Probe Placement Problem

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Outline

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Introduction

- Design of monitoring infrastructures : placement, maintenance and operation of monitors at all nodes in a network is not cost-efficient
- Messaging cost (master-slave): $O(n)$, n = number of monitored nodes
- Sorting time: $O(m.nlog(n.m))$ to $O((n.m)^2)$, m = number of records per node
- Monitoring tasks (often) involves a subset of entities $<<$ total number of entities
- $\rightarrow$ Tune the number and placement of probes (required to realize a given measurement task)

Preliminaries

- Network topology modeled as undirected graph $G = (V, E)$
  - V: vertex set
  - E: edge set
- Path $p(s, t)$ from node s (source) to t (destination): node sequence $[v_0(= s), v_1, \ldots, v_{i-1} = u, v_i, \ldots, v_n(= t)]$ such that v_i is adjacent to v_{i-1}, $(v_{i-1}, v_i) \in E(G)\ \forall i$
  - Length of path $p(s, t)$: number of edges the path traverses from s to t
  - Distance $d(s, t)$: cost of a minimum cost path $p(s, t)$ from s to t
- $P(s, t)$: set of all paths $p(s, t)$ from node s to t

Problem

When monitoring network path (segments), probe placement shall account for $P(s, t)$ dynamics (traffic/load, topology) as determined by the routing decisions at each $v \in V$.
$\rightarrow$ Probe placement implies "tracking" of routing path changes.

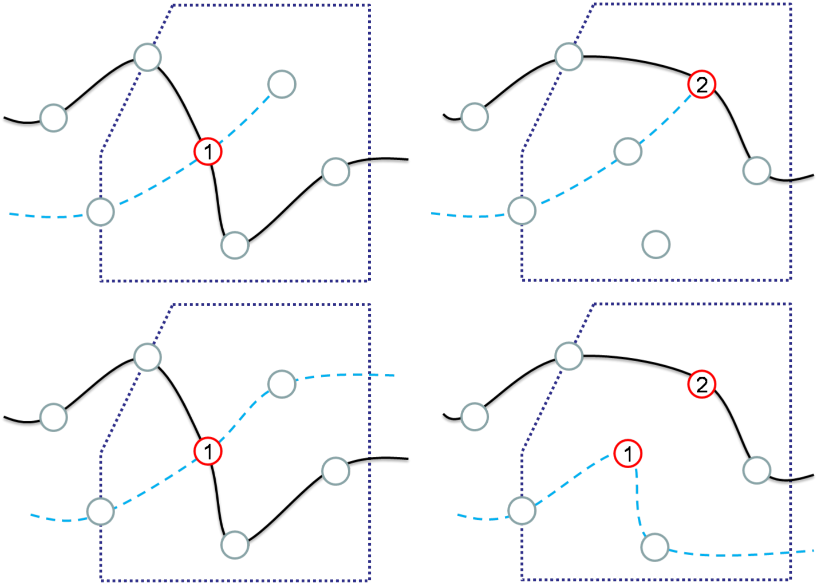
A solution (to the general problem) consists of two parts

1. A set of locations where to deploy monitors (in particular thus the number of probes)
2. A set of entities that are to be monitored over time (to realize a certain measurement task)

How does this relates to **autonomics**

- As objective functions are known: minimize input data processing cost
- Anticipate/predict probe placement

Example



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Multi-period probe deployment/placement and monitoring decision problem

Cost of a solution

- Probe installation cost: c
- Probe maintenance cost: m
- Measurement costs, e.g., link selection or path crossing given link from which probe capacity utilization is derived

Monitoring deployment/placement problem

Time dimension: probe maintenance cost

Measurement cost problem

Upper bound on the number of probes and simultaneous events (frequency vs. accuracy)

Input data

- Directed graph $G = (V, E)$
- Set of periods $\Pi = 1, \ldots, P$
- For each period $p \in \Pi$, demand matrix D^p
- For each link (i, j)
  - Probe capacity κ_{ij}
  - Probe installation cost c_{ij}
  - Probe maintenance cost m_{ij} (here we assume, $m_{ij} < c_{ij}$)
  - Bound M on the number of probes

Variables

- $y_{ij}^{p} \in 0, 1$: indicates if link (i, j) is newly monitored at period p
- $z_{ij}^{p} \in 0, 1$: indicates if monitoring of link (i, j) is maintained at period p
- $x_{ij}^{tp} \in 0, 1$: indicates if node j is the next hop for node i to destination t at period p
- $w_{i}^{tp} \in 0, 1$: indicates if the next hop for node i to destination t changed between periods p and $p + 1$
- $f_{ij}^{tp} \geq 0$: amount of traffic on link (i, j) to destination t at period p

Monitoring cost function

- For each period $p \in \Pi$, traffic load on link (i,j) during period p: $l_{ij}^{p} = \sum_{t \in V} f_{ij}^{tp}$
- Monitoring cost function depends on how close load l_{ij}^{p} is to the probe capacity κ_{ij}, l_{ij}^{p} associated to each edge $(i,j) \in E$
  - Passive case: cost $\sim$ link/path load (cost of extraction)
  - Active case: cost $\sim$ number of paths (cost of insertion)
- Monitoring cost per unit of traffic for each link (i,j) and each period p defined by an increasing convex function of its utilization: $\phi(\kappa_{ij}, l_{ij}^{p})$
- Note: any increasing convex cost function $\phi(\kappa_{ij}, l_{ij}^{p})$ can be considered (here assumed piecewise linear)

