# Speed-ups of Elliptic Curve-Based Schemes

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Results based on work conducted at Certicom Research

IETF-78
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The Netherlands
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# Outline

- ECDSA signature scheme
- Fast ECDSA signature scheme
- Speed-ups:
  - ECDSA fast verification
  - ECDSA certificate verification and ECC-based key agreement (ECDH, ECMQV)
  - Batch ECDSA verification
- How to get from ECDSA to Fast ECDSA
- How an IETF standard could help
- IPR aspects

# ECDSA signature scheme

**System-wide parameters**

Elliptic curve of prime order $n$ with generator $G$. Hash function $h$.

**Signature generation**

INPUT: Message $m$, private key $d$.
OUTPUT: Signature $(r, s)$.

ACTIONS:

1. Compute $e := h(m)$.
2. Select random $k \in [1,n-1]$.
3. Compute $R := kG$ and map $R$ to $r$.
4. Compute $s := k^{-1}(e + d\ r) \bmod n$.
5. If $r \notin [1,n-1]$ or $s \notin [1,n-1]$, go to #2.
6. Return $(r, s)$.

**Initial set-up**

Signer A selects private key $d \in [1,n-1]$ and publishes its public key $Q = dG$.

**Signature verification**

INPUT: Message $m$, signature $(r, s)$;
Public signing key $Q$ of Alice.
OUTPUT: Accept or reject signature.

ACTIONS:

1. If $r \notin [1,n-1]$, return 'reject'.
2. If $s \notin [1,n-1]$, return 'reject'.
3. Compute $e := h(m)$.
4. Compute $R' := s^{-1} (e\ G + r\ Q)$.
5. Check that $R'$ maps to $r$.
   If verification succeeds, return 'accept'; otherwise return 'reject'.

Non-repudiation: Verifier knows the true identity of the signing party, since the public signing key $Q$ is bound to signing party Alice.

# Fast ECDSA signature scheme

**System-wide parameters**

Elliptic curve of prime order $n$ with generator $G$. Hash function $h$.

**Initial set-up**

Signer A selects private key $d \in [1,n\text{-}1]$ and publishes its public key $Q = dG$.

**Signature generation**

INPUT: Message $m$, private key $d$.
OUTPUT: Signature $(R, s)$.

ACTIONS:

1. Compute $e := h(m)$.
2. Select random $k \in [1,n\text{-}1]$.
3. Compute $R := kG$ and map $R$ to $r$.
4. Compute $s := k^{-1}(e + d\,r) \bmod n$.
5. If $r \notin [1,n\text{-}1]$ or $s \notin [1,n\text{-}1]$, go to #2.
6. Return $(R, s)$.

**Signature verification**

INPUT: Message $m$, signature $(R, s)$;
Public signing key $Q$ of Alice.
OUTPUT: <u>Accept</u> or <u>reject</u> signature.

ACTIONS:

1. If $r \notin [1,n\text{-}1]$, return '<u>reject</u>'.
2. If $s \notin [1,n\text{-}1]$, return '<u>reject</u>'.
3. Map $R$ to $r$.
4. Compute $e := h(m)$.
5. Check that $R = s^{-1}(e\,G + r\,Q)$.
   If verification succeeds, return '<u>accept</u>'; otherwise return '<u>reject</u>'.

<u>Non-repudiation:</u> Verifier knows the true identity of the signing party, since the public signing key $Q$ is bound to signing party Alice.

# Fast ECDSA and speed-ups

Speed-ups for prime curves and binary non-Koblitz curves:
– NIST prime curves, 'Suite B' curves, Brainpool curves, GOST (RFC 5832)
– NIST random binary curves

**Fast verification of ECDSA signatures** ([2]):
40% speed-up compared to ordinary approach

**ECDSA certificate verification + Static ECDH/ECMQV** ([7]):
Speed-up incremental cost ECDSA verify compared to separate approach:
2.4x speed-up (compared to ordinary ECDSA verify)
1.7x (compared to Fast ECDSA verify)
Simple side channel resistance virtually for free

**Batch verification of ECDSA signatures** ([3]):
Dependent on number of signatures involved

# Part I –
# Accelerated Verification of ECDSA Signatures

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# Fast ECDSA signature scheme

## Computational aspects

| Ordinary signature verification | Fast signature verification |
|---|---|
| **ACTIONS:**<br>**...**<br>**3. Compute $R' := (e\ s^{-1})\ G + (r\ s^{-1})\ Q$.**<br>**4. Check that $R'$ maps to $r$.**<br>**...** | **ACTIONS:**<br>**...**<br>**2. Map $R$ to $r$.**<br>**4. Check that $R = (e\ s^{-1})\ G + (r\ s^{-1})\ Q$.**<br>**...** |

**Ordinary signature verification**

Compute expression $R' := (e\ s^{-1})\ G + (r\ s^{-1})\ Q$.

Cost: *full-size* linear combination of *known* point $G$ and *unknown* point $Q$.

**Fast signature verification**

Evaluate expression $\Delta := s^{-1}\ (e\ G + r\ Q) - R$ and check that $\Delta = O$,
by verifying instead

$$\mu\ \Delta := (\mu\ e\ s^{-1})\ G + (\mu\ r\ s^{-1})\ Q - \mu\ R = O \text{ for suitable } \mu \in [1,n\text{-}1].$$

Cost: *half-size* combination of *known* points $G$, $G'$ and *unknown* points $Q$, $R$.

