# Genuine and optimized entanglement-based quantum networks

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# **Overview**

Introduction & Motivation

#### Entanglement-based quantum networks

Top-down approach to quantum networks - stack model Optimized networks

#### Genuine quantum networks – superposed tasks

Superposed measurements, sending, addressing & state generation Coherent randomized benchmarking

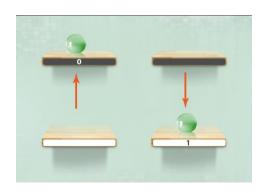
#### Other relevant network protocols for long-range communication

Entanglement-assisted entanglement purification Long-range big quantum data transmission Delocalized information in quantum networks

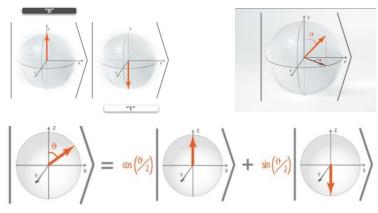
# Introduction and motivation

#### **Entanglement** as a resource

In contrast to classical systems, quantum systems can be in superposition of two states







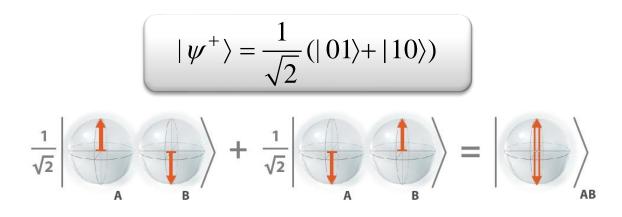
Quantum: superposition of 0 or 1

System of two qubits: superposition of all 4 basis states  $\rightarrow$  entanglement

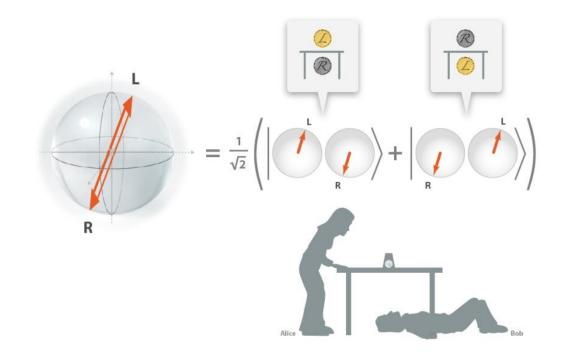
$$\left( |\psi^{+}\rangle = \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle) \right) \qquad \qquad \frac{1}{\sqrt{2}} \left| \frac{1}{\sqrt{2}} |\psi^{+}\rangle + \frac{1}{\sqrt{2}} |\psi^{+}\rangle = \left| \psi^{+}\rangle \right|_{AB}$$

Measurement results are random, but fully (anti)correlated

#### **Entanglement is a resource !**



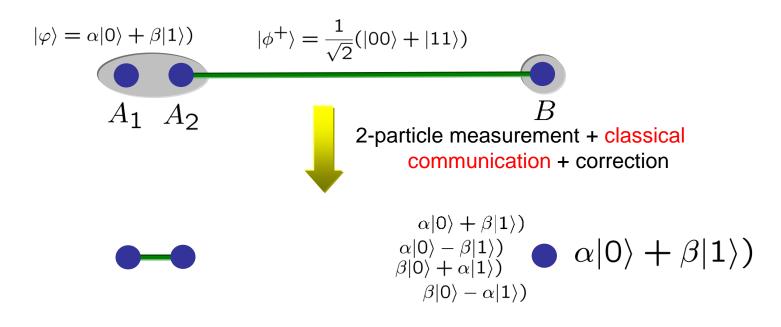
#### **Rotating coin**



### "Spooky action at a distance"

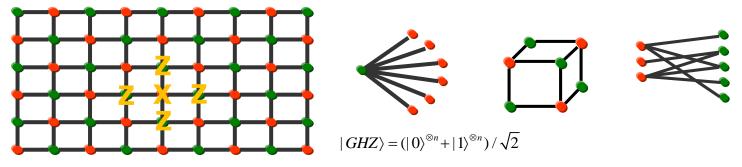


#### Teleportation – transmission of unknown quantum information



## Multipartite states: Graph states and their manipulation

Briegel and Raussendorf, M. Hein et al



Class of multipartite quantum states corresponding to graphs G=(V,E) edges  $\rightarrow$  interaction pattern  $|\psi_G\rangle = \prod_{(i,j)\in E} CZ_{ij} |+\rangle^{\otimes n}$ 

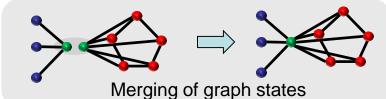
Important because there exist an efficient description (stabilizer formalism) GHZ states, codewords of error correction codes, cluster states for MBQC, ....

