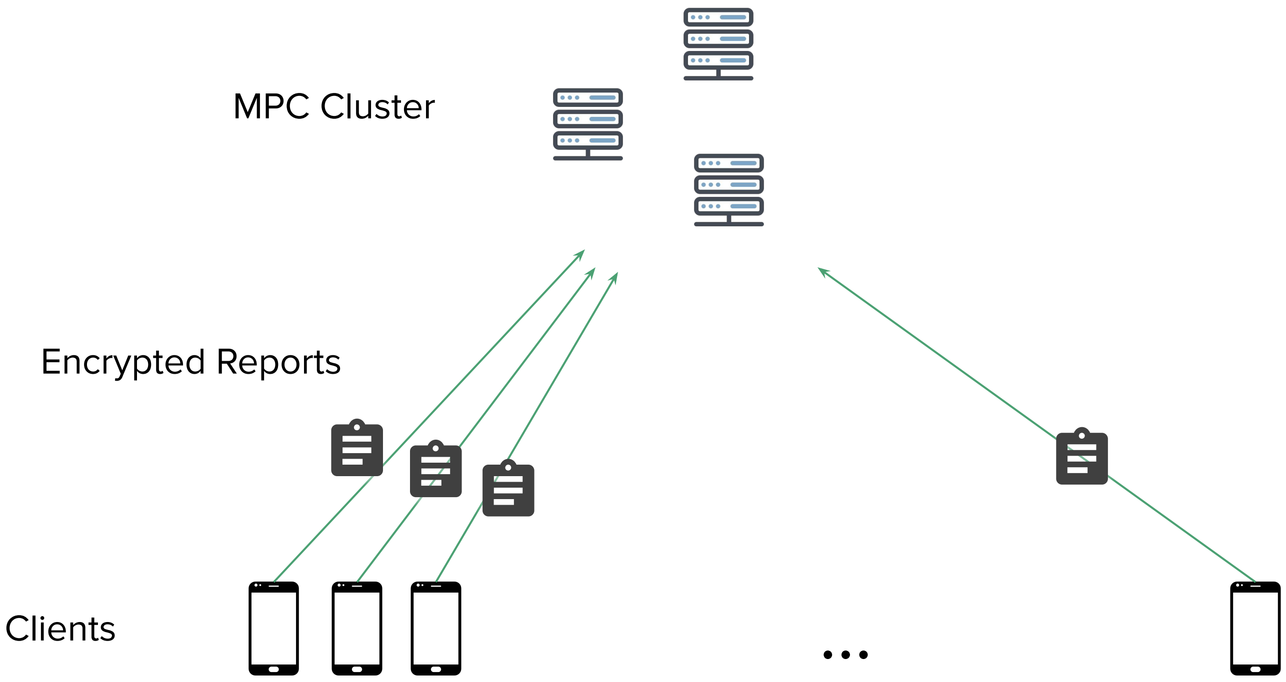

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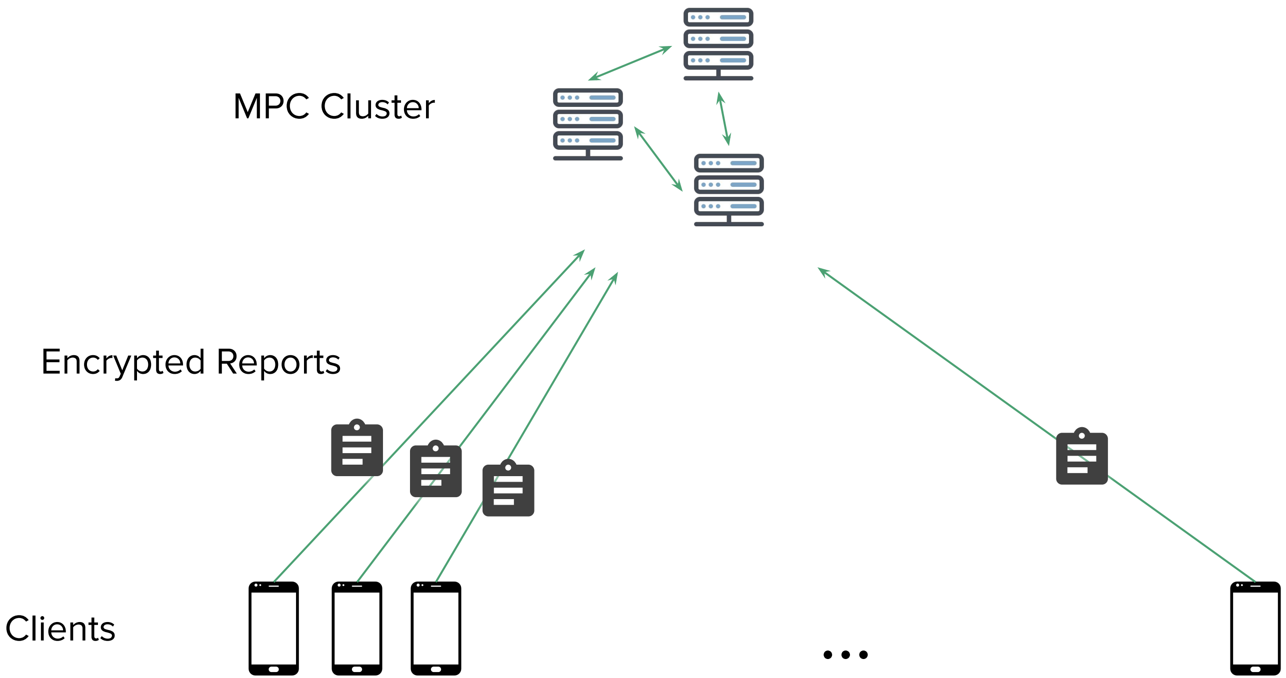