# Example

**Verification cost ECDSA scheme vs. Fast ECDSA scheme**
- Curve: NIST prime curve P-384 with 192-bit security (Suite B)
- Integer representation: NAF, joint sparse form (JSF)
- Coordinate system: Jacobian coordinates

| P-384 curve | ECDSA Verify | |
|---|---|---|
| ECC operations | Ordinary | Fast |
| – Add | 194 | 196 |
| – Double | 384 | 192 |
| – Total[1] | 459 | 328 |

[1]Normalized (double/add ratio: 0.69)

| | | |
|---|---|---|
| RIM Blackberry[2] | 221 ms | 158 ms |

[2]Platform: ARM7TDMI (50 MHz)

**Speed-up cost Fast ECDSA verify**
compared to ordinary approach: 1.4x

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# Cost of signature verification

**Verification cost of ECDSA signature vs. RSA signatures**

- RSA: public exponent $e = 2^{16}+1$
- ECDSA: NIST prime curves
- Platform: HP iPAQ 3950, Intel PXA250 processor (400 MHz)

| Security level (bits) | Verification cost (ms) | | | Ratio fast ECDSA verify vs. RSA verify |
|---|---|---|---|---|
| | RSA[2] | ECDSA | | |
| | | ordinary[2] | fast[3] | |
| 80 | 1.4 | 4.0 | 2.9 | 0.5x faster |
| 112 | 5.2 | 7.7 | 5.5 | 0.9x faster |
| 128 | 11.0 | 11.8 | 8.4 | 1.3x faster |
| 192 | 65.8 | 32.9 | 23.5 | 2.8x faster |
| 256 | 285.0 | 73.2 | 52.3 | 5.4x faster |

[1]Excluding (fixed) overhead of identification data

[2]Certicom Security Builder [3]Estimate

**Conclusion**

Efficiency advantage of RSA signatures over ECDSA signatures is vanishing

# Part II –
# Combined Verification and Key Computation

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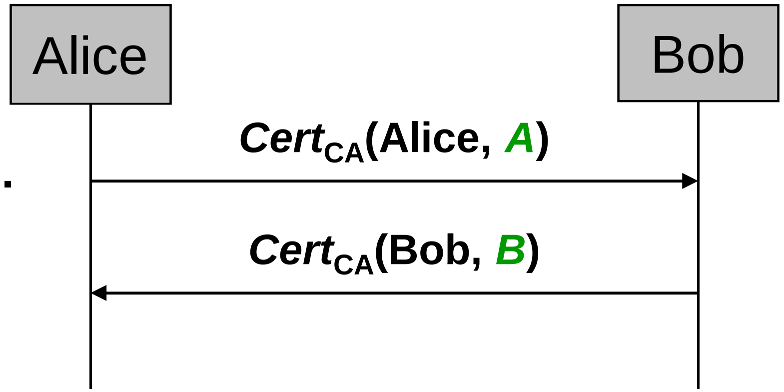
# Key agreement schemes

**Authenticated Diffie-Hellman (static ECDH)**

**ACTIONS:**
**1. Verify $Cert_{CA}$(Bob, $B$).**
**2. Compute $K:=aB$.**




**ACTIONS:**
**1. Verify $Cert_{CA}$(Alice, $A$).**
**2. Compute $K:=bA$.**

**Properties**

- Key agreement: Both parties arrive at same key $K$, since $K=abG=aB=bA$.
- Key authentication: Each party knows the true identity of the key sharing party, since keys $A$ and $B$ *are* bound to parties Alice and Bob.

# Computational aspects (1)

**Step 2: ECDH key computation (key establishment)**

**ACTIONS (Alice):**
**1. Verify $Cert_{CA}$(Bob,$B$).**
**2. Compute $K$:=$aB$.**

Compute expression $K := aB$,

where $a$ is Alice's private key;
$B$ is Bob's public key (derived from his certificate).

**Step 1: ECDSA certificate verification (key authentication)**

Evaluate expression $s^{-1}(e\,G + r\,Q) - R = O$,

where $e$ is hash value of certificate info (including Bob, $B$);
$Q$ is public key of certificate authority;
($r$, $s$) is ECDSA signature over certificate info.

Question: Can one combine these steps?

Answer: YES!

# Example (1)

**Static ECDH with ECDSA certificates**

- Curve: NIST prime curve P-384 with 192-bit security (Suite B)
- Integer representation: NAF, joint sparse form (JSF)
- Coordinate system: Jacobian coordinates

| P-384 curve | ECDH key | ECDSA (incremental cost) | | |
|---|---|---|---|---|
| | | Separately | | Combined with ECDH |
| ECC operations | | Ordinary | Fast | |
| – Add | 128 | 194 | 196 | 195 |
| – Double | 384 | 384 | 192 | – |
| – Total[1] | 393 | 459 | 328 | 195 |

[1]Normalized (double/add ratio: 0.69)

**Speed-up incremental cost ECDSA verify**

compared to separate approach: 2.4x (ordinary ECDSA verify)

1.7x (Fast ECDSA verify)

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# Cost of certificate verification

**Incremental verification cost of ECDSA certificates vs. RSA certificates**

- RSA: public exponent $e = 2^{16}+1$
- ECDSA, ECDH: NIST prime curves
- Platform: HP iPAQ 3950, Intel PXA250 processor (400 MHz)

| Security level (bits) | Certificate size[1] (bytes) | | Ratio ECC/RSA certificates | Verify – incremental cost (ms) | | | Ratio ECDSA verify vs. RSA verify |
|---|---|---|---|---|---|---|---|
| | ECDSA | RSA | | RSA[2] | ECDSA | | |
| | | | | | ordinary[2] | combined[3] | |
| 80 | 72 | 256 | 4x smaller | 1.4 | 4.0 | 1.7 | 0.8x faster |
| 112 | 84 | 512 | 6x smaller | 5.2 | 7.7 | 3.2 | 1.6x faster |
| 128 | 96 | 768 | 8x smaller | 11.0 | 11.8 | 4.9 | 2.2x faster |
| 192 | 144 | 1920 | 13x smaller | 65.8 | 32.9 | 13.7 | 4.8x faster |
| 256 | 198 | 3840 | 19x smaller | 285.0 | 73.2 | 30.5 | 9.3x faster |

[1]Excluding (fixed) overhead of identification data [2]Certicom Security Builder [3]Estimate

**Conclusion**

Efficiency advantage of RSA certificates with DH-based schemes is no more

# ECDSA vs. Fast ECDSA

**Security of Fast ECDSA**

Both schemes are equally secure: ECDSA has signature ($r$, $s$) if and only if Fast ECDSA has signature ($R$, $s$) where $R$ maps to $r$.