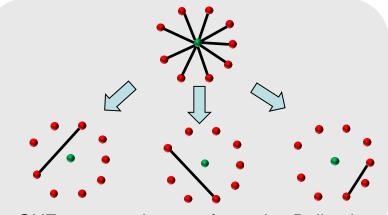
$$K_{j} = \sigma_{x}^{(j)} \otimes_{k \in N_{j}} \sigma_{z}^{(k)} \qquad \qquad K_{j} |\psi_{G}\rangle = (+1) |\psi_{G}\rangle$$

Graph states can be merged (CZ + Pauli measurement) Vertices can be deleted (Z-measurement) Graph can be changed (local complementation)

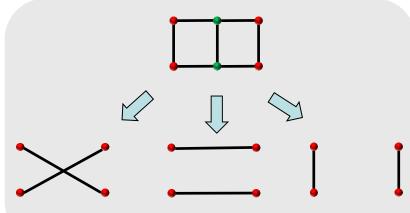
#### Generate other graph states from a given one

### **Manipulation of graph states**

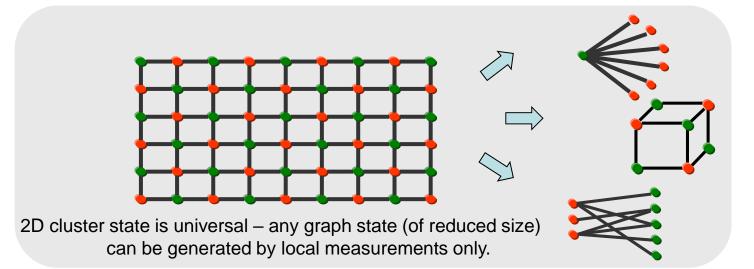




GHZ state can be transformed to Bell pair



2 Bell pairs can be generated from 6-qubit state

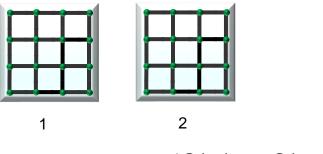


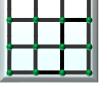
### **Applications of distributed entanglement**

→ Quantum key distribution (cryptography)

→ Generation of (multipartite) entanglement multi-user applications – secret sharing/voting/conference key agreement clock synchronization distributed sensing

→ distributed (cloud) computation & remote quantum computation



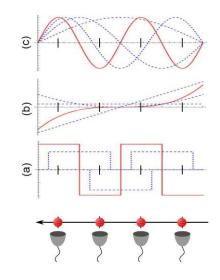


m

m\*O(2<sup>n</sup>) vs. O(2<sup>nm</sup>)

Connected small quantum processors are exponentially more powerful

→ distributed sensing of spatially correlated quantities (e.g. gradient of field)



Entanglementbased quantum networks

> A. Pirker, J. Wallnöfer and W. Dür, New J. Phys. 20, 053054 (2018) A. Pirker and W. Dür, New J. Phys. 21, 033003 (2019)

# **Quantum networks**

Quantum devices connected by channels goal: transmit quantum information, prepare entangled states

Usual approach: classical control plane that controls quantum devices Design protocols to distribute & utilize entanglement

Routing of entanglement Long-distance quantum communication

 $\rightarrow$  quantum repeaters to overcome channel noise and imperfections



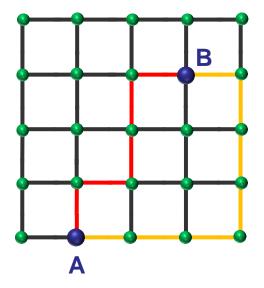
A quantum network is more than a variant of a classical network that transmits quantum information !

# **Bottom-up** approach

Generate entanglement between end-nodes on demand → Routing of entanglement

Similar as in classical networks – recieve request, use given resources (channels, intermediate nodes) to connect end users

Direct transmission of quantum information vs. preparation of entangled state



Routing of entanglement is probabilistic (loss, entanglement purification to overcome noise) Large overhead in resources (time, memory)

Challenge: How to deal with noise and imperfections in channels and local control operations

### **Top-down approach: entanglement-based networks**

A. Pirker, J. Wallnöfer and W. Dür, New J. Phys. 20, 053054 (2018); A. Pirker and W. Dür, New J. Phys. 21, 033003 (2019);

#### **Dynamic phase**

Generate desired (multipartite) entangled resource states Same as in bottom-up approach but: several (known) target states → optimize strategy No time pressure – resource state generation can be done beforehand

#### **Static phase**

Store and maintain entangled states, manage log in/off & failure of devices Requires long-term quantum memory, protocols to manage

#### **Adaptive phase**

Locally manipulate resource states to generate desired target state Requires protocols to manage

Network states can be generated beforehand – i.e. before the request!

### **Top-down approach: entanglement-based networks**

A. Pirker, J. Wallnöfer and W. Dür, New J. Phys. 20, 053054 (2018); A. Pirker and W. Dür, New J. Phys. 21, 033003 (2019)

Network topology is independent of topology of underlying physical network  $\rightarrow$  optimize networks  $\leftarrow$ 

Requests can be fullfilled without delay – all required entanglement is available

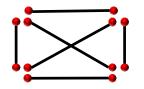
Which resource states should be stored? Bell pairs shared between all pairs of parties suffice, but multipartite states allow for storage advantage

#### **Resource states for entanglement-based networks**

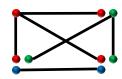
Identify resource states

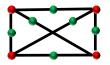
at level of single network (link layer) and between networks (network layer) that allow to perform all desired task: simultanous pairwise communication between arbitrary pairs, generate arbitrary graph state, etc.