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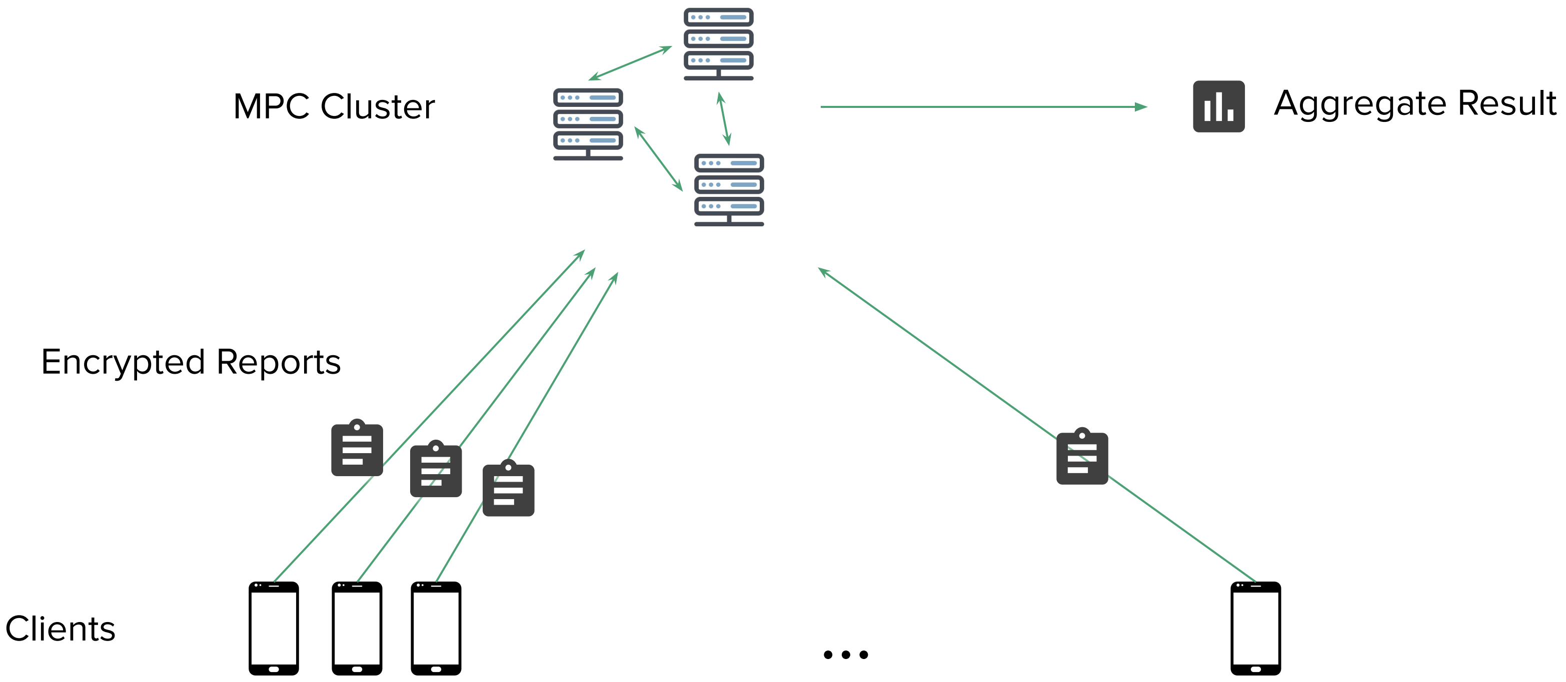
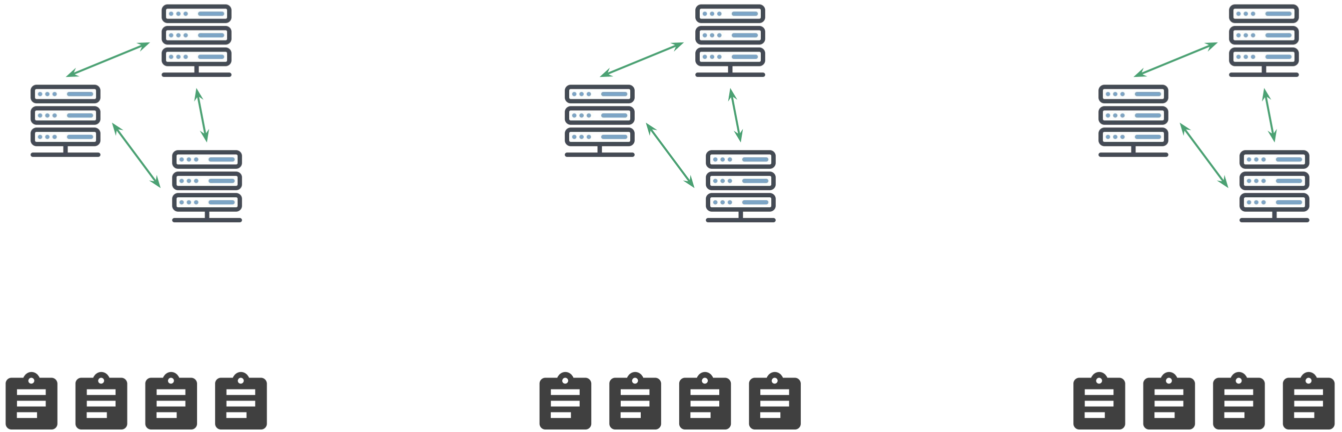