**ECDSA signature verification**

- Convert ECDSA signature ($r$, $s$) to Fast ECDSA signature ($R$, $s$)
- Verify Fast ECDSA signature ($R$, $s$)

Note:

- Conversion generally yields *pair* ($R$, $-R$) of *candidate points* that map to $r$.
- Verification involves trying out all those candidate points not discarded based on some side constraints (the so-called *admissible points*).

How to ensure only one admissible point:

- Generate ECDSA signature with $k$ such that y-coordinate of $R$:=$kG$ can be prescribed. (If necessary, change the sign of $k$.)
- Use the fact that ($r$, $s$) is a valid ECDSA signature if and only if ($r$, $-s$) is.

Conversion of ECDSA to Fast Verify friendly format: via simple post-processing

“Friendly ECDSA” ☺

# Friendly ECDSA scheme

**System-wide parameters**

Elliptic curve of prime order $n$ with generator $G$. Hash function $h$.

**Initial set-up**

Signer A selects private key $d \in [1,n\text{-}1]$ and publishes its public key $Q = dG$.

**Signature generation**

INPUT: Message $m$, private key $d$.
OUTPUT: Signature $(r, s)$.

ACTIONS:

1. Compute $e := h(m)$.
2. Select random $k \in [1,n\text{-}1]$.
3. Compute $R := kG$ and map $R$ to $r$.
4. Compute $s := k^{-1}(e + d\, r) \bmod n$.
5. If $r \notin [1,n\text{-}1]$ or $s \notin [1,n\text{-}1]$, go to #2.
6. Return $(r, s)$ if $y$-coordinate $R$ even; return $(r, -s)$ otherwise.

**Signature verification**

INPUT: Message $m$, signature $(r, s)$; Public signing key $Q$ of Alice.
OUTPUT: Accept or reject signature.
ACTIONS:

1. If $r \notin [1,n\text{-}1]$, return 'reject'.
2. Map $r$ to $R$ (only one of $R$ or $-R$ valid, since $y$-coordinate of $R$ or $-R$ odd).
3. If $s \notin [1,n\text{-}1]$, return 'reject'.
4. Compute $e := h(m)$.
5. Check that $R = s^{-1}(e\, G + r\, Q)$. If verification succeeds, return 'accept'; otherwise return 'reject'.

Anyone can do this post-processing

Anyone can do this pre-processing



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# How to get to Friendly ECDSA

**Existing ECDSA signatures**

–Anyone can post-process legacy ECDSA certificate and put into friendly format

–This could be device that participates into key agreement (ECDH + signed exponents)

**New ECDSA signatures**

–Generate in friendly format

–If verifier knows, he can always get speed-ups

explicit method: use, e.g., new OID with PKIX, etc.

implicit method: facilitate in IETF drafts currently in pipeline (no need for new OIDs)

–If verifier does not know, he can guess (best: +40%, worst: -12%, avg.: +8%)

<u>Note:</u>

–Devices that do not implement speed-ups will not notice, since compatible format

–Possible to move towards implementing verification speed-ups over time (one can change one's mind)

# IPR – where is it?

Potential IPR strings attached to following techniques:

– Accelerated verification of ECDSA signatures
– Combined ECDSA signature verification and ECC-based key agreement (e.g., ECDH with ECDSA signed exponents)

Ref: https://datatracker.ietf.org/ipr/1363/

Hence, making techniques *optional* to use for those who choose to do so

Ideal scenario:
Everyone facilitates others to fully benefit from speed-ups should they choose to do so.

# Further reading


1. ANSI X9.63-2001, *Public Key Cryptography for the Financial Services Industry: Key Agreement and Key Transport Using Elliptic Curve Cryptography*, American National Standard for Financial Services, American Bankers Association, November 20, 2001.
2. A. Antipa, D.R. Brown, R. Gallant, R. Lambert, R. Struik, S.A. Vanstone, 'Accelerated Verification of ECDSA Signatures,' in *Proceedings of Selected Areas in Cryptography – SAC2005*, B. Preneel, S. Tavares, Eds., Lecture Notes in Computer Science, Vol. 3897, pp. 307-318, New York: Springer, 2006.
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4. FIPS Pub 186-3, *Digital Signature Standard (DSS)*, Federal Information Processing Standards Publication 186-3, US Department of Commerce/National Institute of Standards and Technology, Gaithersburg, Maryland, USA, June 2009.
5. D.R. Hankerson, A.J. Menezes, S.A. Vanstone, *Guide to Elliptic Curve Cryptography*, New York: Springer, 2003.
6. NIST SP800-56a, *Recommendation for Pair-wise Key Establishment Schemes Using Discrete Logarithm Cryptography*, March 8, 2007.
7. R. Struik, 'Batch Computations Revisited: Combining Key Computations and Batch Verifications,' to be presented at SAC 2010, Waterloo, ON, Canada, August 12-13, 2010.