 $\rightarrow$  solutions: O(n<sup>2</sup>) Bell states; GHZ states of size n,n-1,...2; fully decorated graph state



Bell pairs shared between all pairs of parties





GHZ-states of n, n-1,  $\dots$  , 2 qubits

Decorated, fully connected graph state

can be used to generate arbitrary graph state

#### Quantum network stack model for entanglement-based networks

A. Pirker and W. Dür, New J. Phys. 21, 033003 (2019)

Introduce independent layers to abstract problem and allow for standards – advance different layers independently

Network stack	Network device		Protocol	Goal/Responsibility	Auxiliary
Network layer	×	Router	Region Routing Hierarchical Regions Reliable Regions	Enable for inter-network graph state requests	otocols
Link layer	<b>t</b> tt	Switch	State linking Reliable state linking Quantum network configuration	Generate arbitrary graph states on request in a network	t distillation pr ability (Ping)
Connectivity layer	<b>→</b>	Repeater	Encoded direct communication Repeater protocols	Ensure point-to-point or point-to- multipoint connectivity	ement dist Reachabili
Physical layer		Channel		Physically connect quantum networking devices	Entanglement ( Reacha

*Physical layer:* direct transmission of quantum information (e.g. via transmission of photons through optical fibers or free space). Deals with quantum channels & interfaces between memory qubits and transmission qubits

*Connectivity layer:* responsible to establish entangled states between nodes (handle noise and imperfections); use entanglement purification, quantum repeaters and encoded transmission

*Link layer:* single quantum network that consists of multiple nodes and devices. Coordinates the generation of network state to fulfill arbitrary tasks and requests (dynamic phase).

*Network layer:* connect multiple networks to enable and coordinate inter-network requests. Uses (virtual) inter-network state is to connect network via routers; hierarchical grouping is possible

#### Quantum network stack model for entanglement-based networks

Network devices

Quantum hub, switch, router, ... - implement using entangled resource states

Resource states

Which resource states can be used? Optimize?

Design protocols

to generate & transform network state  $\rightarrow$  desired target state. State linking protocol, region routing, ...

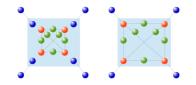
Methods and protocols to ensure reliability

Decorated states to cope with unexpected node failure, symmetrization Quantum ping – ensure that entanglement is still there with sufficiently high fidelity

#### **NETWORK DEVICES**

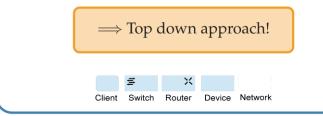
Quantum clients: Solely connect to quantum network devices and share entanglement with them. Available operations are restricted to single qubits.

Quantum switches: Connect clients and other devices via a device state and a network state, and use *connect* and *merge* for requests of clients.

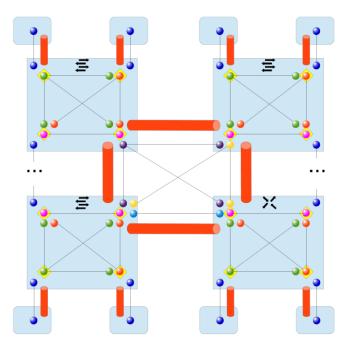


Quantum routers: Same functionality as switches, but connect in addition also quantum networks.

Quantum networks: Connects devices via a network state which is available prior to requests  $\implies$  state is such that arbitrary graph states may be generated.



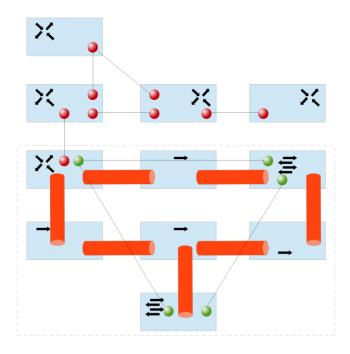
# Entanglement-based network using GHZ states



3 switches and a router connect in a network via GHZ states of decreasing size (black lines indicate entanglement). Internally each of these networking devices use again GHZ states of decreasing size to connect three clients each via Bell-pairs.

Observe that the entanglement structure of the network is different from the physical channel conguration (orange tubes).

#### Link layer and network layer



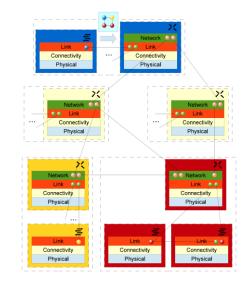
#### Setting of layer 3 and 4:

The network devices of layer 3 request and combine the entangled states from layer 2 to create the network state (green nodes).

On layer 4, quantum routers connect quantum networks via multipartite entangled quantum states (red nodes).

States are consumed when fulfilling client requests.

#### Working of quantum network stack



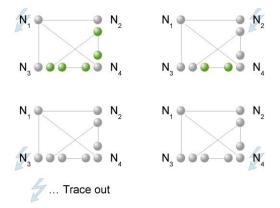


A client of the switch in the blue network requests to share a four qubit cluster state with a client located at a switch in the yellow, and two clients located at two different switches in the red network.

Routers generate a virtual network state shared between different networks, which is then manipulated and combined with internal network states to form the desired target state.