Sharding MPC Clusters



Challenge: How to partition reports across shards, s.t. all reports of the same client end up in the same shard?

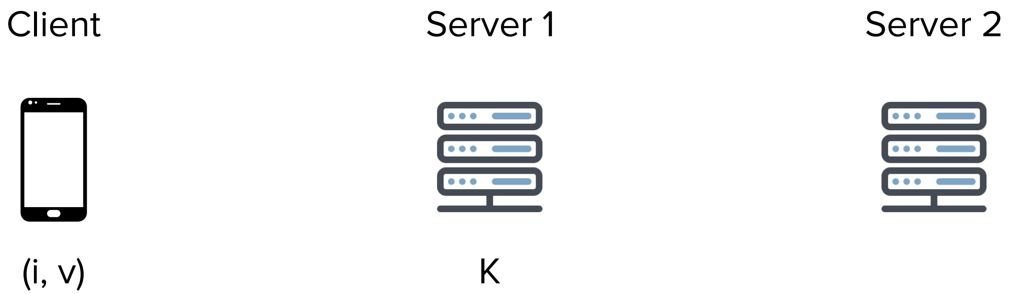
Goals

- Inputs from the same client end up in the same shard
- Low communication overhead and round complexity
- Partitioning must not affect correctness / utility of downstream computation

Assumptions

- Bound M on the number of contributions per client
- Lots of clients (billions), few shards (thousands)

Blueprint: Partitioning from Distributed OPRFs



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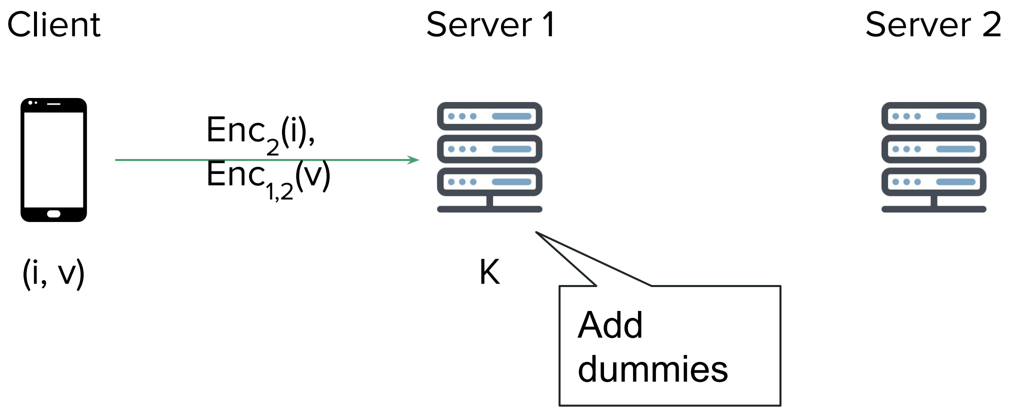

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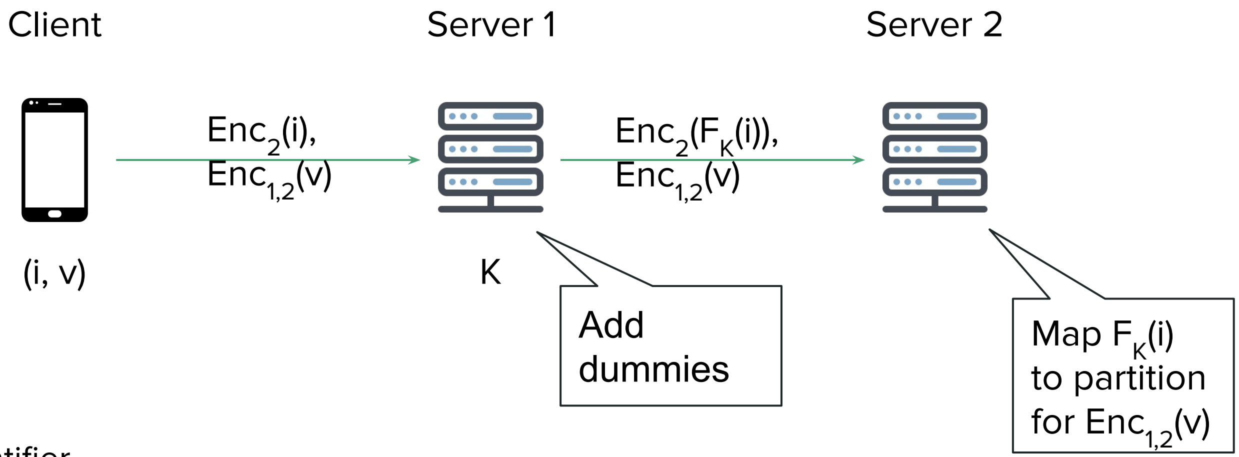