Back-up slides with more technical detail

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# Part I –
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# Outline

- ECDSA signature scheme
- Fast ECDSA signature scheme
- Computational aspects
  - Simultaneous multiplication
  - Extended Euclidean Algorithm
- Examples
  - Fast ECDSA verification
  - ECDSA verification
  - Comparison with RSA signatures
- Conclusions

# ECDSA signature scheme

**System-wide parameters**

**Elliptic curve of prime order $n$ with generator $G$. Hash function $h$.**

**Initial set-up**

**Signer A selects private key $d \in [1,n-1]$ and publishes its public key $Q = dG$.**

**Signature generation**

**INPUT: Message $m$, private key $d$.**
**OUTPUT: Signature $(r, s)$.**

**ACTIONS:**

1. **Compute $e := h(m)$.**
2. **Select random $k \in [1,n-1]$.**
3. **Compute $R := kG$ and map $R$ to $r$.**
4. **Compute $s := k^{-1}(e + d\,r) \bmod n$.**
5. **If $r, s \in [1,n-1]$, return $(r, s)$; otherwise, go to Step 2.**

**Signature verification**

**INPUT: Message $m$, signature $(r, s)$; Public signing key $Q$ of Alice.**
**OUTPUT: <u>Accept</u> or <u>reject</u> signature.**

**ACTIONS:**

1. **Compute $e := h(m)$.**
2. **Check that $r, s \in [1,n-1]$. If verification fails, return 'reject'.**
3. **Compute $R' := s^{-1}(e\,G + r\,Q)$.**
4. **Check that $R'$ maps to $r$. If verification succeeds, return '<u>accept</u>'; otherwise return '<u>reject</u>'.**

<u>Non-repudiation:</u> Verifier knows the true identity of the signing party, since the public signing key $Q$ is bound to signing party Alice.

# Fast ECDSA signature scheme

**System-wide parameters**

Elliptic curve of prime order $n$ with generator $G$. Hash function $h$.

**Initial set-up**

Signer A selects private key $d \in [1,n-1]$ and publishes its public key $Q = dG$.

**Signature generation**

INPUT: Message $m$, private key $d$.
OUTPUT: Signature $(R, s)$.

ACTIONS:

1. Compute $e := h(m)$.
2. Select random $k \in [1,n-1]$.
3. Compute $R := kG$ and map $R$ to $r$.
4. Compute $s := k^{-1}(e + d\,r) \bmod n$.
5. If $r, s \in [1,n-1]$, return $(R, s)$; otherwise, go to Step 2.

**Signature verification**

INPUT: Message $m$, signature $(R, s)$; Public signing key $Q$ of Alice.
OUTPUT: Accept or reject signature.

ACTIONS:

1. Compute $e := h(m)$.
2. Map $R$ to $r$.
3. Check that $r, s \in [1,n-1]$. If verification fails, return 'reject'.
4. Check that $R = s^{-1}(e\,G + r\,Q)$. If verification succeeds, return 'accept'; otherwise return 'reject'.

Non-repudiation: Verifier knows the true identity of the signing party, since the public signing key $Q$ is bound to signing party Alice.

# Fast ECDSA signature scheme

## Computational aspects

| Ordinary signature verification | Fast signature verification |
| --- | --- |
| **ACTIONS:**<br>**...**<br>**3. Compute $R' := (e\, s^{-1})\, G + (r\, s^{-1})\, Q$.**<br>**4. Check that $R'$ maps to $r$.**<br>**...** | **ACTIONS:**<br>**...**<br>**2. Map $R$ to $r$.**<br>**4. Check that $R = (e\, s^{-1})\, G + (r\, s^{-1})\, Q$.**<br>**...** |

**Ordinary signature verification**

Compute expression $R' := (e\, s^{-1})\, G + (r\, s^{-1})\, Q$.

Cost: *full-size* linear combination of *known* point $G$ and *unknown* point $Q$.

**Fast signature verification**

Evaluate expression $\Delta := s^{-1}\, (e\, G + r\, Q) - R$ and check that $\Delta = O$.

Cost: *full-size* linear combination of *known* point $G$ and *unknown* point $Q$.

Seemingly no computational advantages over traditional approach … ☹

# Computational aspects (1)

One can do better, though! ☺

**Fast signature verification**

Evaluate expression $\Delta := (e\, s^{-1})\, G + (r\, s^{-1})\, Q - R$ and check that $\Delta = O$.

**Equivalent test**

Check that $\mu\, \Delta := (\mu\, e\, s^{-1})\, G + (\mu\, r\, s^{-1})\, Q - \mu\, R = O$ for any $\mu \in [1, n\text{-}1]$.

or:

Check that $\mu\, \Delta := (\mu\, e\, s^{-1})\, G + \lambda\, Q - \mu\, R = O$, where $r / s \equiv \lambda / \mu \pmod{n}$.

**Optimum choice**

Write $r / s \equiv \lambda / \mu \pmod{n}$, where $\lambda$ and $\mu$ have size *half* the bit-length of $n$.

<u>Note:</u> This can be done efficiently using the Extended Euclidean Algorithm.

**Why speed-up?**

Speed-up due to getting rid of <u>half</u> of so-called point doubles.

# Computational aspects (2)

**Fast signature verification**

Check that $\mu\,\Delta := (\mu\, e\, s^{-1})\, G + \lambda\, Q - \mu\, R = O$, where $r / s \equiv \lambda / \mu \pmod{n}$ and where $\lambda$ and $\mu$ have size *half* the bit-length of $n$.

**Details:**

Pre-compute $G_1 := t\, G$, where $t \approx \sqrt{n}$. Let $G_0 := G$.

Write $r / s \equiv \lambda / \mu \pmod{n}$, where $\lambda$ and $\mu$ have size *half* the bit-length of $n$.