Problem formulation

Objective: minimize the sum of installation, maintenance costs and monitoring costs

MILP formulation

$$\min \sum_{p \in \Pi} \sum_{(i,j) \in E} \left(c_{ij} y_{ij}^{p} + m_{ij} z_{ij}^{p} + \phi(\kappa_{ij}, \sum_{t \in V} f_{ij}^{tp}) \right) \quad (1)$$

- First term: installation cost
- Second term: maintenance cost
- Last term: piecewise linear monitoring cost

Constraints (1)

Constraint (i) and (ii): conservation constraints (aggregated per destination) ensuring that monitored flow requirement given by matrix D^p routed at period p

Note: aggregation $\rightarrow$ decrease substantially number of variables and conservation constraints

$$\sum_{j:(i,j)\in E} f_{ij}^{tp} - \sum_{j:(j,i)\in E} f_{ji}^{tp} = D^p(i,t) \; i,t \in V, i \neq t, p \in \Pi \tag{2}$$

$$\sum_{j:(j,t)\in E} f_{jt}^{tp} = \sum_{s\in V} D^p(s,t) \; t \in V, p \in \Pi \tag{3}$$

Constraints (2)

- Constraint (iii): a probe can be used for monitoring link (i, j) at period p only if it is installed or maintained open at period p
- Constraint (iv): a probe is not both installed and maintained during the same period p
- Constraint (v): a probe can be maintained for monitoring link (i, j) at period p only if it was installed during the previous period $p - 1$
- Constraint (vi): no probe installed before the first period starts

$$x_{ij}^{tp} \leq y_{ij}^{p} + z_{ij}^{p} \; (i, j) \in E, t \in V, p \in \Pi \tag{4}$$

$$y_{ij}^{p} + z_{ij}^{p} \leq 1 \; (i, j) \in E, t \in V, p \in \Pi \tag{5}$$

$$z_{ij}^{p} \leq y_{ij}^{p-1} + z_{ij}^{p-1} \; (i, j) \in E, p \in \Pi, p \geq 2 \tag{6}$$

$$z_{ij}^{1} = 0 \; (i, j) \in E \tag{7}$$

Constraints (3)

- Constraint (vii): a monitoring flow can be sent/monitoring input can be received on a link (i, j) for given destination t at given period p only if corr. next-hop is in routing table, where $C_{ij}^{tp} = \min\left(\kappa_{ij}, \sum_{s \in V} D^p(s,t)\right)$ is a tight upper bound on the monitoring capacity for link (i, j) to destination t at period p
- Constraint (viii): a monitoring flow can be sent/monitoring input can be received only on links where probes are installed, and the monitoring capacity not exceeded, where $C_{ij}^{p} = \min\left(\kappa_{ij}, \sum_{s,t \in V} D^p(s,t)\right)$ tight upper bound on the monitoring capacity for link (i, j) at period p

$$f_{ij}^{tp} \leq C_{ij}^{tp} x_{ij}^{tp} \; (i,j) \in E, t \in V, p \in \Pi \tag{8}$$

$$\sum_{t \in V} f_{ij}^{tp} \leq C_{ij}^{p}(y_{ij}^{p} + z_{ij}^{p}) \; (i,j) \in E, p \in \Pi \tag{9}$$

Constraints (4)

- Constraint (ix): exactly one next-hop is selected by each node towards each destination t at each period p.
- Constraint (x): count the number of decision changes between periods
- Constraint (xi): bound by M the number decision changes that can be monitored

$$\sum_{j:(i,j)\in E} x_{ij}^{tp} = 1 \; i,t \in V, i \neq t, p \in \Pi \tag{10}$$

$$x_{ij}^{t(p+1)} - x_{ij}^{tp} \leq w_i^{tp} \; (i,j) \in E, t \in V, p \in \Pi \setminus P \tag{11}$$

$$\sum_{p\in\Pi\setminus P} \sum_{t\in V} \sum_{i\in V} w_i^{tp} \leq M \tag{12}$$

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Case: preliminary results

Geant topology: 22 Nodes, 36 Links, Avg/max degree: 3.27/8, Diameter: 5

Topology	Changes	Installation cost	Monitoring cost	Total cost	Time
GEANT	39	85462	667280	752742	222
	12	85157	667439	752596	317
	10	85123	667935	753058	621
	5	84929	669011	753940	274
	4	84929	669146	754075	310
	3	84929	669339	754268	351
	2	90634	664001	754635	319
	1	96619	659818	756437	361
	0	85208	673026	758234	82

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Analysis

- The problem becomes rapidly intractable for large networks and large number of paths
  - One of the major limitations of the proposed model results from the large number of variables and constraints, that makes the optimization problem quickly non-tractable by off-the-shelf solvers
- Requires to consider decomposition method: installation (y_{ij}^{p}), maintenance (z_{ij}^{p}) and change (w_{i}^{tp}) variables are kept in the master problem while decisions (x_{ij}^{tp}, f_{ij}^{tp}) are projected out and used only in subproblems
- Next steps
  - Distributed decomposition method
  - Include adaptation of sampling rate
  - Predictive scenarios

Acknowledgments

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