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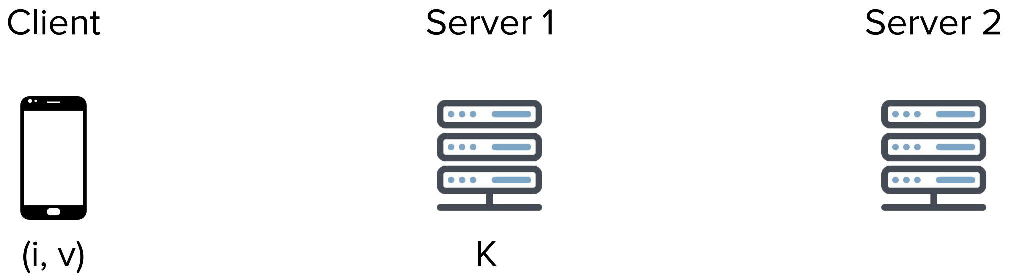
Dense Partitioning: OPRF Output = Shard ID

Assume there are exactly S shards, and let [S] be the range of F_K.



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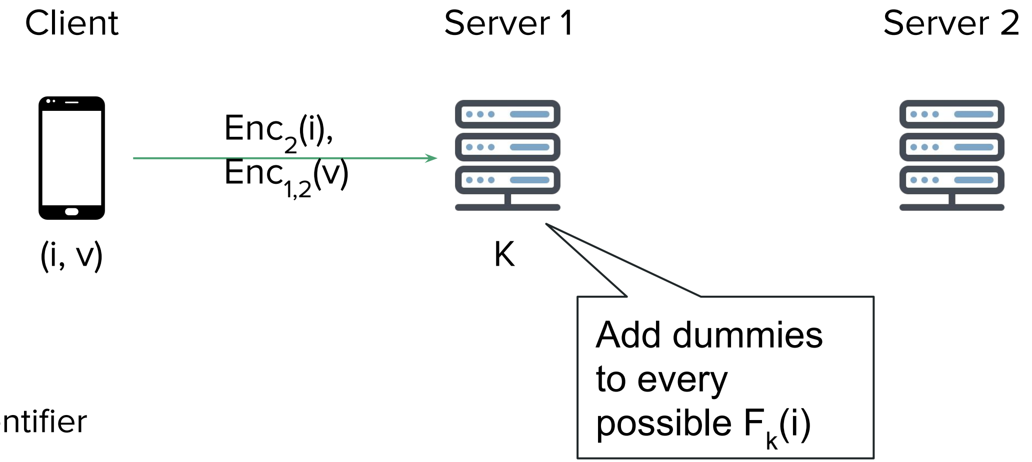

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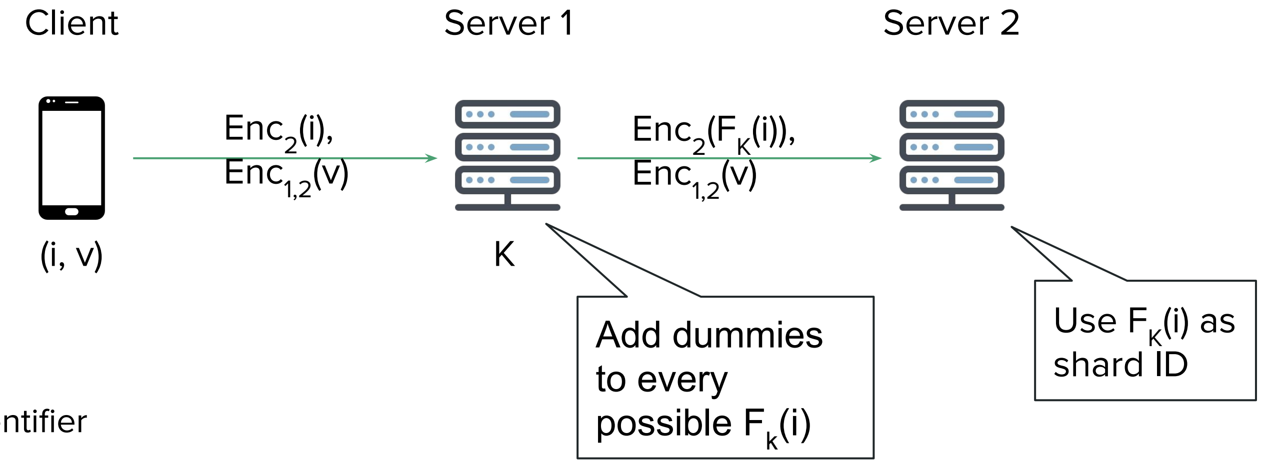