Write $\mu\, e\, s^{-1} \equiv \alpha_0 + \alpha_1\, t \pmod{n}$, where $\alpha_0$, $\alpha_1$ have size half the bit-length of $n$.

Evaluate $\mu\Delta := (\mu\, e\, s^{-1})\, G + \lambda\, Q - \mu\, R$

$$= \alpha_0\, G_0 + \alpha_1\, G_1 + \lambda\, Q - \mu\, R$$

Cost: *half-size* combination of *known* points $G_0$, $G_1$ and *unknown* points $Q$, $R$.

**Ordinary signature verification**

Compute expression $R' := (e\, s^{-1})\, G + (r\, s^{-1})\, Q$.

Cost: *full-size* linear combination of *known* point $G$ and *unknown* point $Q$.

# Computational aspects (3)

**Optimum choice**

Write $r / s \equiv \lambda / \mu \pmod{n}$, where $\lambda$ and $\mu$ have size *half* the bit-length of $n$.

This can be done efficiently using the Extended Euclidean Algorithm.

**Extended Euclidean Algorithm (EEA)**

**INPUT:** Positive integers $a$ and $n$ with $a \leq n$.

**OUTPUT:** $d = \gcd(a, n)$ and integers $x$, $y$ satisfying $a\,x + n\,y = d$.

**ACTIONS:**

1. $(u, v) \leftarrow (a, n)$; $X \leftarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$;
2. while $u \neq 0$ do
   {
   $q \leftarrow v$ div $u$; $(u, v) \leftarrow (v \bmod u, u)$; $X \leftarrow \begin{pmatrix} -q & 1 \\ 1 & 0 \end{pmatrix} X$
   }
3. $(d, x, y) \leftarrow (v, x_{21}, x_{22})$.

**Invariant:**

$$a\,x_{11} + n\,x_{12} = u$$
$$a\,x_{21} + n\,x_{22} = v$$

Let $a := r\,s^{-1} \pmod{n}$.

Use Ext. Euclidean Algorithm to compute $\gcd(a, n)$. (which is 1, since $n$ is prime.)

Abort algorithm once $u < \sqrt{n}$.

(Most likely, $|x_{11}|$ is also close to $\sqrt{n}$.)

Set $\lambda := u$ and $\mu := x_{11}$.

# Example

**Verification cost ECDSA scheme vs. Fast ECDSA scheme**

• Curve: NIST prime curve P-384 with 192-bit security (Suite B)
• Integer representation: NAF, joint sparse form (JSF)
• Coordinate system: Jacobian coordinates

| P-384 curve | ECDSA Verify | |
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| | | |
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| RIM Blackberry[2] | 221 ms | 158 ms |

[2]Platform: ARM7TDMI (50 MHz)

**Speed-up cost Fast ECDSA verify**
compared to ordinary approach: 1.4x

# ECDSA vs. Fast ECDSA

**Security of Fast ECDSA**

Both schemes are equally secure: ECDSA has signature ($r$, $s$) if and only if Fast ECDSA has signature ($R$, $s$) where $R$ maps to $r$.

**ECDSA signature verification**

- Convert ECDSA signature ($r$, $s$) to Fast ECDSA signature ($R$, $s$)
- Verify Fast ECDSA signature ($R$, $s$)

Note:

- Conversion generally yields *pair* ($R$, $-R$) of *candidate points* that map to $r$.
- Verification involves trying out all those candidate points not discarded based on some side constraints (the so-called *admissible points*).

How to ensure only one admissible point:

- Generate ECDSA signature with $k$ such that y-coordinate of $R:=kG$ can be prescribed. (If necessary, change the sign of $k$.)
- Use the fact that ($r$, $s$) is a valid ECDSA signature if and only if ($r$, $-s$) is.

# Cost of signature verification

**Verification cost of ECDSA signature vs. RSA signatures**

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- ECDSA: NIST prime curves
- Platform: HP iPAQ 3950, Intel PXA250 processor (400 MHz)

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[1]Excluding (fixed) overhead of identification data
[2]Certicom Security Builder [3]Estimate

**Conclusion**

Efficiency advantage of RSA signatures over ECDSA signatures is vanishing

# Conclusions

Fast ECDSA signature scheme attractive:

- Security: Same security as original ECDSA signature scheme
- Efficiency: Considerable speed-up possible for non-Koblitz curves
  - NIST prime curves, ‘Suite B’ curves, Brainpool curves: 40% speed-up
  - NIST random binary curves: 40% speed-up

Efficiency results applicable to ordinary ECDSA signature scheme:

- ECDSA and Fast ECDSA have same cost if only 1 admissible point
  - Append 1 bit of side info to ECDSA signature to distinguish ($R$, $-R$)
  - Agree on particular way of generating ECDSA signatures such that only one of points $R$ and $–R$ is admissible
- ECDSA can still use Fast ECDSA if more than 1 admissible point
  - Roughly 8% average speed-up for curves mentioned above