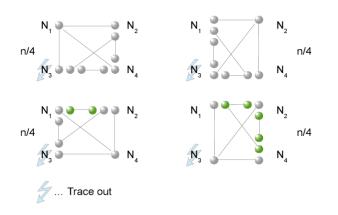
#### **Reliability**

GHZ states are fragile under loss/node disconnect

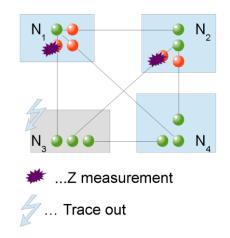


Methods to ensure functionality upon node failures or disconnecting devices:

#### Symmetrice resource states



#### Add shield qubits



# Optimized quantum networks

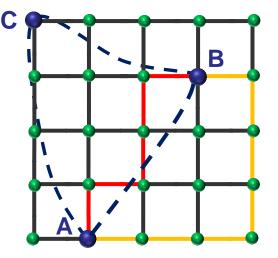
Jorge Miguel-Ramiro, Alexander Pirker and Wolfgang Dür, E-print: arXiv:2107.10275

# **Optimized quantum networks**

Geometry of entanglement-based network is determined by shared multipartite resource states - can be different than topology of physical network/channels !

# Can use this to optimize geometry of entanglement-based network to minimize storage requirements

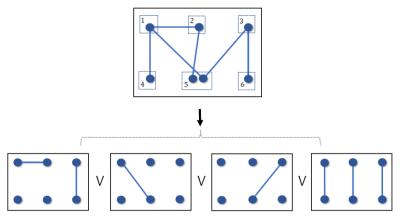
Physical network: nodes and channels are usually given and cannot be designed at will



Entanglement can be shared between any set of parties/nodes — independent of their location/distance!

# **Optimized quantum networks**

Functionality of network defined by set of requests (i.e. which states should be possible to generate from the resource state)

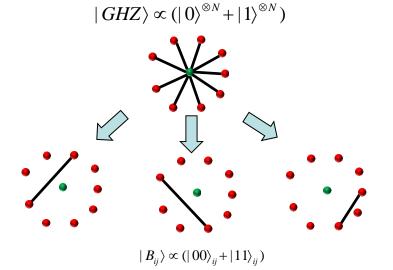


Requests: all possible graph states (includes multiple bipartite links) {p<sub>j</sub>,  $|\psi_{Gj}\rangle$ }

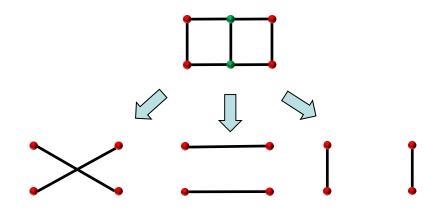
We are interested in resource state of minimal size that is capable to fulfill all requests

Topology of entanglement-based network is adjusted to desired functionality

# **Reduce size of resource state**

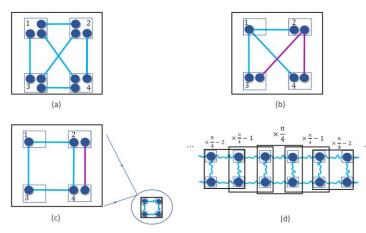


Can use single GHZ state (store 1 qubit per node) to generate Bell-pair between any pair of parties



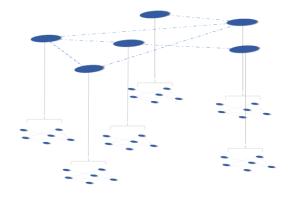
Can use 6 qubit state to generate two Bell-pairs between among 4 parties

(quantum network coding - Eisert/Pappa, Van Meter)



Full connectivity (i.e. all possible pairwise interactions in parallel) can be achieved via: (a) Bell pairs (12); (b) GHZ states of reduced size (7); (c) Butterfly state/network coding state (6)

#### Step 1: Clustering algorithm to identify connected nodes/cluster

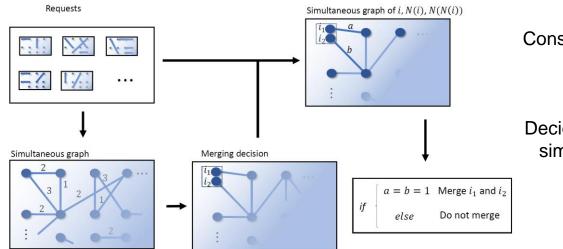


Use graph laplacian + cumulative matrix (# times link appears) to separate resource network state in different layers

Each of the identified clusters can be considered as single node for higher layer – hierarchically applied

→ Gives topology of entanglement-based network – independent of physical connections

#### **Step 2: Merging algorithm to reduce size of resource state**



Consider all Bell pairs that appear in one target configuration

Chose vertex Decide two Bell pairs can be merged based on simultaneous matrix (# parallel requests for given link) of neighborhood