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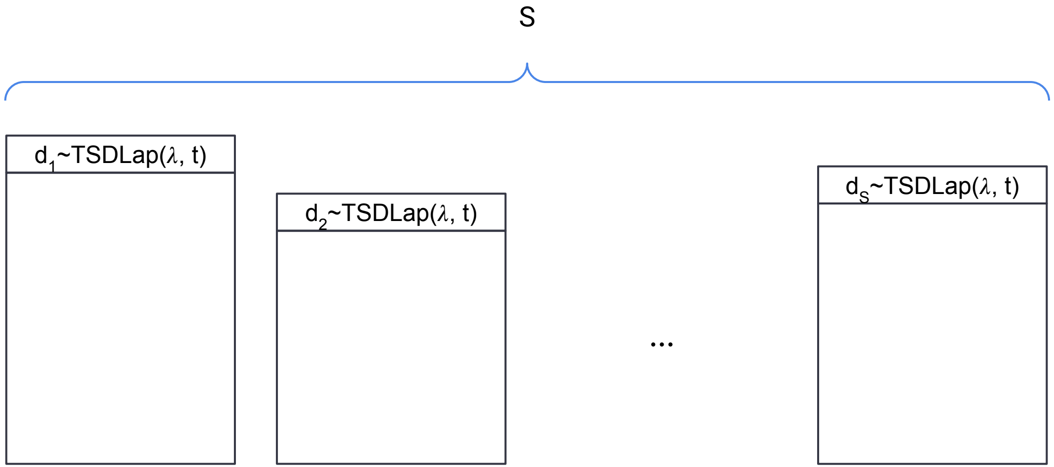
Dense Partitioning: Adding Dummies

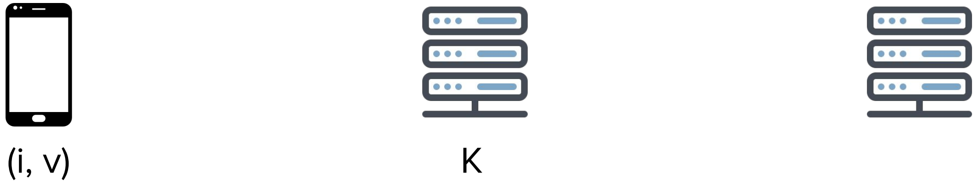
M: Upper bound on the number of ciphertexts with the same index / from the same client
S: Number of shards
TSDLap(λ, t): Truncated, shifted, discrete Laplace distribution with mean t and scale λ

Expected #dummies per bucket for $\varepsilon = 0.5$ and $\delta = 10^{-11}$: 49M per server



Sparse Partitioning: OPRF Output = Random Client ID

- If the OPRF codomain is large enough to make collisions unlikely, we can use the OPRF outputs as a pseudorandom client identifier.
- Allows *local* per-client aggregation (e.g., using Homomorphic Encryption)



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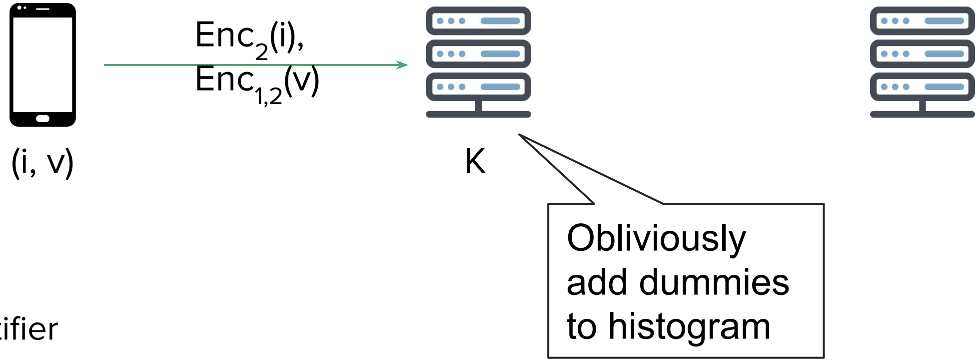

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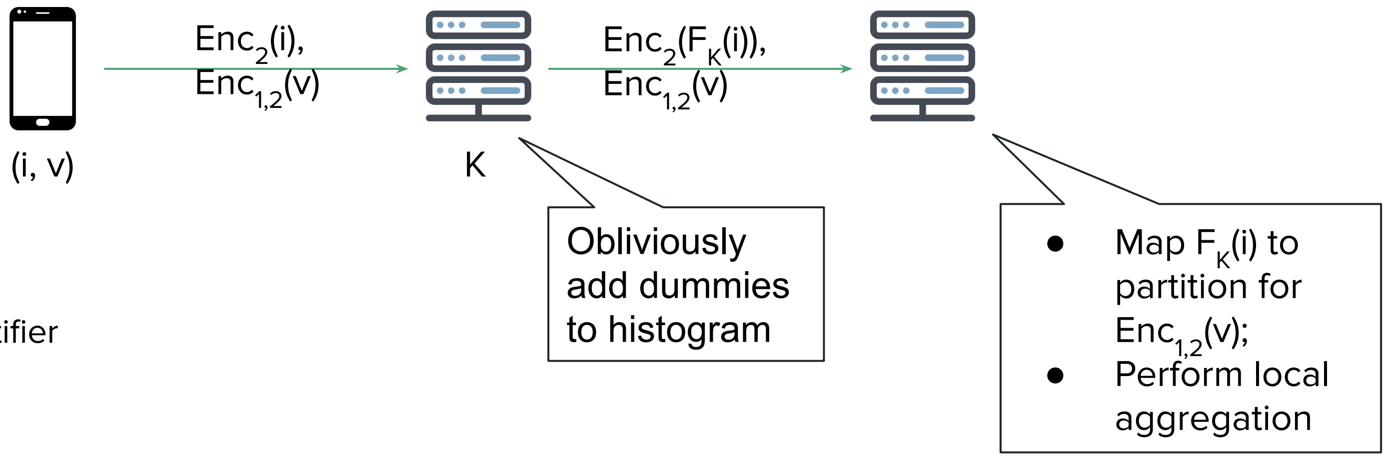