Efficiency advantage of RSA signatures over ECDSA signatures is vanishing

# Part II – Combined Verification and Key Computation

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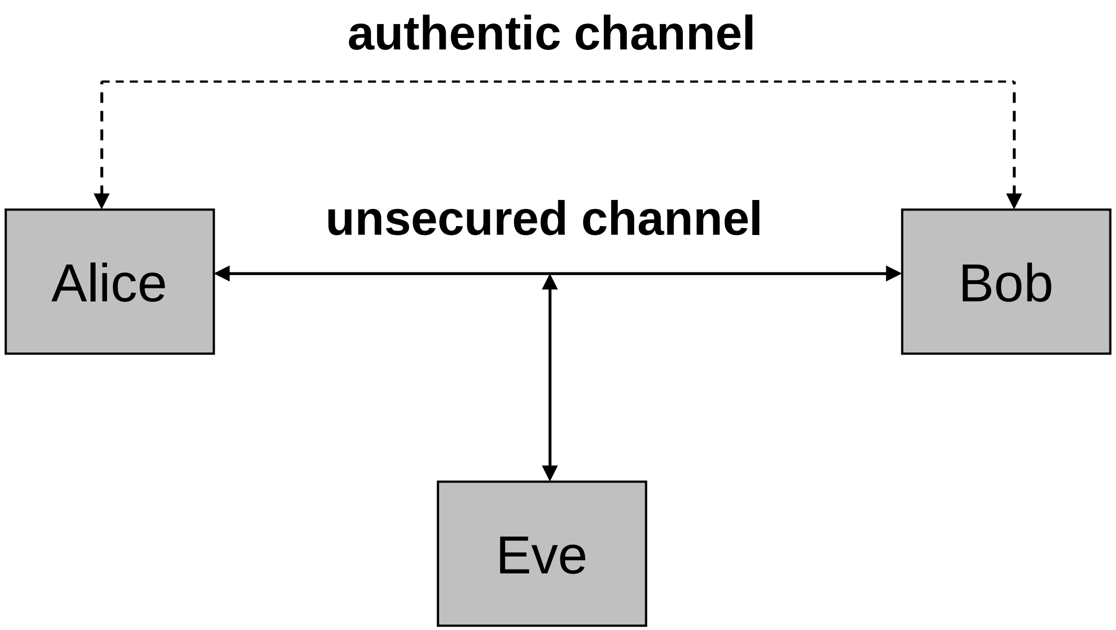
# Outline

- Public key cryptography
  - Key agreement schemes
  - Signature schemes
- Computational aspects
  - Key computation
  - Certificate verification
  - Combined key computation and certificate verification
- Examples
  - Static Diffie-Hellman with ECDSA certificates
  - ECMQV with ECDSA certificates
  - Comparison with RSA certificates
- Conclusions

# Public key cryptography

**Communication model**

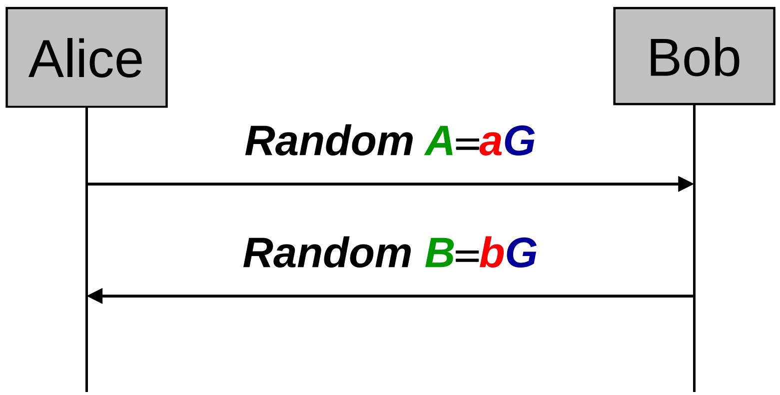
Communicating parties a priori share authentic information



# Key agreement schemes

## Anonymous Diffie-Hellman (ephemeral ECDH)

**ACTIONS:**
**1. Select $a \in_R [1,n\text{-}1]$.**
**2. Compute $A := aG$.**
**3. Compute $K := aB$.**




**ACTIONS:**
**1. Select $b \in_R [1,n\text{-}1]$.**
**2. Compute $B := bG$.**
**3. Compute $K := bA$.**

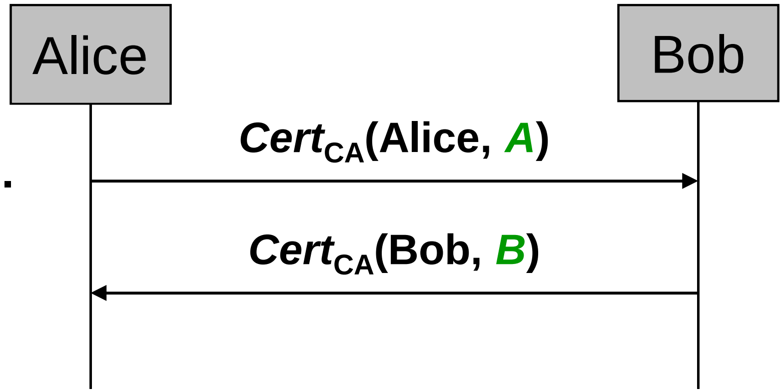
## Properties

- Key agreement: Both parties arrive at same key $K$, since $K = abG = aB = bA$.
- No key authentication: Neither party knows the true identity of the key sharing party, since keys $A$ and $B$ are *not* bound to parties Alice and Bob.

# Key agreement schemes

**Authenticated Diffie-Hellman (static ECDH)**

**ACTIONS:**
**1. Verify $Cert_{CA}$(Bob, $B$).**
**2. Compute $K{:=}aB$.**




**ACTIONS:**
**1. Verify $Cert_{CA}$(Alice, $A$).**
**2. Compute $K{:=}bA$.**

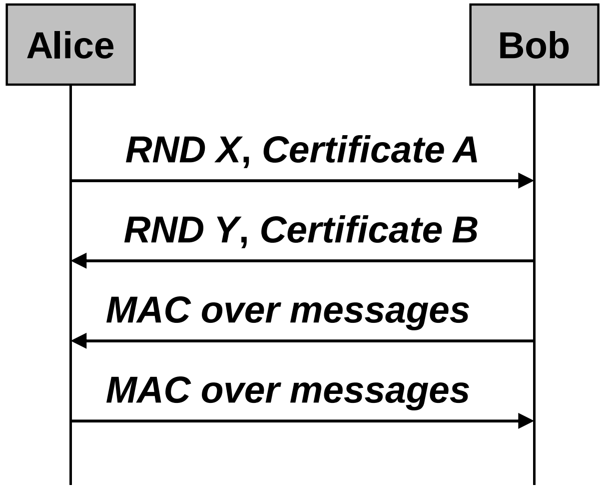
**Properties**

- Key agreement: Both parties arrive at same key $K$, since $K=abG=aB=bA$.
- Key authentication: Each party knows the true identity of the key sharing party, since keys $A$ and $B$ *are* bound to parties Alice and Bob.