→ Reduces storage requirements

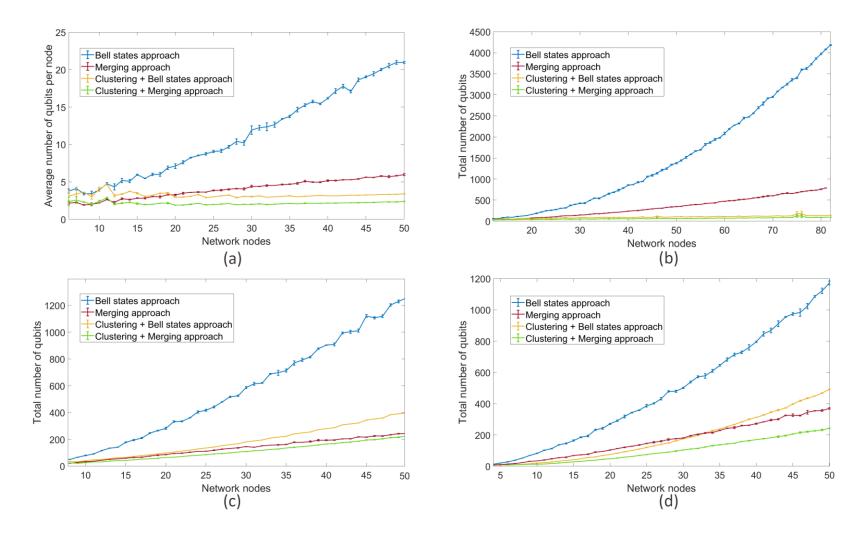


Fig. 6. Performance of the clustering and the clustering+merging (in each cluster and layer) algorithm, under different conditions and compared with directly providing the necessary Bell pairs (blue) or only applying the merging algorithm (red). Requests are taken randomly but with modified probability to cause heterogeneity in the connections. (a) Average number of qubits per node for m = n random different requests and higher probability for connections among groups of 4 nodes. (b) Several rounds of clustering with m = 2n requests and higher probability for connections among groups of 4 nodes. (c) Total network storage required for m = 75 requests and higher probability for connections among groups of 4 nodes. (d) Total

# Genuine quantum networks

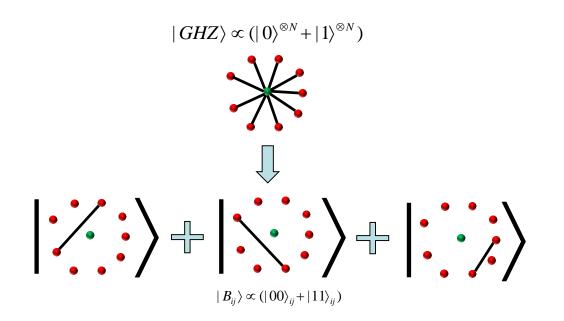
Jorge Miguel-Ramiro, Alexander Pirker and Wolfgang Dür, npj Quantum Information 7, 135 (2021)

# **Genuine quantum networks**

Perform different tasks (state generation, state manipulation, sending, addressing, ...) in coherent superposition

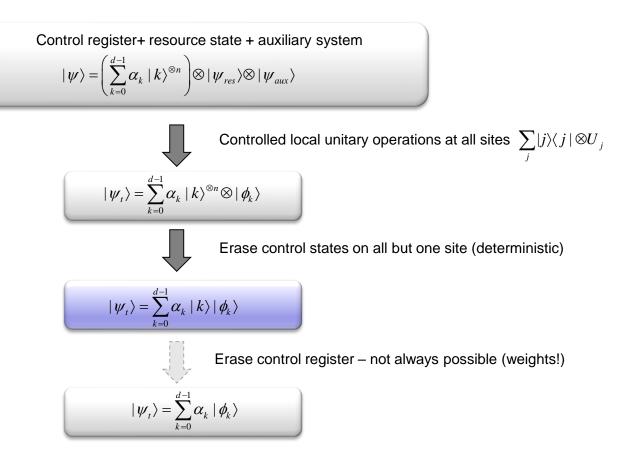
Lifts quantum network to genuine quantum level - new possibilities

Requires quantum control plane (via shared entanglement)



## **Superposed state generation**

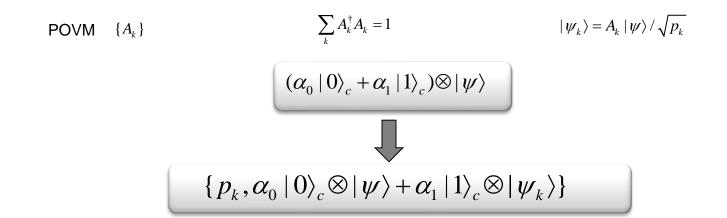
Quantum control register:



What happens if controlled measurements/controlled classical tasks are involved ?

# **Superposed measurements**

Consider coherent superposition to perform a measurement - or not on initial state  $|\psi\rangle$ 



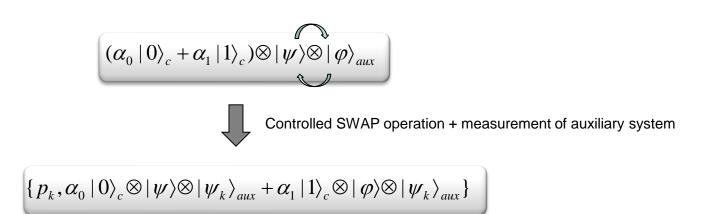
Strong indications that this is impossible:

- process is non-linear in coefficients (QM is linear) Specifying action on pure states leads to non-unique prediction for mixed states

Remark: If one includes measurement outcome (as register), then one actually has an incoherent mixture!