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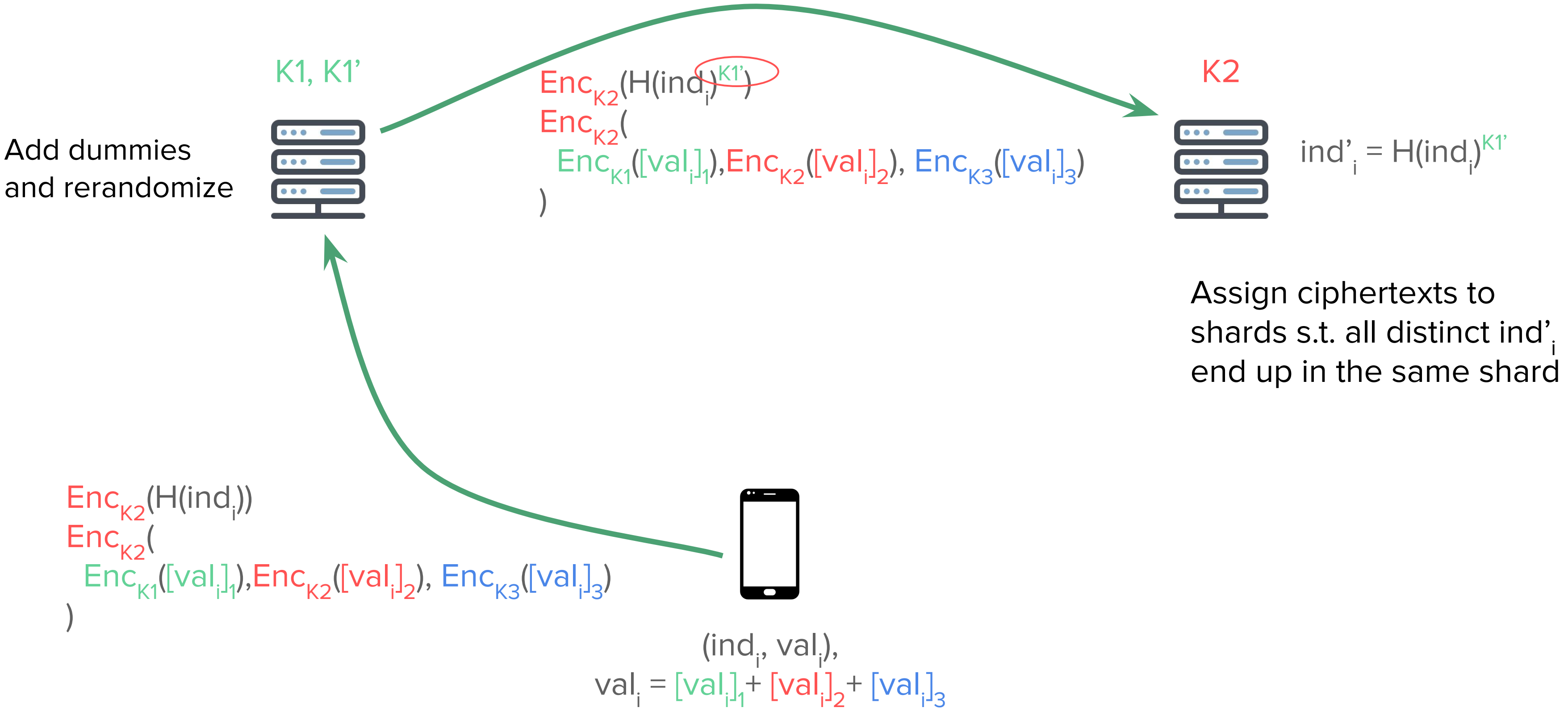
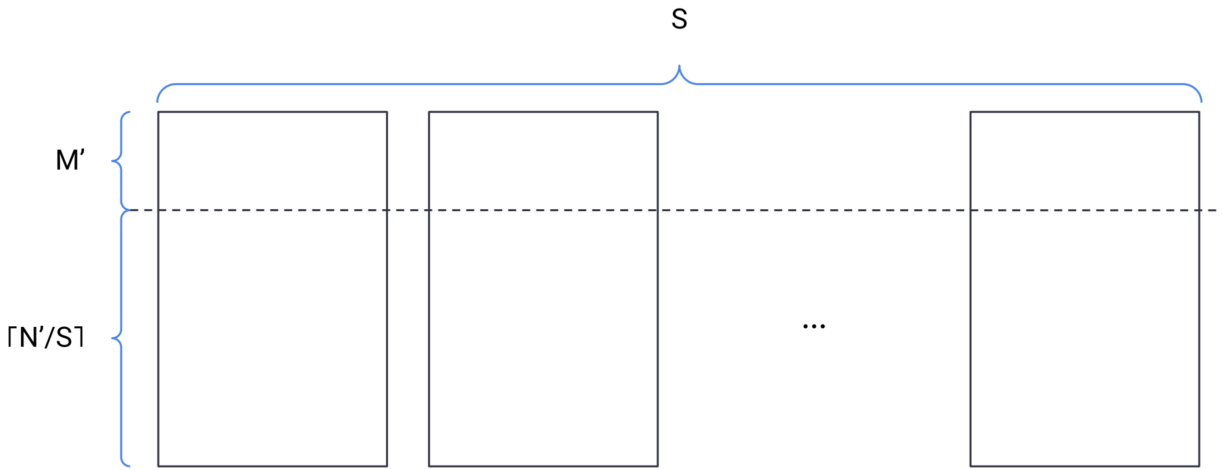
Sparse Partitioning Protocol



