Summary

- recent results have shown LDPC-staircase/triangle codes are:
  - very close to ideal codes
  - one order of magnitude faster than Reed-Solomon over GF($2^8$) (using Rizzo’s reference codec)

…in many use-cases

- made possible by
  - hybrid Zyablov Iterative decoding/Gaussian elimination scheme, and...
  - the new N1 parameter
Summary… (cont’)

● N1 parameter

○ number of “1s” in each column of the parity check matrix during the first step of the algorithm

\[
\begin{pmatrix}
S_1 & \ldots & S_6 & P_7 & \ldots & P_9 \\
0 & 1 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 0 & 0 & 1 \\
1 & 0 & 1 & 1 & 1 & 0 \\
\end{pmatrix}
\]

add N1 “1s”

○ N1 was fixed and equal to 3 until 08 version, but now:

• N1 belongs to \{3; 10\} (N1=3 remains the default)
• N1 is set by the encoder…
• …and communicated to the decoder (in EXT_FTI or FDT)
• increases the density of the matrix… and the probability it is invertible!

LDPC performances

● depend on:

○ decoding scheme used

○ N1 parameter with LDPC-staircase codes (# triangle)

● more specifically

○ the decoder has to solve a system of linear equations

○ possible with Zyablov Iterative Decoding (ID) scheme

• fast but sub-optimal erasure recovery

○ or Gaussian elimination (GE)

• optimal erasure recovery but more costly

○ or intelligent variants of ID

• see Raptor/RFC 5053 and associated US patent 6,856,263
**LDPC performances… (cont’)**

- …or with a **hybrid ID/GE scheme**
  - recommended for small to medium sized objects
  - start decoding with ID
    - it’s perhaps sufficient…
    - if not, it will anyway simplify the system
  - finish with GE (e.g., if it’s known that no additional symbols will be received)
    - works on the system simplified by the ID, not the original one!

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**Erasure recovery results**

- example: LDPC-staircase, various N1 values

![Graph showing erasure recovery results](graph.png)

- $Pr=1 \Rightarrow$ cannot be decoded
- $Pr<<1 \Rightarrow$ high decoding probability

$max \text{ theoretical limit} (\text{depends on code rate})$

- Decoding failure probability
- Loss probability (%)
Erasure recovery results… (cont’)

● LDPC-staircase results (N1=5, k=1,000)

<table>
<thead>
<tr>
<th>code rate</th>
<th>average overhead</th>
<th>overhead for a failure proba ≤ 10⁻⁴</th>
</tr>
</thead>
<tbody>
<tr>
<td>2/3 (=0.66)</td>
<td>0.63%</td>
<td>2.21%</td>
</tr>
<tr>
<td>2/5 (=0.4)</td>
<td>2.04%</td>
<td>4.41%</td>
</tr>
</tbody>
</table>

• then results further improve as the code rate decreases
  • not shown here, see [SPSC08]
  • means that small-rate codes are feasible...

• results remain excellent with smaller objects
  • no need to artificially increase the number of symbols…
    • symbols groups are no longer needed ➔ use G=1 (default)
  • this is the opposite with ID!

Decoding complexity results

● complexity depends on:

• block size (GE complexity increases)

• loss rate (is ID sufficient or should GE be used too?)

• N1 parameter with LDPC-staircase codes
  (the linear system complexity increases with N1)
Decoding complexity results… (cont’)

- example: LDPC-staircase, code rate 2/3, k=1,000
  - the higher N1, the more complex the decoding
  - yet with N1=5, between 32 to 10 times faster than RS(2^8)

- we see that:
  - decoding complexity isn’t prohibitive at all with objects that are a few thousands of symbols long 😄
  - it requires a careful implementation though
    - take into account the specific parity check matrix structure
  - these results are not the ultimate ones and we should be able to further reduce the decoding complexity…

![Graph showing decoding complexity results]

- ID sufficient
- GE needed more and more often
- 32.4 times faster than RS
- sustainable decoding speed (Mbps)
- with RS: 54Mbps
- still 10.2 times faster
- loss probability(%)
To conclude

- with small/medium sized objects
  - prefer hybrid decoding
  - use $G=1$ (no symbol grouping), it's useless now
  - with larger objects, fall back to ID

- optimal LDPC-triangle codes performances
  - achieved with $N1=3$ (default) for ID or hybrid decoding

- optimal LDPC-staircase codes performances
  - require an appropriate $N1$ value
    - $N1=3$ (default) is the best for ID
    - $N1=4$ or 5 is recommended with hybrid decoding

References

- “Improving the Decoding of LDPC Codes for the Packet Erasure Channel with a Hybrid Zyablov Iterative Decoding/Gaussian Elimination Scheme”, INRIA Research Report RR-6473, March 2008
  [http://hal.inria.fr/inria-00263682/en/](http://hal.inria.fr/inria-00263682/en/)

  [http://hal.inria.fr/inria-00291656/en/](http://hal.inria.fr/inria-00291656/en/)