# **Superposed measurements**

However, one can mimick the action of a controlled measurement on a *known* state by always performing the measurement – either on target state, or an auxiliary state. Perform controlled-SWAP + measurement on auxilairy state

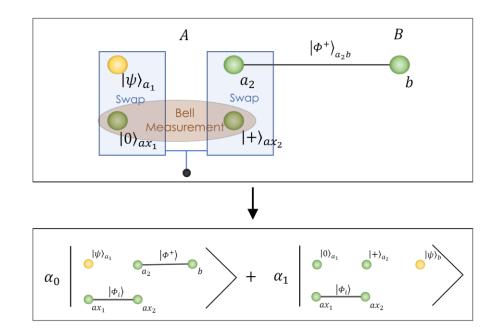


probability to obtain certain outcome need to match for states  $|\psi\rangle$  and  $|\varphi\rangle_{aux}$ 

This is useful for our porpose – as we are mainly interested in measurements on parts of entangled states (with equal spectra and can chose appropriate auxiliary state)

# **Controlled sending (teleportation)**

Jorge Miguel-Ramiro, Alexander Pirker and Wolfgang Dür, E-print: arXiv:2005.00020

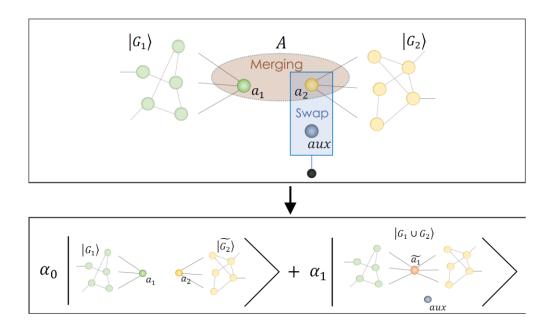


Superposition of teleporting an unknown state - or not

Use controlled-SWAP with two auxiliary qubits – this allows one to preserve entangled state if no teleportation is performed

# **Controlled merging of graph states**

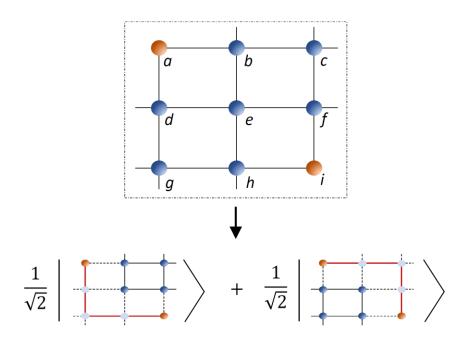
Jorge Miguel-Ramiro, Alexander Pirker and Wolfgang Dür, E-print: arXiv:2005.00020



Superposition of merging two graph states - or not

Similar method works to manipulate graph states by local measurements in a superposed way

### **Controlled transmission over different paths**



#### Superposition of sending q-information among path 1 or path 2 (or through channel 1 or channel 2)

G. Rubino, L. A. Rozema, D. Ebler, H. Kristjánsson, S. Salek, P. Allard Guérin, A.A. Abbott, C. Branciard, Č. Brukner, G. Chiribella, P. Walther, Phys. Rev. Research 3, 013093 (2021); Experimental Quantum Communication Enhancement by Superposing Trajectories

Caleffi and Cacciapuoti; Kristjánsson and Chiribella, Rubino, Brukner et al etc. (quantum switch)

#### Possible advantages of superposed states & tasks

Superposition of different target states:

$$|\psi_{t}\rangle = \sum_{k=0}^{d-1} \alpha_{k} |k\rangle |\phi_{k}\rangle$$

$$|\Psi\rangle = \frac{1}{2} \sum_{i=0}^{3} |2\rangle_i |\text{GHZ}\rangle_{N/i}$$

$$\left|\psi\right\rangle = \frac{1}{\sqrt{2}}\left|0\right\rangle_{c}\left|\mathrm{GHZ}\right\rangle_{1234} + \frac{1}{\sqrt{2}}\left|1\right\rangle_{c}\left|C_{1D}\right\rangle_{1234}\right\rangle$$

Can access different kinds of entanglement from superposition (different entanglement features of each possible target state – coherent superposition state – incoherent mixture)

Only superposed state is robust against loss of 2 qubits

#### Superposition of paths/channels:

Sending q-info in a superposed way through two noise channels leads to improved transmission as compared to any of the channels May also provide additional robustness against loss

Kristjánsson and Chiribella, Rubino, Brukner et al, Caleffi and Cacciapuoti etc. (quantum switch)

Superposition of different destinations, error correction codes, cryptography protocols,



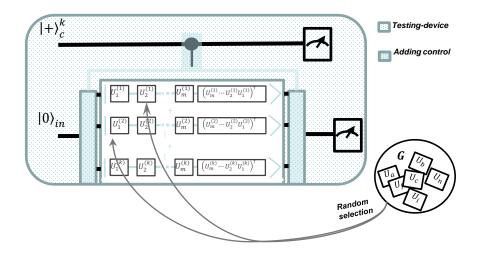
Jorge Miguel-Ramiro, Alexander Pirker and Wolfgang Dür, Phys. Rev. Research 3, 033038 (2021)

### **Standard randomized benchmarking**

Apply sequence of different length of randomly selected gates to determine average gate fidelity – sample from different realizations