Assigning Ciphertexts to Shards

N’: Number of ciphertexts after adding dummies
M’: Upper bound on the number of ciphertexts with the same index
S: Number of shards

Observation: As long as M’ << ⌈N’/S⌉, the overhead will be small in practice.
But: N’ might still be significantly larger than N. For $\varepsilon = 0.5$ and $\delta = 10^{-11}$, $N'/N = 1.1$



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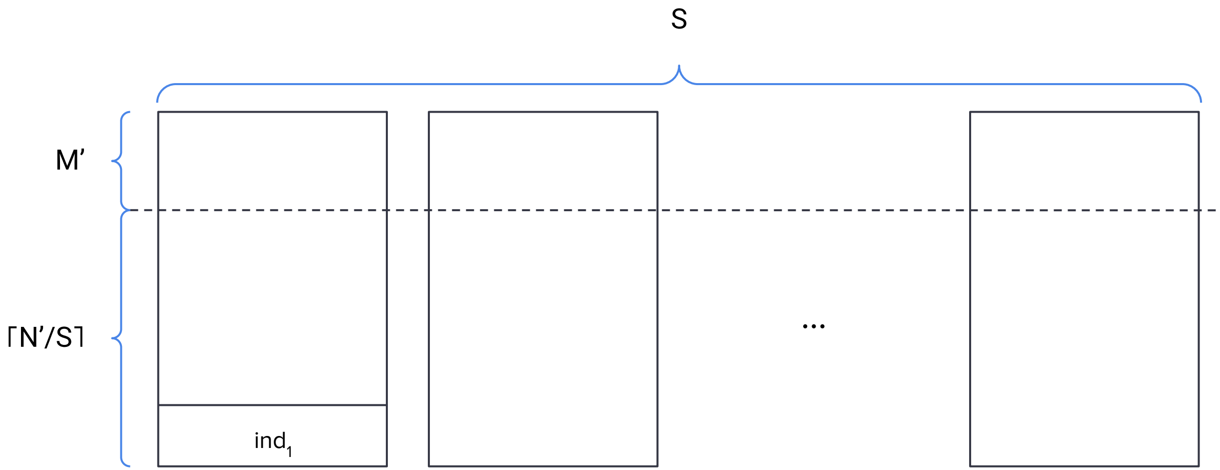

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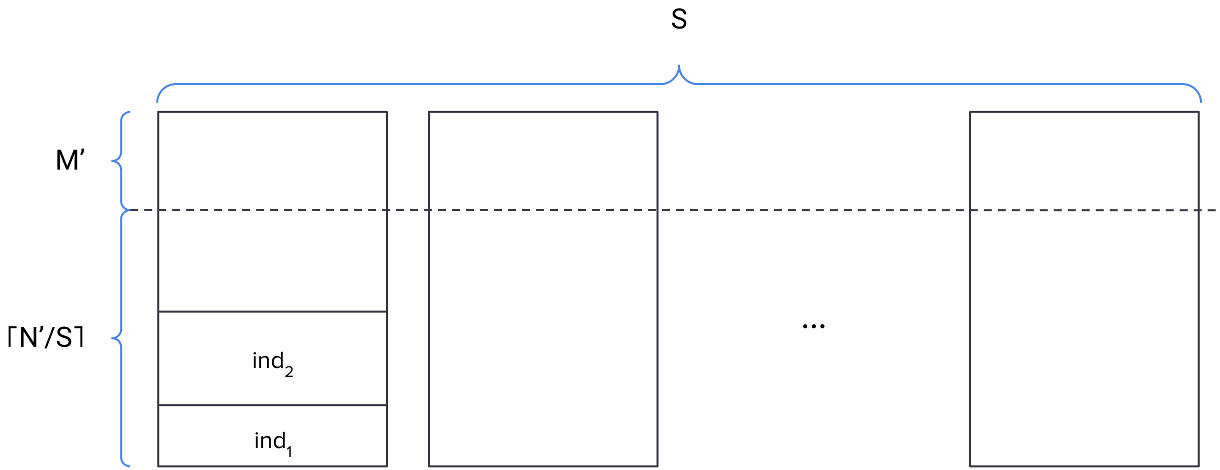