# Key agreement schemes

## General protocol format

**Step 1: Key contributions**
Each party randomly generates a short-term (ephemeral) public key pair and communicates the ephemeral public key to the other party (but not the private key).
**Step 2: Key establishment**
Each party computes the shared key based on static and ephemeral public keys received from the other party and static and ephemeral private keys it generated itself.
**Step 3: Key authentication**
Each party verifies the authenticity of the static key of the other party.
**Step 4: Key confirmation**
Each party evidences possession of the shared key to the other party. This also confirms its true identity to the other party.



# Key agreement schemes

## Computational aspects

Offline fixed point multiplication

**Step 1: Key contributions**

Each party randomly generates a short-term (ephemeral) public key pair and communicates the ephemeral public key to the other party (but not the private key).

**Step 2: Key establishment**

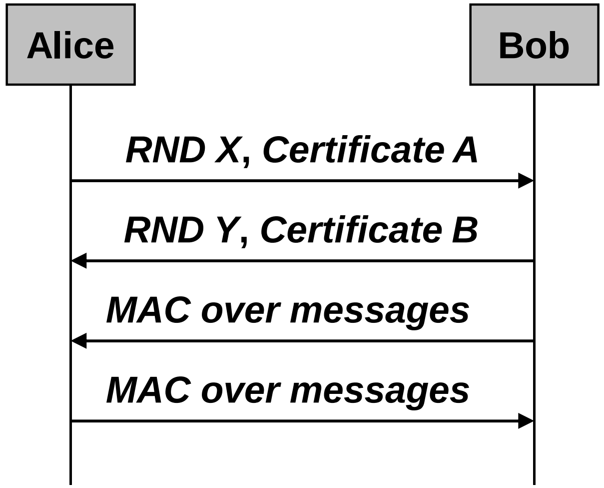
Each party computes the shared key based on static and ephemeral public keys received from the other party and static and ephemeral private keys it generated itself.

**Step 3: Key authentication**

Each party verifies the authenticity of the static key of the other party.

**Step 4: Key confirmation**

Each party evidences possession of the shared key to the other party. This also confirms its true identity to the other party.




Online variable point multiplication

Online verification of public key certificate

# ECDSA signature scheme

**ECDSA signature verification**

**INPUT: Message $m$, signature $(r, s)$;**
**Public signing key $Q$ of Alice.**
**OUTPUT: Accept or reject signature.**

**System-wide parameters**

**Elliptic curve with generator $G$.**
**Hash function $h$.**

**Ordinary signature verification**

**ACTIONS:**

...
1. Compute $e := h(m)$.
2. Compute $R' := (e\, s^{-1})\, G + (r\, s^{-1})\, Q$.
3. Check that $R'$ maps to $r$.

...

**Fast signature verification**

**ACTIONS:**

...
1. Compute $e := h(m)$.
2. Reconstruct $R$ from $r$.
3. Check that $R = (e\, s^{-1})\, G + (r\, s^{-1})\, Q$.

...

ECDSA verification: Check equation $s^{-1}(e\, G + r\, Q) - R = O$.

Non-repudiation: Verifier knows the true identity of the signing party, since the public signing key $Q$ is bound to signing party Alice.

# Computational aspects (1)

**Step 2: ECDH key computation (key establishment)**

**ACTIONS (Alice):**
**1. Verify $Cert_{CA}$(Bob,$B$).**
**2. Compute $K$:=$aB$.**

Compute expression $K := aB$,

where $a$ is Alice's private key;
$B$ is Bob's public key (derived from his certificate).

**Step 3: ECDSA certificate verification (key authentication)**

Evaluate expression $s^{-1}\,(e\,G + r\,Q) - R = O$,

where $e$ is hash value of certificate info (including Bob, $B$);
$Q$ is public key of certificate authority;
$(r, s)$ is ECDSA signature over certificate info.

Question: Can one combine these steps?

Answer: YES!

# Computational aspects (2)

**Step 2: ECDH key computation (key establishment)**

Compute expression $K := aB$.

**ACTIONS (Alice):**
**1. Verify $Cert_{CA}$(Bob,$B$).**
**2. Compute $K$:=$aB$.**

**Step 3: ECDSA certificate verification (key authentication)**

Evaluate expression $\Delta := s^{-1}\,(e\,G + r\,Q) - R$ and check that $\Delta = O$.

**Step 2 and Step 3 combined: Combined verification and key computation**

Compute expression $K' := \underbrace{aB}_{\text{Key expression}} + \underbrace{\lambda}_{\text{Randomizer}}\,(\underbrace{s^{-1}(e\,G + r\,Q) - R}_{\text{Verification expression}})$ instead.

More generally, compute $K' := K + \lambda\,\Delta$ instead.

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# Computational aspects (3)

**Step 2 and Step 3 combined: Combined verification and key computation**

Compute expression $K' := \underbrace{aB}_{\text{Key expression}} + \underbrace{\lambda}_{\text{Randomizer}} (\underbrace{s^{-1}(e\,G + r\,Q) - R}_{\text{Verification expression}})$ instead.

More generally, compute $K' := K + \lambda\,\Delta$ instead.