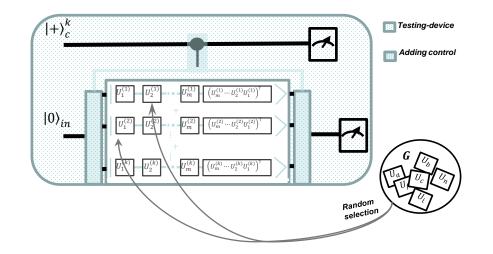
Works for specific groups (e.g. Clifford) – but limited applicability, scaling problem

### **Coherent randomized benchmarking**



Apply coherent superposition of different gate sequences

### **Coherent randomized benchmarking**



Coherence  $\rightarrow$  extra accessible information  $\rightarrow$  larger flexibility, improved efficiency

Can apply method to many more gate sets and individual gates ! Favourable scaling with system size

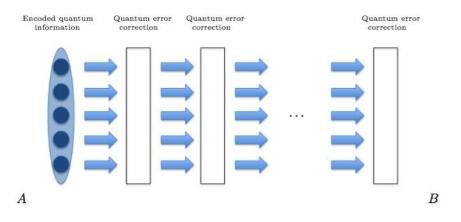
Can add control by external device – and still use same gates to sample from! Pre-calibrated external device that uses other degrees of freedom – "factory testing"

## Other protocols for long-distance communication

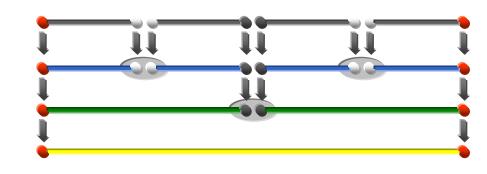
# Long-rang big quantum data transmission

M. Zwerger, A. Pirker, V. Dunjko, H.J. Briegel and W. Dür, Phys. Rev. Lett. 120, 030503 (2018); J. Wallnöfer, A. Pirker, M. Zwerger and W. Dür, Scientific Reports 9, 314 (2019); Multipartite state generation in quantum networks with optimal scaling

### Previous repeater protocols for long-distance communication



- I. Based on quantum error correction
- Tolerable noise: channel:  $O(10^{-2})$ gates:  $O(10^{-5})$ 
  - E. Knill, and R. Laflamme, arXiv: quant-ph/9608012 (1996);
    - S. Muralidharan, J. Kim, N. Lütkenhaus, M. D. Lukin, and L. Jiang, Phys. Rev. Lett. 112, 250501 (2014).



- II. Based on iteration of purification & swapping
- Tolerable noise: channel: O(10<sup>-1</sup>)

gates:  $\mathcal{O}(10^{-2})$ 

H.J. Briegel, W. Dür, J.I. Cirac, and P. Zoller, PRL 81, 5932 (1998)

#### III. Based on combination of error correction and entanglement purification

L. Jiang, J. M. Taylor, K. Nemoto, W. J. Munro, R. Van Meter, and M. D. Lukin, Phys. Rev. A 79, 032325, (2009) Encoded Bell pairs that are connected in parallel

#### **Measurement-based quantum communication:**

#### alternative approach that leads to much better error thresholds (>10% per particle)

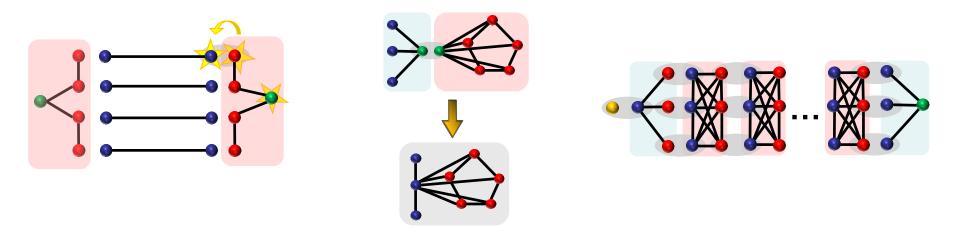
M. Zwerger, H. J. Briegel and W. Dür, Phys. Rev. Lett. 110, 260503 (2013). Universal and optimal error thresholds for measurement-based entanglement purification" M. Zwerger, H. J. Briegel and W. Dür, Applied Physics B, 122:50, (2016), `` Measurement-based Quantum Communication"

#### Measurement-based implementation:

"teleportation" through resource graph states – can implement all Clifford circuits & Pauli measurements in a single step deterministically, only input & output particles

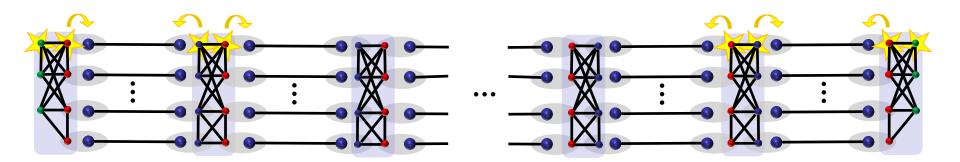
Entanglement purification and error correction (encoding/decoding/syndrome) are of this type

ightarrow high error threshold and efficient implementation  $\leftarrow$ 



### Long-rang big quantum data transmission

M. Zwerger, A. Pirker, V. Dunjko, H.J. Briegel and W. Dür, Phys. Rev. Lett. 120, 030503 (2018); J. Wallnöfer, A. Pirker, M. Zwerger and W. Dür, Scientific Reports 9, 314 (2019); Multipartite state generation in quantum networks with optimal scaling



idea: replace the nested entanglement purification and swapping by non-nested scheme (hashing) and subsequent swapping