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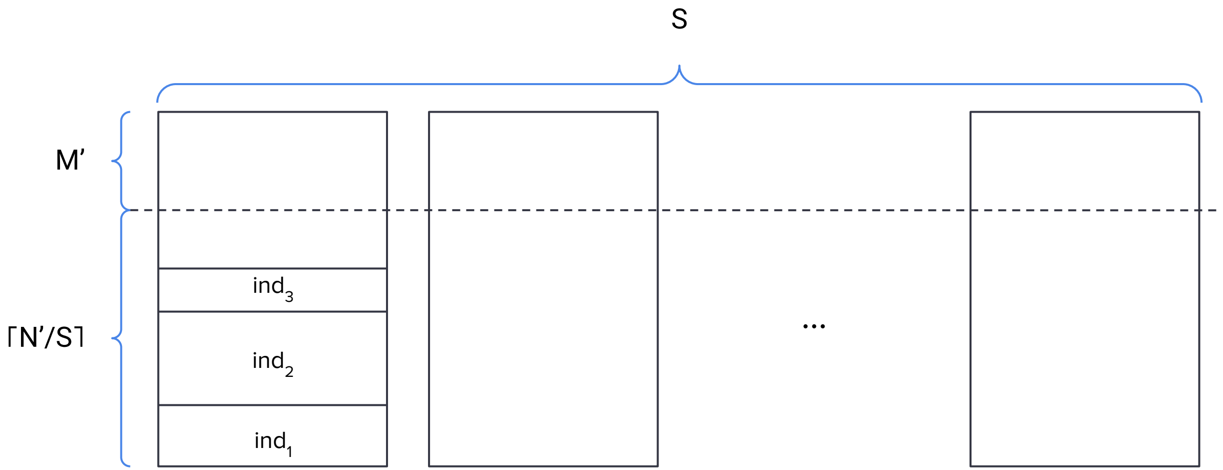

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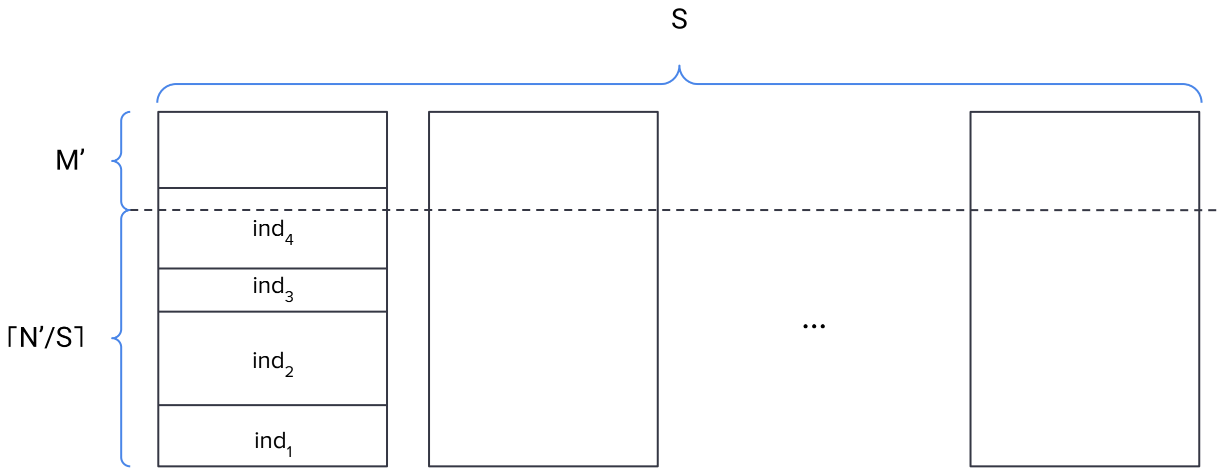

How to make sparse histogram private without seeing it?

Server 1 can add dummy contributions in two ways:

- duplicate existing clients' indices, replacing values with Enc(0)
  - Useful to hide the exact count of indices that are **common**
- add new fake indices with value 0
  - Useful to hide the exact count of indices that are **rare**

Our approach:

1. Choose Threshold T
2. For each multiplicity i < T, add i fake indices ~Laplace times
3. Duplicate each ciphertext ~NegativeBinomial times

Conclusion

- Distributed OPRFs allow for efficient sharding protocols.
- When the number of shards is much smaller than the number of clients, the overhead is negligible.
- For a slightly larger (10%) overhead, we can enable local aggregation at one of the servers. Example application: Sparse histogram computation [1].

[1] Bell, James, Adrià Gascón, Badih Ghazi, Ravi Kumar, Pasin Manurangsi, Mariana Raykova, and Phillipp Schoppmann. "Distributed, Private, Sparse Histograms in the Two-Server Model." In Proceedings of the 2022 ACM SIGSAC Conference on Computer and Communications Security, pp. 307-321. 2022.

Next Steps

- General interest from the working group in secure partitioning?
- Other protocols or settings where this might be useful?
- Do we need additional properties (e.g., keep the order of inputs)?