**Why does this work?**

Alice can only compute $K'$ correctly if certificate is 'correct' (i.e., $\Delta = O$); otherwise, $K'$ is random (since then $\Delta \neq O$).

**Property**

Implicit key authentication: Each party knows the true identity of the key sharing party, if any, since keys $A$ and $B$ are bound to parties Alice and Bob and either party can only compute a shared key if that binding is 'correct'.

# Computational aspects (4)

**Step 2: ECDH key computation (key establishment)**

Compute expression $K := aB$.

<u>Cost:</u> full-size multiple of *unknown* point $B$.

**Step 3: ECDSA certificate verification (key authentication)**

Check expression $s^{-1}(e\,G + r\,Q) = R$.

<u>Cost:</u> linear combination of *known* point $G$ and *unknown* point $Q$.

**Step 2 and Step 3 combined: Combined verification and key computation**

Compute expression $K' := aB - \lambda\,R + (\lambda\,e\,s^{-1})\,G + (\lambda\,r\,s^{-1})\,Q$.

<u>Cost:</u> linear combination of *known* point $G$ and *unknown* points $B$,$Q$, and $R$.

**Why speed-up?**

Speed-up due to getting rid of <u>half</u> of so-called point doubles.

# Example (1)

**Static ECDH with ECDSA certificates**

- Curve: NIST prime curve P-384 with 192-bit security (Suite B)
- Integer representation: NAF, joint sparse form (JSF)
- Coordinate system: Jacobian coordinates

| P-384 curve | ECDH key | ECDSA (incremental cost) | | |
|---|---|---|---|---|
| | | Separately | | Combined with ECDH |
| ECC operations | | Ordinary | Fast | |
| – Add | 128 | 194 | 196 | 195 |
| – Double | 384 | 384 | 192 | – |
| – Total[1] | 393 | 459 | 328 | 195 |

[1]Normalized (double/add ratio: 0.69)

**Speed-up incremental cost ECDSA verify**
compared to separate approach: 2.4x (ordinary ECDSA verify)
1.7x (Fast ECDSA verify)

# Example (2)

**ECMQV with ECDSA certificates**

- Curve: NIST prime curve P-384 with 192-bit security (Suite B)
- Integer representation: NAF, joint sparse form (JSF)
- Coordinate system: Jacobian coordinates

| P-384 curve | ECMQV key | ECDSA (incremental cost) | | |
|---|---|---|---|---|
| | | Separately | | Combined with ECMQV |
| ECC operations | | Ordinary | Fast | |
| – Add | 194 | 194 | 196 | 196 |
| – Double | 384 | 384 | 192 | – |
| – Total[1] | 459 | 459 | 328 | 196 |

[1]Normalized (double/add ratio: 0.69)

**Speed-up incremental cost ECDSA verify**

compared to separate approach: 2.3x (ordinary ECDSA verify)
1.7x (Fast ECDSA verify)

# Example (3)

**Static ECDH and ECMQV with ECDSA certificates**

| P-384 curve Total ECC operations[1] | Key computation | Key computation + ECDSA (total cost) | | |
|---|---|---|---|---|
| | | ECDSA separately | | ECDSA combined |
| | | Ordinary | Fast | |
| **ECDH** | 393 | 852 | 721 | 588 |
| **ECMQV** | 459 | 918 | 787 | 655 |

[1]**Normalized (double/add ratio: 0.69)**

**Speed-up total cost ECDH + ECDSA**
compared to separate approach: +45% (ordinary ECDSA verify)
+23% (Fast ECDSA verify)

**Speed-up total cost ECMQV + ECDSA**
compared to separate approach: +40% (ordinary ECDSA verify)
+20% (Fast ECDSA verify)

# Cost of certificate verification

**Incremental verification cost of ECDSA certificates vs. RSA certificates**

- RSA: public exponent $e = 2^{16}+1$
- ECDSA, ECDH: NIST prime curves
- Platform: HP iPAQ 3950, Intel PXA250 processor (400 MHz)

| Security level (bits) | Certificate size[1] (bytes) | | Ratio ECC/RSA certificates | Verify – incremental cost (ms) | | | Ratio ECDSA verify vs. RSA verify |
|---|---|---|---|---|---|---|---|
| | ECDSA | RSA | | RSA[2] | ECDSA ordinary[2] | ECDSA combined[3] | |
| 80 | 72 | 256 | 4x smaller | 1.4 | 4.0 | 1.7 | 0.8x faster |
| 112 | 84 | 512 | 6x smaller | 5.2 | 7.7 | 3.2 | 1.6x faster |
| 128 | 96 | 768 | 8x smaller | 11.0 | 11.8 | 4.9 | 2.2x faster |
| 192 | 144 | 1920 | 13x smaller | 65.8 | 32.9 | 13.7 | 4.8x faster |
| 256 | 198 | 3840 | 19x smaller | 285.0 | 73.2 | 30.5 | 9.3x faster |

[1]Excluding (fixed) overhead of identification data [2]Certicom Security Builder [3]Estimate

**Conclusion**

Efficiency advantage of RSA certificates with DH-based schemes is no more

# Conclusions

Combined computation of ECDH-key and ECDSA verification attractive:

- Security: Same security as underlying DH-based key agreement scheme or ECDSA signature scheme
- Efficiency: Considerable speed-up for all NIST prime curves
  - ECDH + ECDSA: up to 45% speed-up total online cost
  - ECMQV + ECDSA: up to 40% speed-up total online cost
  - ECDSA: up to 2.4x speed-up incremental ECDSA cost
- Implementation security: Simple side channel resistance virtually for free

Incremental cost of signature verification is the right metric:

- Efficiency advantage of RSA certificates with ECDH scheme is no more
  - Break-even point already at roughly 80-bit security level