Bell pairs with near unit fidelity are created in all segments via hashing and then swapped - this can be done in a single step

Ultrafast, constant overhead, large error thresholds

### **Comparison of repeater protocols**

scheme	I. Knill&Laflamme (1996)	II. Briegel, Dür, Cirac, Zoller (1998)	III. Jiang, Taylor, Nemoto, Munro, Van Meter & Lukin (2009)	IV. Zwerger, Pirker, Dunjko, Briegel & Dür (2017)
based on	QEC	Bell pairs & two- way EDP	Bell pairs & QEC	Bell pairs & hashing
Scaling of resources	O(polylog(L))	O(poly(L))	O(polylog(L))	constant
Distribution time dominated by	transmission time processing time	waiting times	time to create elementary pairs	time to create elementary pairs
constraint on loss	YES (50%)	no	no	no

Rates for continental & intercontinental distances (10<sup>3</sup>-10<sup>4</sup> segments) Our scheme IV: overhead s=2 Scheme II: rate per storage qubit about 10<sup>6</sup>-10<sup>9</sup> smaller Scheme III: rate per storage qubit about 10<sup>2 -</sup> 10<sup>3</sup> smaller



Ferran Riera-Sabat, Pavel Sekatski, Alexander Pirker and Wolfgang Dür, Phys. Rev. Lett. 127, 040502 (2021) Ferran Riera-Sabat, Pavel Sekatski, Alexander Pirker and Wolfgang Dür, Phys. Rev. A 104, 012419 (2021)

#### **Entanglement-assisted entanglement purification**

Problem from connectivity layer – produce few highly entangled qubit pairs from many copies

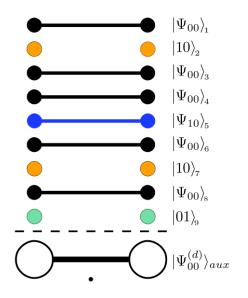
#### Noisy ensemble:

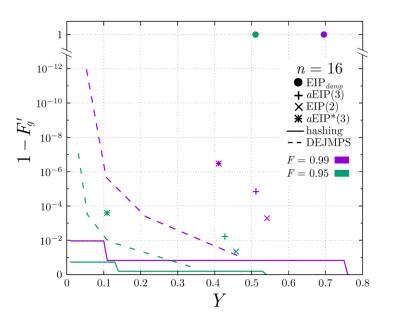
Mixture of desired entangled states and (undesirable) error states unknown number of errors at unknown position

#### Main idea:

Design protocol to learn first number of errors, then positions in controlled way. This information is not locally accessible

 $\rightarrow$  use auxiliary entanglement of high dimension that is returned at end of process





Delocalized information in quantum networks

Jorge Miguel-Ramiro and Wolfgang Dür, New J. Phys. 22, 043011 (2020)

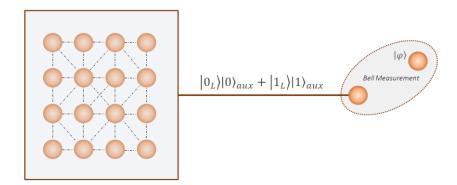
### **Delocalized information in quantum networks**

Store information in encoded/distributed form in the network – qubits of logical state are distributed among different parties

Natural (passive!) protection against failure of network nodes, loss and decoherence during storage and transport.

Built-in security features, such as limited accessible information per network node.

Encoding/Decoding and processing of information using only local operations or limited entanglement



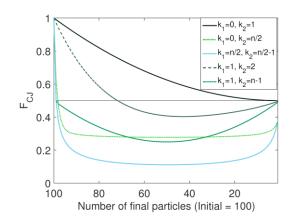
#### **Delocalized information in quantum networks**

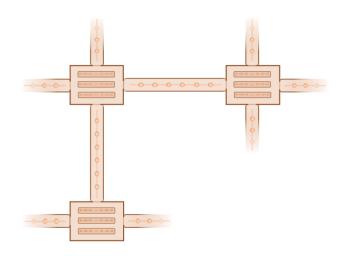
→ Dicke state encoding
→ Correlation space encodings

Storage networks with prototion against loss and node failures, limited entanglement per node

Encoding, transport, decoding (localization of information) – may require extra entanglement

Can built fully functional quantum networks where QI is stored in some region, and can be transmitted & processed solely by local operations





### **Summary**

#### Entanglement-based networks

Design/optimize networks largely independent of underlying physical structure. One can pre-prepare and store required resources, which allows for faster access times and overcome limitations of bottom-up networks → new/shifted challenges to build quantum network ←

Also hybrid approach is possible: add some pre-shared entanglement

Genuine quantum networks:

Lift quantum networks to true quantum level by allowing for coherently controlled operations and task. Classical control plane is replaced by quantum control plane. Superposition of different taks lead to new, largely unexplored possibilities.

#### What (else) can we do with this additional power?